Question 1: Let C be a PRC class and g1, g2, g3, g4 belong to C. Show if h1(x, y ,z) = g1(z, x, y), h2(y) = g2(y, y, y), and h3(w, x, y, z) = h1(g4(3, g3(y, z)), g3(w, y), z), then h1, h2, and h3 also belong to C.

Solution:

To show h_1 , $h_2\, and\, h_3\, belongs\ to\ C$,

1. $h_1(x, y, z) = g_1(U_3^3(x, y, x), U_1^3(x, y, x), U_2^3(x, y, x))$

2. $h_2(y) = g_1(U_1^1(y, y, y), U_1^1(y, y, y), U_1^1(y, y, y))$

3. $h_3(w, x, y, z) = h_1\left(g_4(3, g_3(y, z)), g_3(w, y), z\right)$

Question 2: Let Q(x) = 1 if x is a multiple of 5, and Q(x) = 0, otherwise. Show that Q(x) is primitive recursive.

Solution:

$$Q(x) = \begin{cases} 1 & \text{if } 5 \mid x \\ 0 & \text{otherwise} \end{cases}$$

$$Q(x) \iff (\exists\, t_{\leq x})\,(t \cdot 5 = x)$$

Question 3: Let R(x1, x2) = 1 if x1 , x2 are prime and if x1 + x2 is a multiple of 8, R(x1, x2) = 0 otherwise. Show that R(x1, x2) is primitive recursive.

Solution:

$$R(x_1, x_2) = \begin{cases} 1 & \text{if } x_1, x_2 \text{ are prime} \land 8 \mid (x_1 + x_2) \\ 0 & \ \text{otherwise} \end{cases}$$

$$\text{Prime}(\mathrm{x}_1, x_2) \iff (x_1, x_2) \text{ and } \forall t_{\leq x1*x2}(t = 1 \lor t|x_1 \lor t|x_2 \lor \neg(t|x_1) \lor \neg(t|x_2) \land ((x_1 + x_2) * t = 8))$$

Question 4: Let S(x1, x2) = 1 if the LCM of x1 and x2 is a multiple of 3 and the GCD of x1 and x2 is not a multiple of 3, S(x1, x2) = 0, otherwise. Show that S(x1, x2) is primitive recursive. (LCM: least common multiple; GCD: greatest common divisor.)

Solution:

$$S(x_1, x_2) = \begin{cases} 1 & \text{if } 3|LCM(x_1, x_2) \text{ and } \neg 3|GCD(x_1, x_2) \\ 0 & \text{otherwise} \end{cases}$$

$$S(x_1, x_2) \iff \exists t_{\leq \mathrm{LCM}(}x_1, x_2)(3 * t = LCM(x_1, x_2) \land \neg(3 * t = GCD(x_1, x_2))$$