Macros:

1. inequality ($X_1 < X_2$):

$Z_1 \leftarrow X_1$
$Z_2 \leftarrow X_2$
$Z_3 \leftarrow Y$
$[D]$ $IF Z_1 \neq 0\ GOTO\ A$
$IF Z_2 \neq 0\ GOTO\ B$
$GOTO\ E$
$[A]$ $IF Z_2 \neq 0\ GOTO\ C$
$GOTO\ E$
$[C]$ $Z_1 \leftarrow Z_1 - 1$
$Z_2 \leftarrow Z_2 - 1$
$IF Z_1 \neq 0\ GOTO\ D$
$IF Z_2 \neq 0\ GOTO\ C_1$
$GOTO\ D$
$[C_1]$ $Y \leftarrow Y + 1$
$Z_3 \leftarrow Y$
$GOTO\ E$

Question 1: Write a program in S that computes the function $f_1(x) = 2(x–3)$. If you need any macros, please define the macro expansions. Note that $f_1(x)$ is not defined if $x < 3$.

Solution:

$Z_{11} \leftarrow X$
$Z_{12} \leftarrow 3$
$Z_{13} \leftarrow Z_1 < Z_2$
$IF Z_{13} \neq 0\ GOTO\ B$
$GOTO\ A$
$[B]$ $Z_{14} \leftarrow Z_{14} + 1$
$GOTO\ B$
$[A]$ $Z_{11} \leftarrow X - 3$
$Z_{14} \leftarrow Y + Z_{11}$
$Z_{15} \leftarrow Z_{14}$
$Z_{16} \leftarrow Z_{15} + Z_{15}$
$Y \leftarrow Z_{16}$
$GOTO\ E$

Question 2: Let $f_2(x)$ be the smallest number n such that $x < n^2$. Write a program in S that computes f_2. If you use any macro, you have to provide the corresponding macro expansion.

Solution:

$[A]$ $Z_6 \leftarrow Z_5$
$Z_7 \leftarrow Z_5$
$Z_8 \leftarrow Z_6.Z_7$
$Z_9 \leftarrow X < Z_8$
$IF Z_9 \neq 0\ GOTO\ E$
$GOTO \quad B$

$[B]$ $Z_5 \leftarrow Z_5 + 1$
$Y \leftarrow y + 1$
$GOTO \quad A$

Question 3: Write out a computation of P beginning with the snapshot (1, σ), where σ consists of equations X = 4, Y= 0, Z= 0.

$[A]$ IF$X \neq 0\ GOTO\ B$ (1)
$Y \leftarrow Y - 1$ (2)
$Z \leftarrow Z + 1$ (3)
$IF Z \neq 0\ GOTO\ E$ (4)

$[B]$ $X \leftarrow X - 1$ (5)
$X \leftarrow X - 1$ (6)
$Y \leftarrow Y + 1$ (7)
$Y \leftarrow Y + 1$ (8)
$Z \leftarrow Z + 1$ (9)
$IF Z \neq 0\ GOTO\ A$ (10)

Solution:

(1, $\{X = 4, Y = 0, Z = 0\}$)
(5, $\{X = 3, Y = 0, Z = 0\}$)
(6, $\{X = 2, Y = 0, Z = 0\}$)
(7, $\{X = 2, Y = 1, Z = 0\}$)
(8, $\{X = 2, Y = 2, Z = 0\}$)
(9, $\{X = 2, Y = 2, Z = 1\}$)
(10, $\{X = 2, Y = 2, Z = 1\}$)
(1, $\{X = 2, Y = 2, Z = 1\}$)
(5, $\{X = 1, Y = 2, Z = 1\}$)
(6, $\{X = 0, Y = 2, Z = 1\}$)
(7, $\{X = 4, Y = 3, Z = 1\}$)
(8, $\{X = 0, Y = 4, Z = 1\}$)
(9, $\{X = 0, Y = 4, Z = 2\}$)
(10, $\{X = 0, Y = 4, Z = 2\}$)
(1, $\{X = 0, Y = 4, Z = 2\}$)
(2, $\{X = 0, Y = 3, Z = 2\}$)
(3, $\{X = 0, Y = 3, Z = 3\}$)

Question 4: Write a program P such that for any $n > 0$ and every computation $s_1 = (1,\sigma)$, $s_1, s_2, ..., s_k$ of P that has the equation X = n in σ, k = $an^2 + bn + c$, where $a, b, c > 0$.

Solution:

$Z_{11} \leftarrow a$
$Z_{12} \leftarrow b$
$Z_{13} \leftarrow c$
$Z_{14} \leftarrow n$
$IF Z_{14} \neq 0\ GOTO\ D$
$Y \leftarrow Z_{13}$
$GOTO \qquad E$
$[D] \quad Z_{15} \leftarrow Z_{14}$
$Z_{16} \leftarrow Z_{14}$
$Z_{17} \leftarrow Z_{15}.Z_{16}$
$Z_{18} \leftarrow Z_{11}.Z_{17}$
$Z_{19} \leftarrow Z_{12}.Z_{14}$
$Z_{20} \leftarrow Z_{18} + Z_{19} + Z_{13}$
$Y \leftarrow Z_{20}$