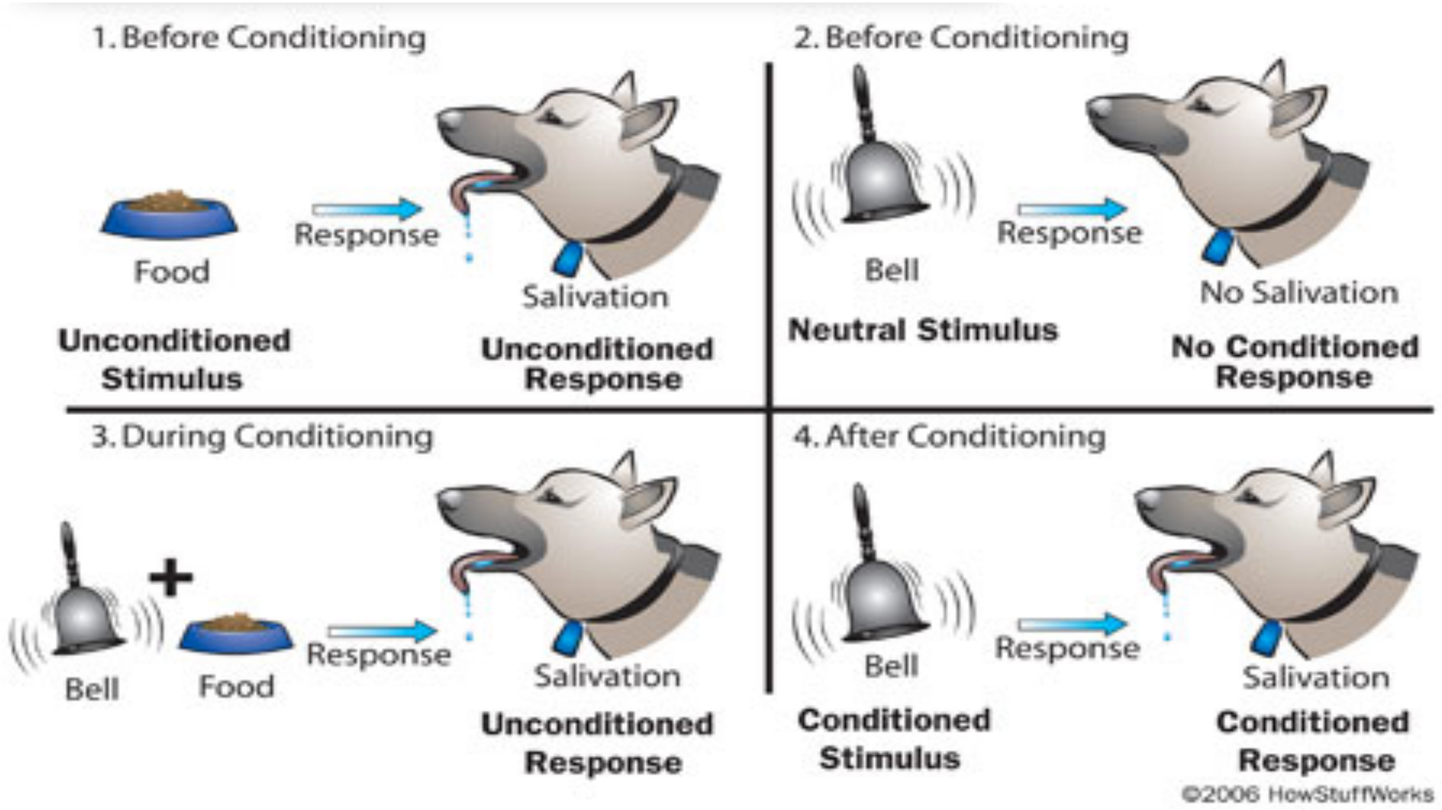
BP神经网络

张敏

18/1/2

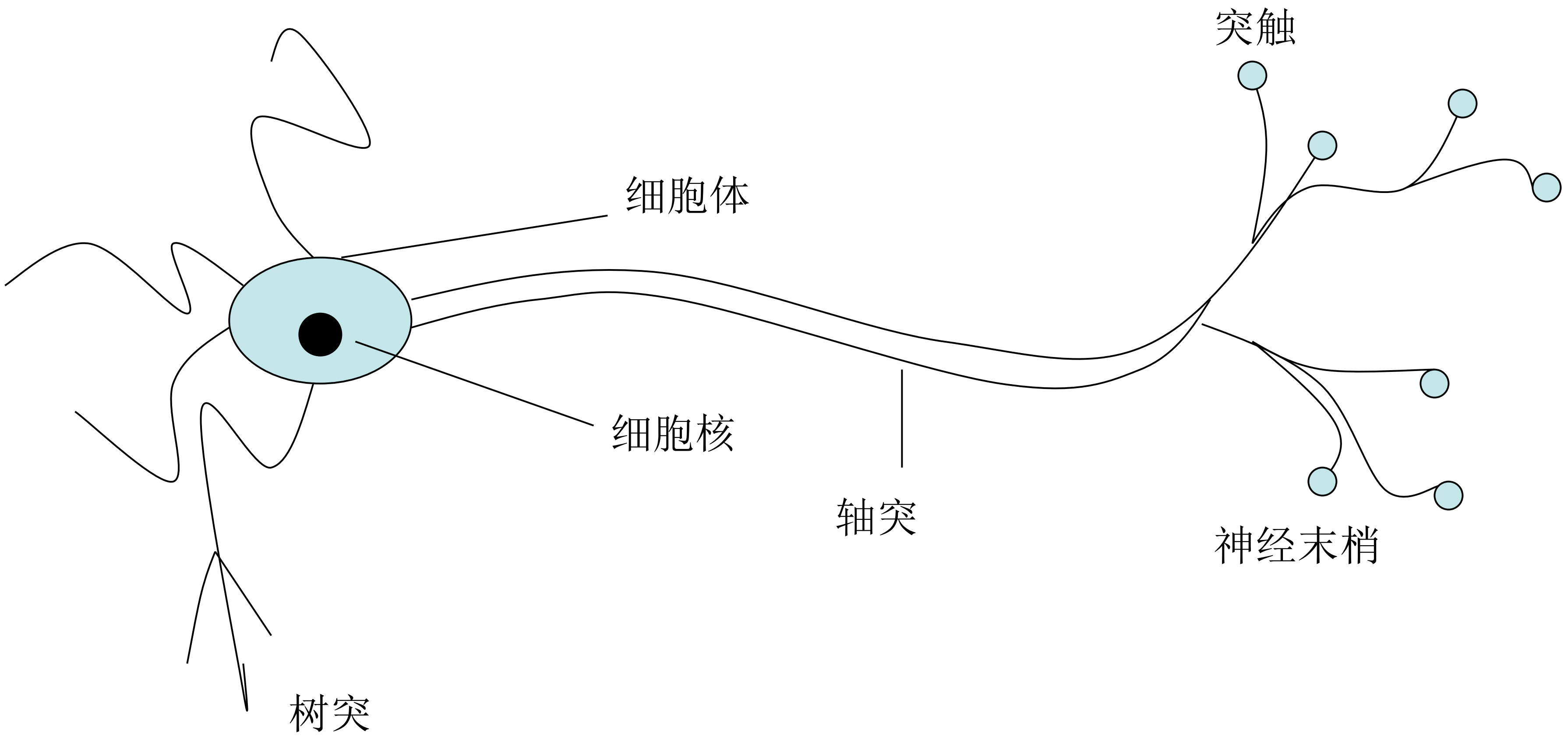
神经网络

巴普洛夫关于神经反射的实验



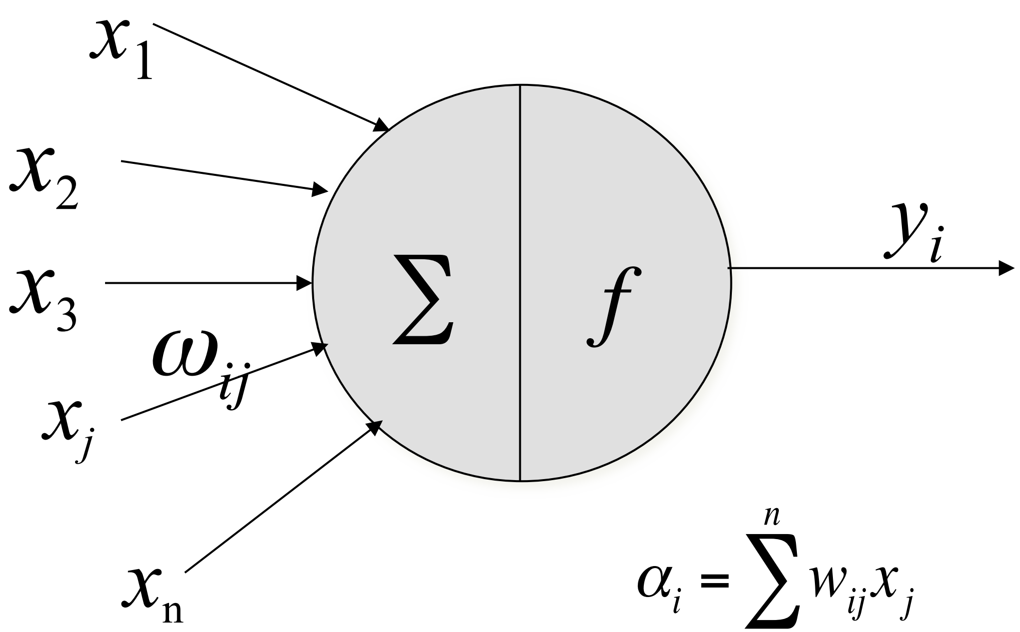
神经网络

生物神经元结构



神经网络

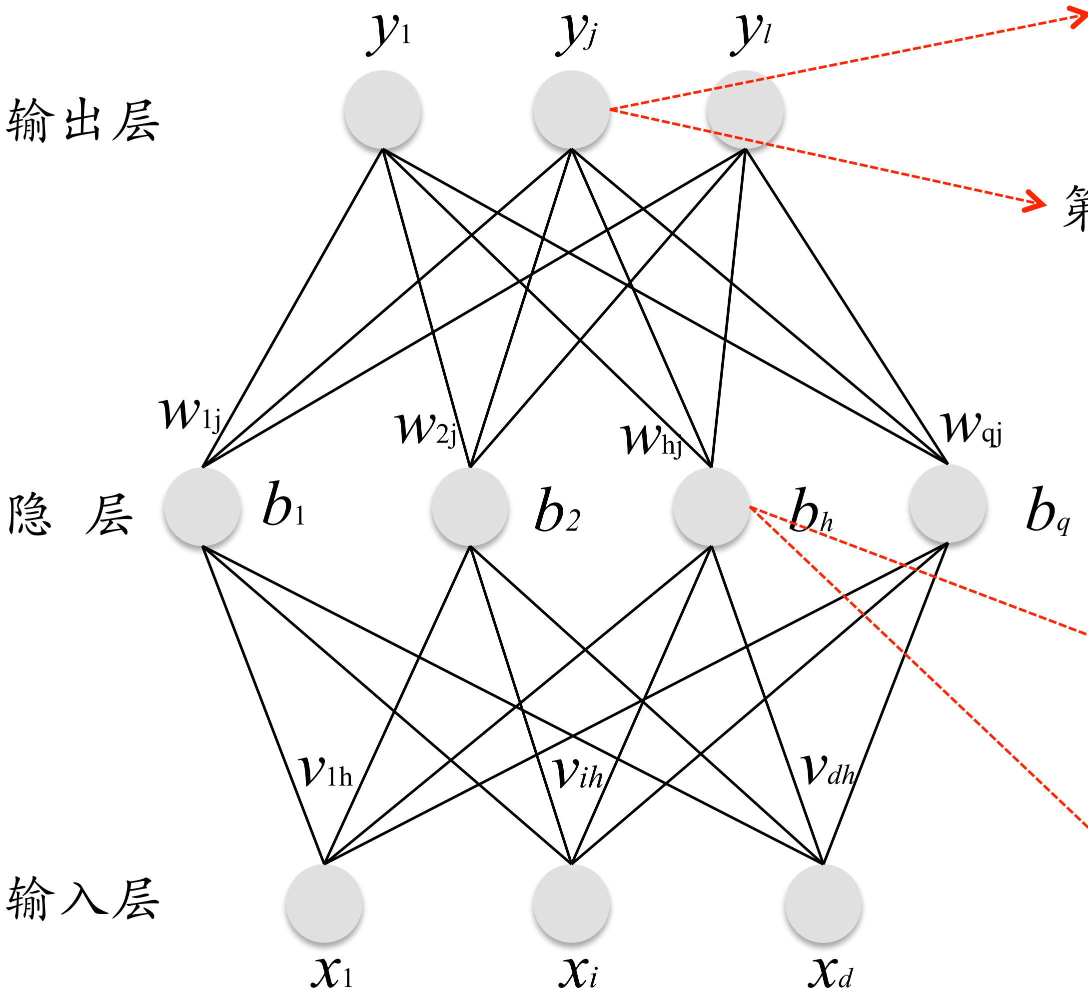
数学神经元结构



$$\alpha_i = \sum_{j=1}^{n} w_{ij} x_j \qquad y_i = f\left(\sum_{j=1}^{n} w_{ij} x_j - \theta\right)$$

x_j为输入信号， f为传递函数，$w_{i,j}$表示与神经元x_j 连接的权值，y_i表示输出值，θ表示阈值

BP网络结构



第j个输出层神经元的输出：

$$\hat{y}_j = f(\beta_j - \theta_j)$$

第j个输出层神经元的输入：

$$\beta_j = \sum_{h=1}^{q} w_{hj} b_h$$

第h个隐层神经元的输出：

$$b_h = f(\alpha_h - \gamma_h)$$

第h个隐层神经元的输入：

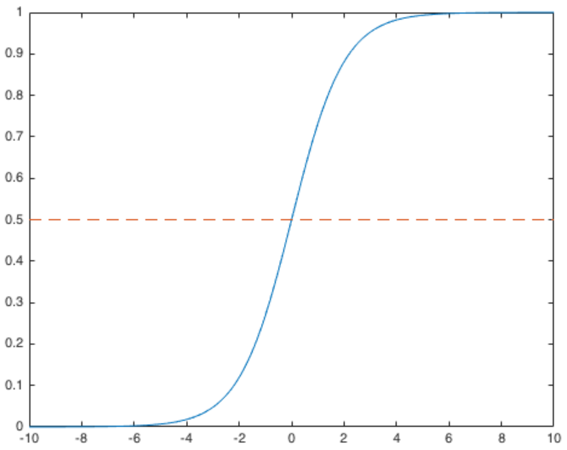
$$\alpha_h = \sum_{i=1}^{d} v_{ih} x_i$$

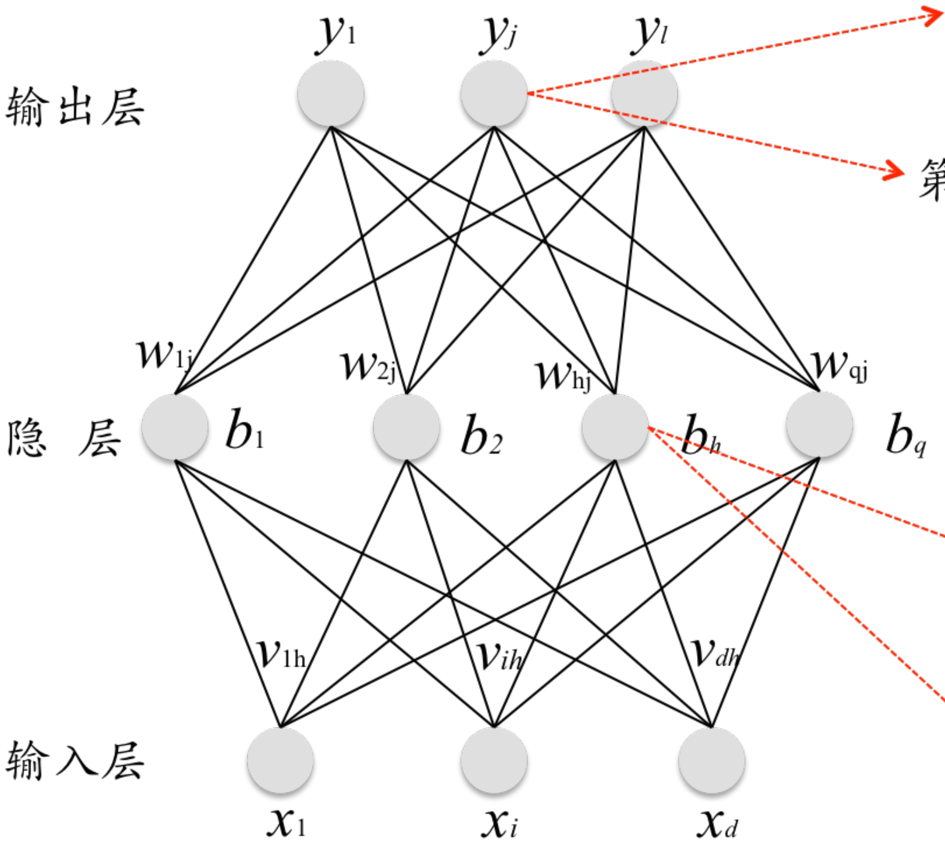
神经网络

BP网络结构

$$E = \frac{1}{2}\sum_{j=1}^{l}(\hat{y}_j - y_j)^2$$

$$f(x) = sigmoid(x) = \frac{1}{1+e^{-x}}$$





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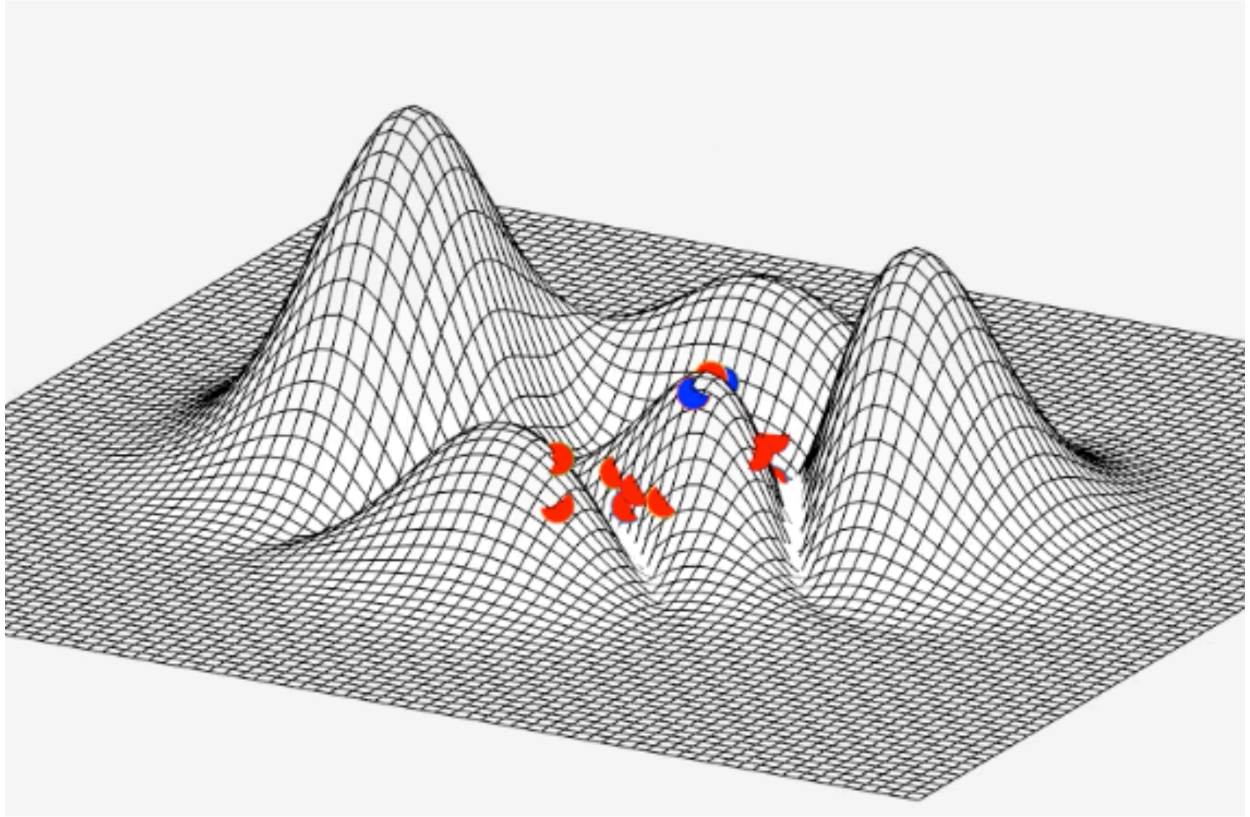
神经网络

BP网络结构

$$E=\frac{1}{2}\sum_{j=1}^{l}(\hat{y}_j-y_j)^2$$

网络训练目标：

找出合适的权值和阈值，使得误差 E 最小



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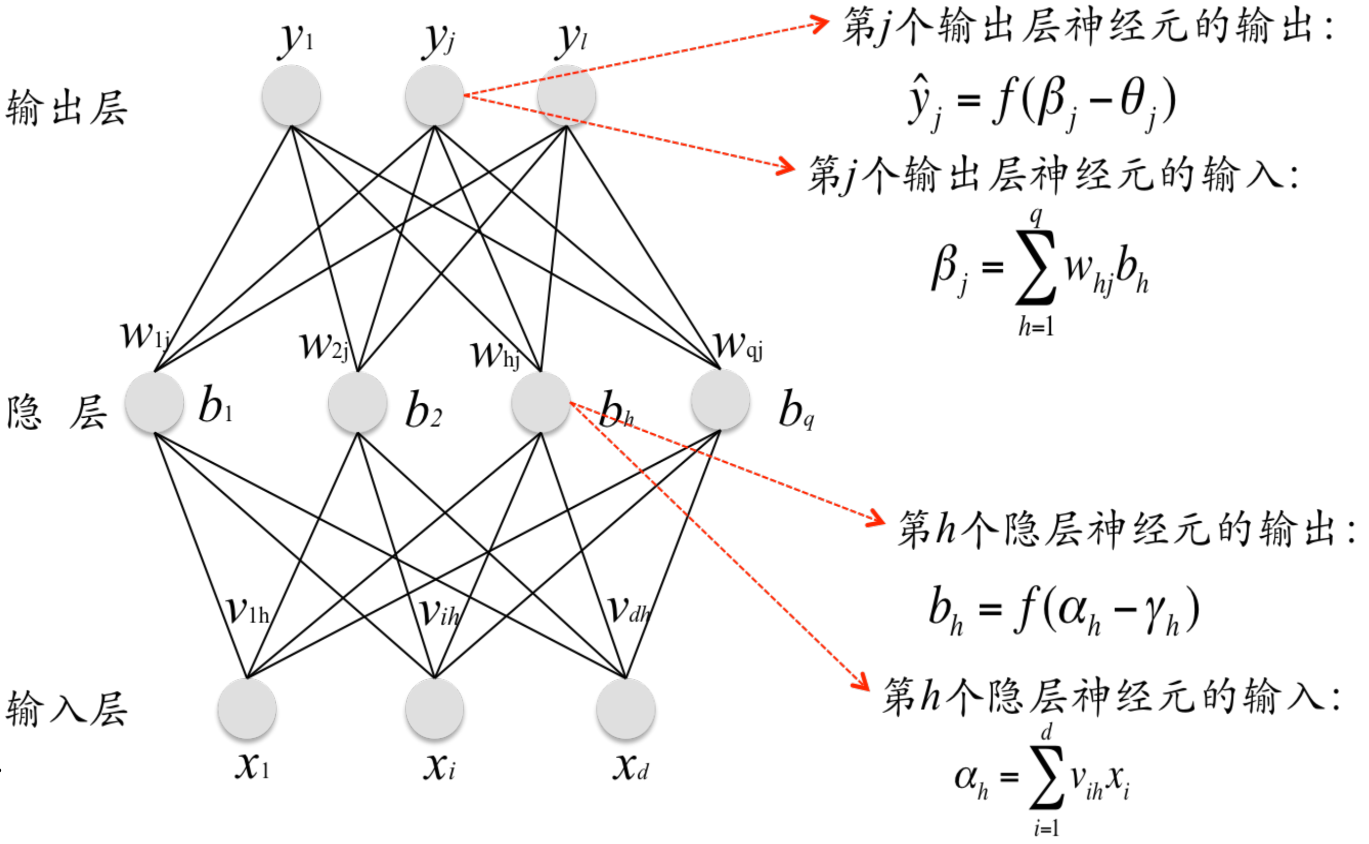
BP网络结构

$$f(x) = sigmoid(x) = \frac{1}{1+e^{-x}}$$

$$f'(x) = f(x)(1 - f(x))$$

$$\hat{y}_j = f(\beta_j - \theta_j)$$

$$E = \frac{1}{2}\sum_{j=1}^{l}(\hat{y}_j - y_j)^2 \dashrightarrow \frac{\partial E}{\partial \hat{y}_j} = \hat{y}_j - y_j$$



$$\Delta w_{hj} = -\eta \frac{\partial E}{\partial w_{hj}} \qquad \frac{\partial E}{\partial w_{hj}} = \frac{\partial E}{\partial \hat{y}_j} \cdot \frac{\partial \hat{y}_j}{\partial \beta_j} \cdot \frac{\partial \beta_j}{\partial w_{hj}}$$

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$$\frac{\partial E}{\partial w_{hj}} = \frac{\partial E}{\partial \hat{y}_j} \cdot \frac{\partial \hat{y}_j}{\partial \beta_j} \cdot \frac{\partial \beta_j}{\partial w_{hj}}$$

$$\frac{\partial \beta_j}{\partial w_{hj}} = b_h \qquad \frac{\partial E}{\partial \hat{y}_j} = \hat{y}_j - y_j$$

$$\frac{\partial \hat{y}_j}{\partial \beta_j} = f'(\beta_j - \theta_j) \qquad f'(x) = f(x)(1 - f(x))$$

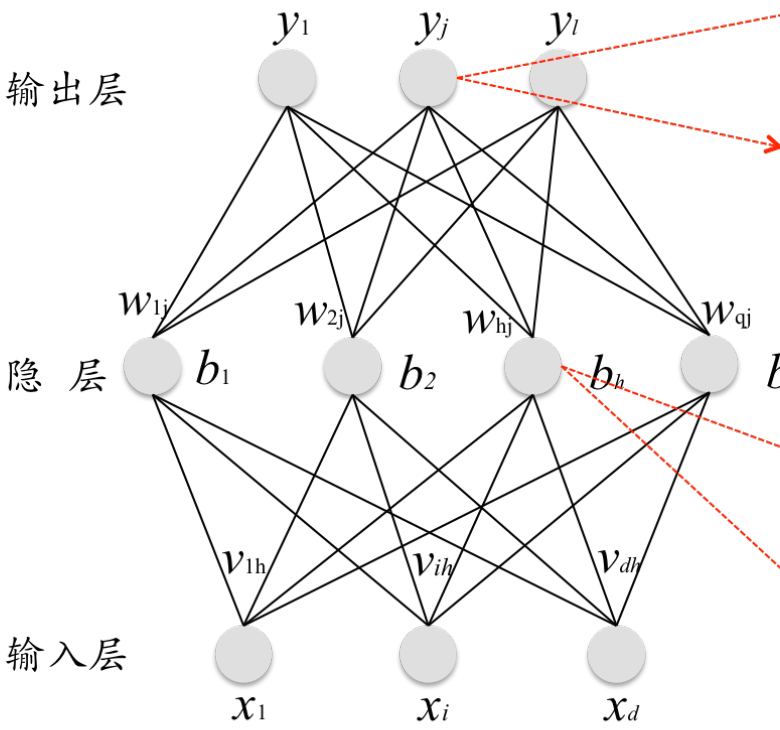
$$= f(\beta_j - \theta_j)(1 - f(\beta_j - \theta_j))$$

$$= \hat{y}_j(1 - \hat{y}_j)$$

$$g_j = -\frac{\partial E}{\partial \hat{y}_j} \cdot \frac{\partial \hat{y}_j}{\partial \beta_j} \qquad \Delta w_{hj} = -\eta \frac{\partial E}{\partial w_{hj}}$$

$$= -(\hat{y}_j - y_j)\hat{y}_j(1 - \hat{y}_j) \qquad = -\eta \frac{\partial E}{\partial \hat{y}_j} \cdot \frac{\partial \hat{y}_j}{\partial \beta_j} \cdot \frac{\partial \beta_j}{\partial w_{hj}} \qquad \Delta w_{hj} = \eta \hat{y}_j(1 - \hat{y}_j)(y_j - \hat{y}_j)b_h$$

$$= \hat{y}_j(1 - \hat{y}_j)(y_j - \hat{y}_j) \qquad = \eta g_j b_h$$



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BP网络结构

$$\Delta w_{hj} = -\eta \frac{\partial E}{\partial w_{hj}}$$

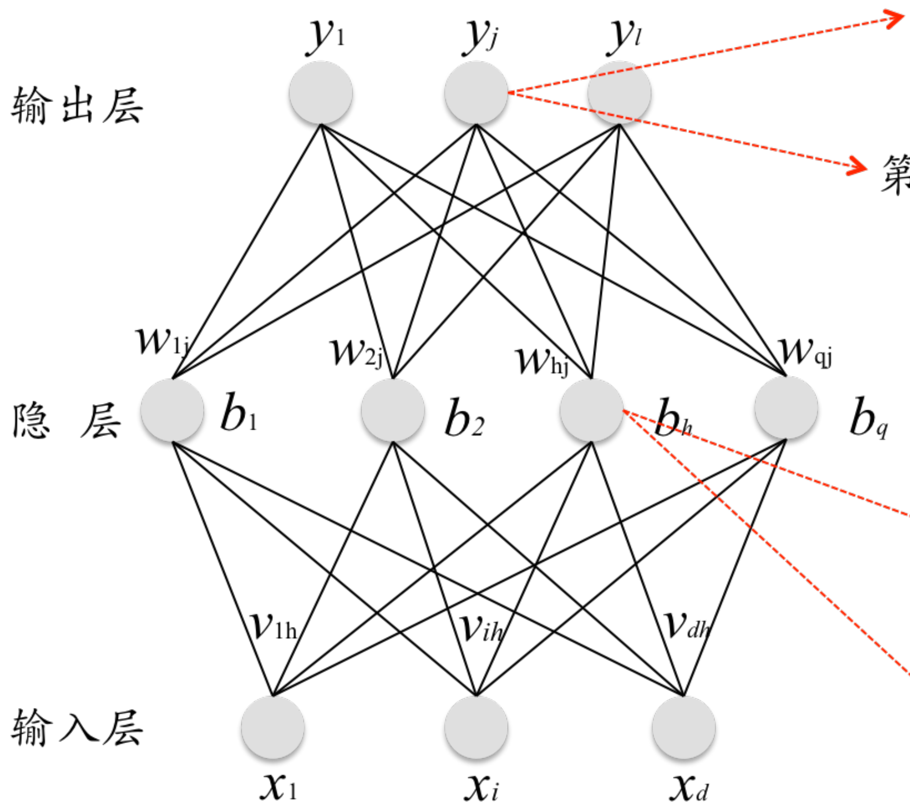
$$= -\eta \frac{\partial E}{\partial \hat{y}_j} \cdot \frac{\partial \hat{y}_j}{\partial \beta_j} \cdot \frac{\partial \beta_j}{\partial w_{hj}}$$

$$= \eta g_j b_h$$

$$= \eta \hat{y}_j (1 - \hat{y}_j)(y_j - \hat{y}_j) b_h$$

$$\Delta \theta_j = -\eta g_j$$

$$= -\eta \hat{y}_j (1 - \hat{y}_j)(y_j - \hat{y}_j)$$



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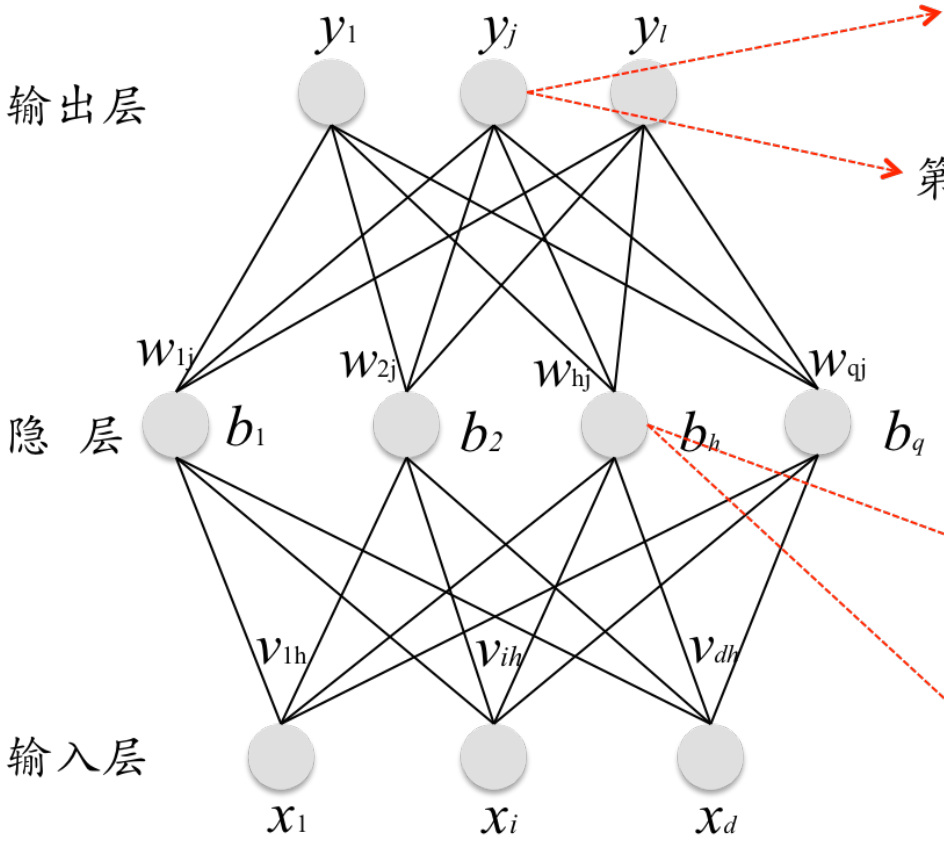
神经网络

BP网络结构

$$\Delta v_{ih} = \eta e_h x_i$$
$$= -\eta \frac{\partial E}{\partial b_h} \cdot \frac{\partial b_h}{\partial \alpha_h} x_i$$
$$= \eta b_h (1 - b_h) \sum_{j=1}^{l} w_{hj} g_j x_i$$

$$\Delta \gamma_h = -\eta e_h$$
$$= \eta \frac{\partial E}{\partial b_h} \cdot \frac{\partial b_h}{\partial \alpha_h}$$
$$= -\eta b_h (1 - b_h) \sum_{j=1}^{l} w_{hj} g_j$$



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神经网络

BP网络结构

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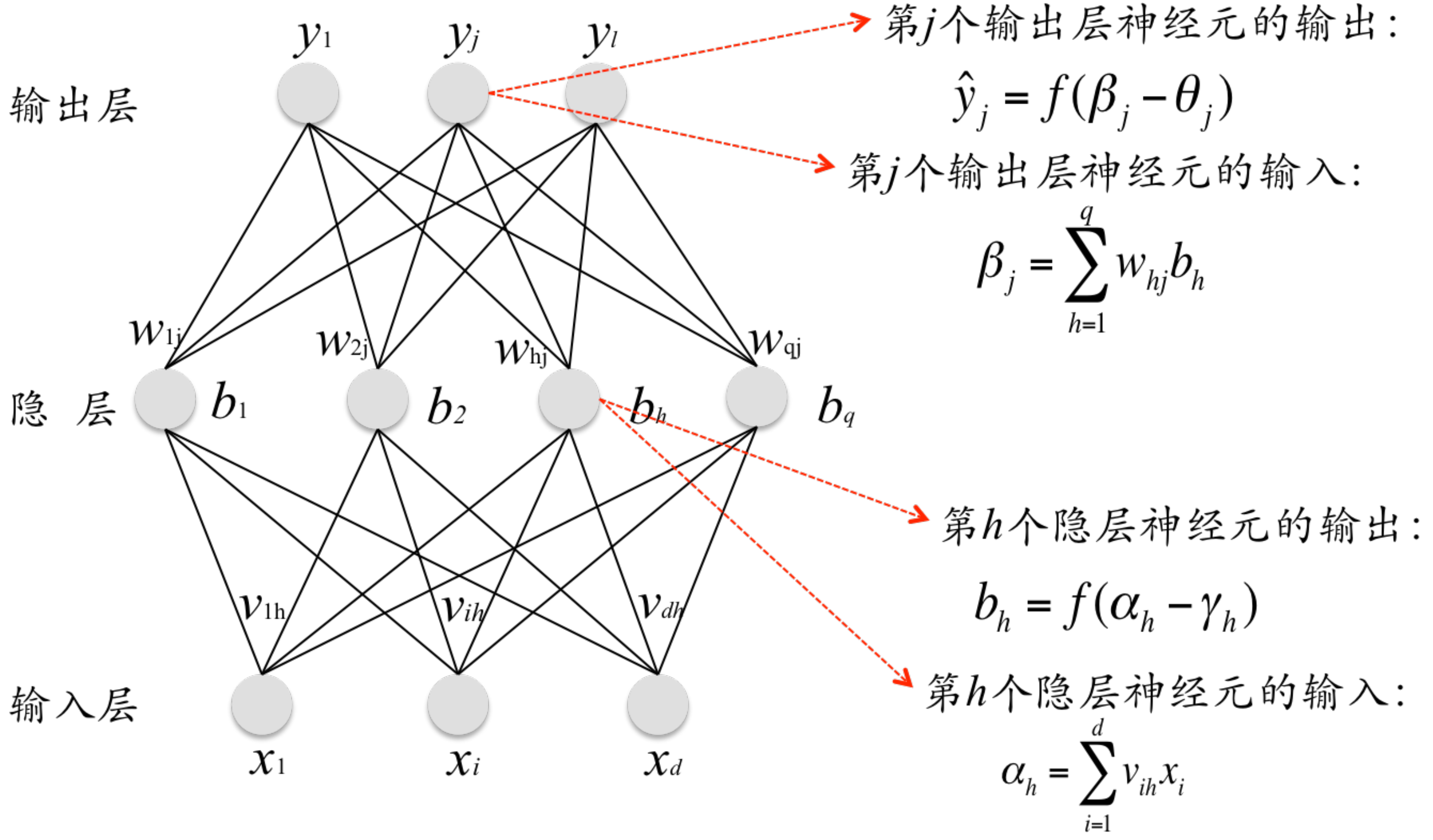
$$\Delta \theta_j = -\eta \hat{y}_j(1-\hat{y}_j)(y_j-\hat{y}_j)$$

$$\Delta v_{ih} = \eta b_h(1-b_h)\sum_{j=1}^{l} w_{hj} g_j x_i$$

$$\Delta \gamma_h = -\eta b_h(1-b_h)\sum_{j=1}^{l} w_{hj} g_j$$



神经网络

网络训练过程

输入：训练集数据、学习速率$yita$

过程：

- 在(0,1)范围内随机初始化网络中所有连接权和阈值
- repeat
  - 根据网络输入和当前参数计算网络输出值y
  - 计算输出层神经元梯度项g_j
  - 计算隐层神经元梯度项e_h
  - 跟新连接权值和阈值
- until达到停止条件
- 输出：连接权值和阈值

神经网络

代码实现

Python（sklearn）

- Net = MLPClassifier(hidden_layer_sizes=10,max_iter=1000).fit(tr_data.ix[:,0:6],tr_data.ix[:,6])
- res = Net.predict(te_data.ix[:,0:6])

R（nnet）

- nnet(x, y, size, softmax = FALSE, maxit = 100)

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Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	class
5.1	3.5	1.4	0.2	setosa
4.9	3	1.4	0.2	setosa
7	3.2	4.7	1.4	versicolor
6.4	3.2	4.5	1.5	versicolor
6.3	3.3	6	2.5	virginica
5.8	2.7	5.1	1.9	virginica
6.5	3	5.8	2.2	?
6.2	2.9	4.3	1.3	?

$Sepal\_length$

$Sepal\_width$ ⇢ 模型 / 系统 ⇢ $class$

$Petal\_length$

$Petal.width$

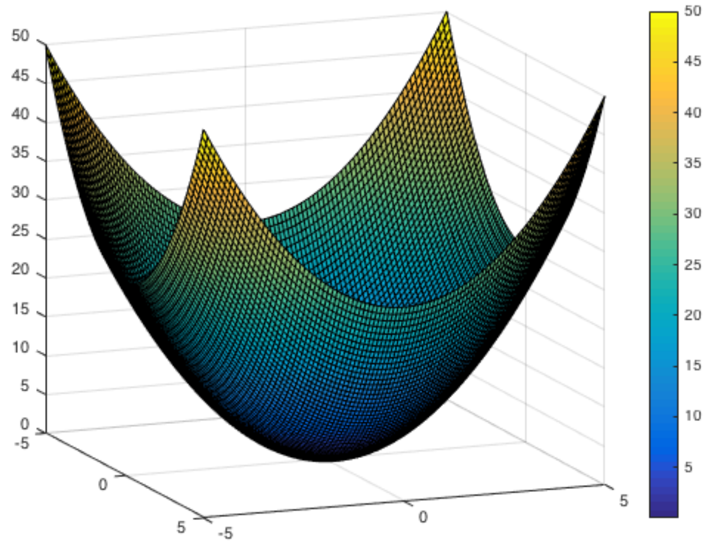
神经网络

附录：BP神经网络自编代码

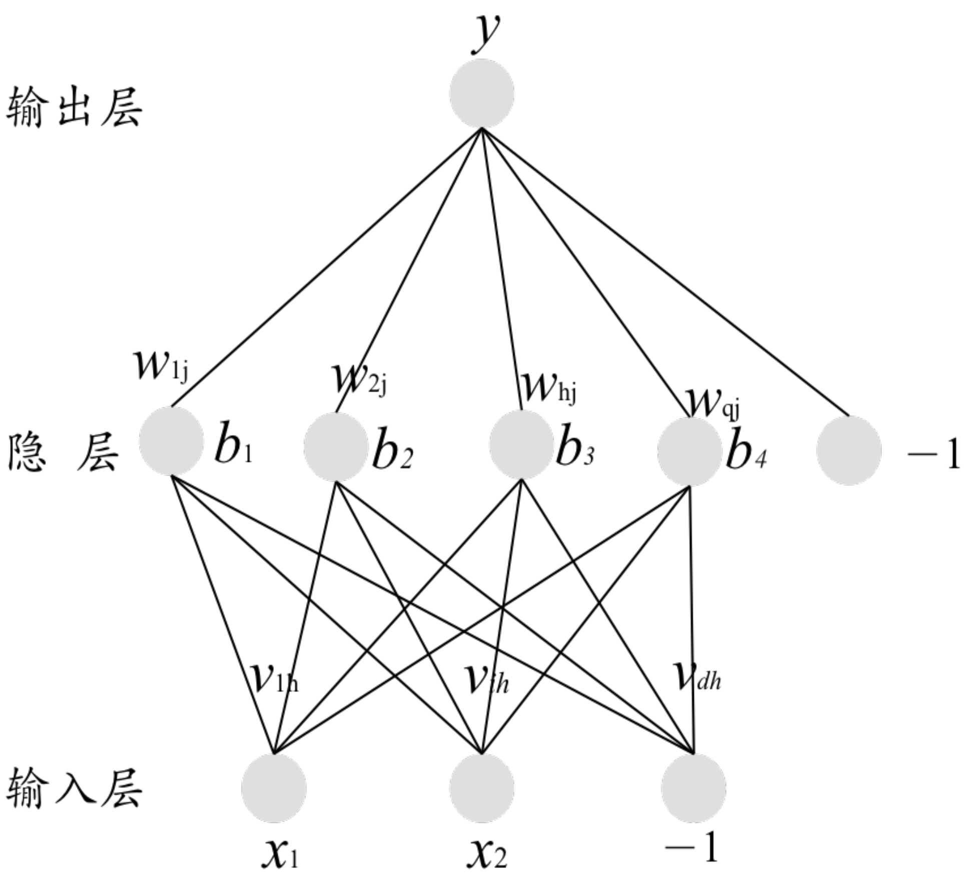
$$y = x_1^2 + x_2^2$$

训练集数据：BPdata_tr.txt

测试集数据：BPdata_te.txt



	x_1	x_2	y
0	0.29	0.23	0.14
1	0.50	0.62	0.64
2	0.00	0.53	0.28
3	0.21	0.53	0.33
4	0.10	0.33	0.12
5	0.06	0.15	0.03
6	0.13	0.03	0.02
7	0.24	0.23	0.11
8	0.28	0.03	0.08
9	0.38	0.49	?
10	0.29	0.47	?



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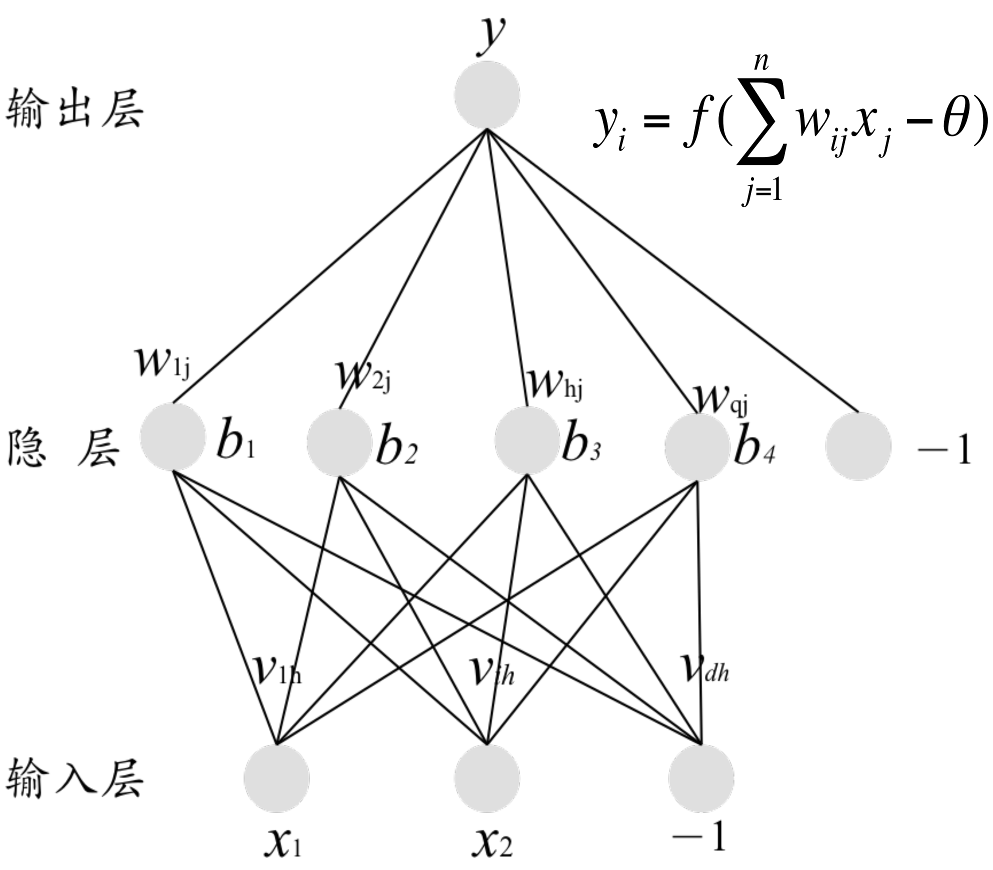
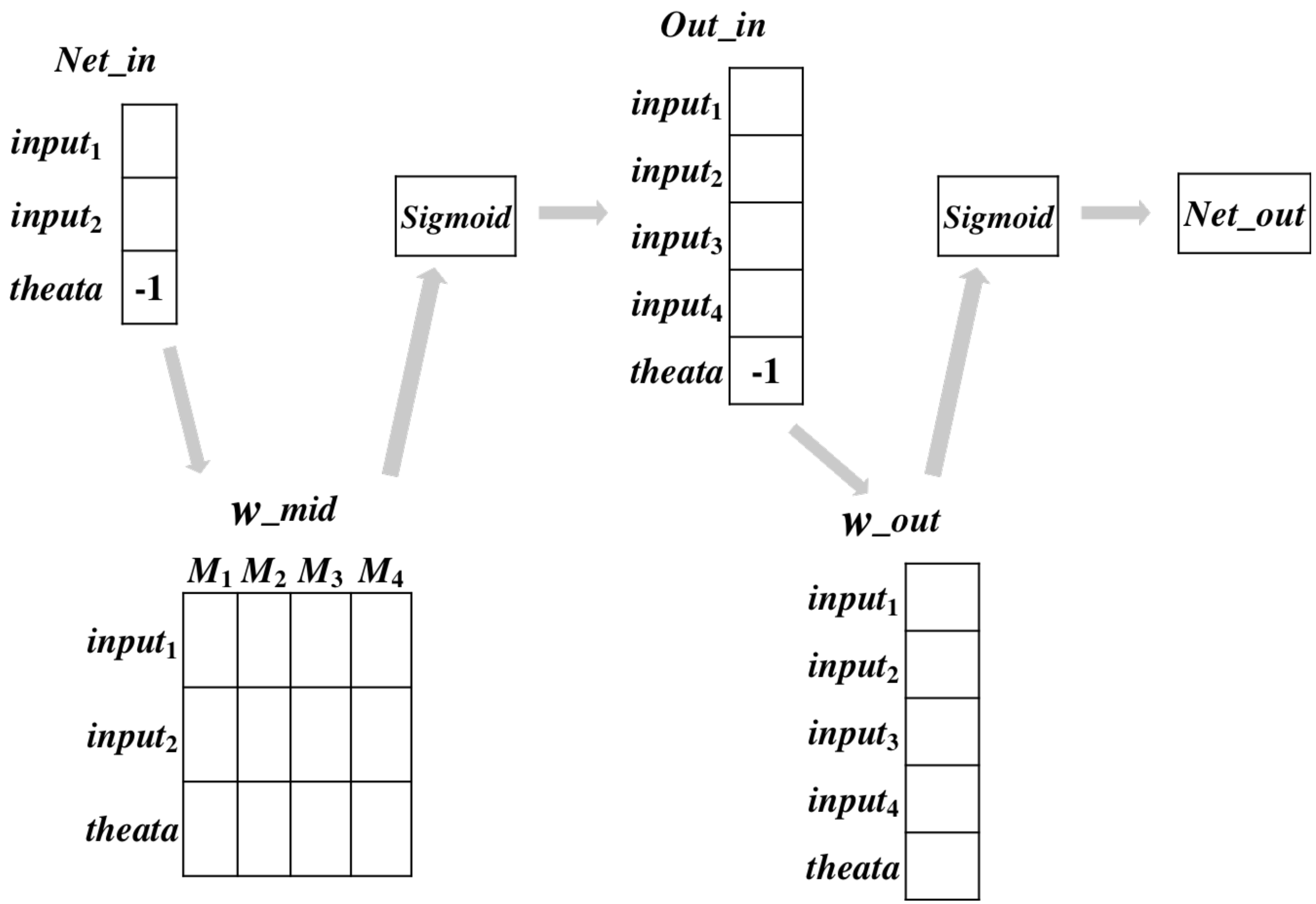
附录：BP神经网络自编代码



$$y_i = f(\sum_{j=1}^{n} w_{ij} x_j - \theta)$$

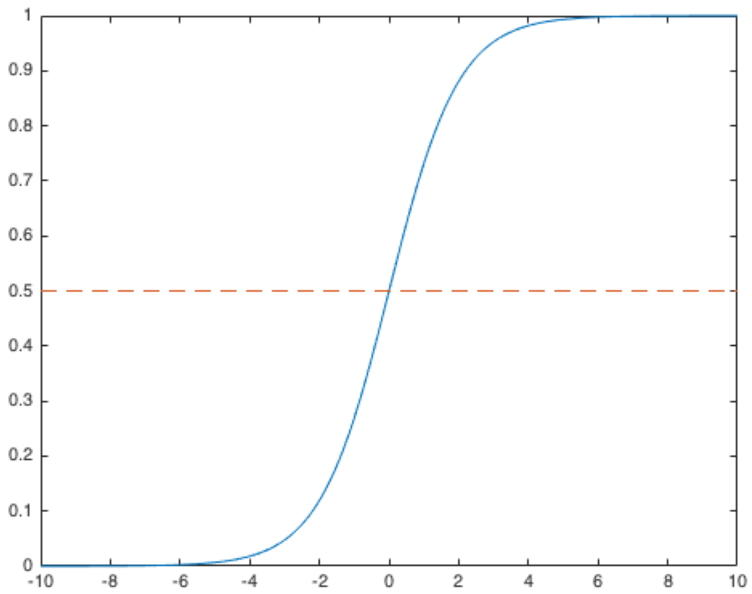


神经网络

附录：BP神经网络自编代码

$$f(x)=sigmoid(x)=\frac{1}{1+e^{-x}}$$

```
def sigmoid(x):  #映射函数
    return 1/(1+math.exp(-x))
import math
import numpy as np
import pandas as pd
from pandas import DataFrame,Seres
```



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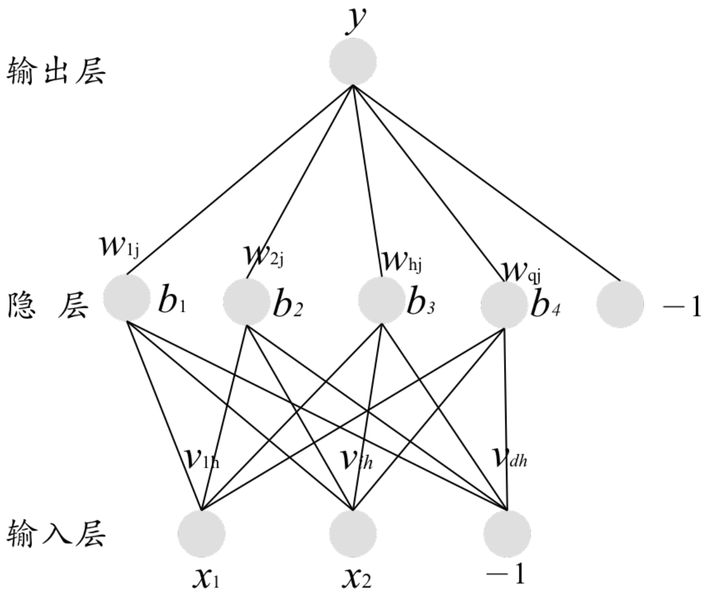
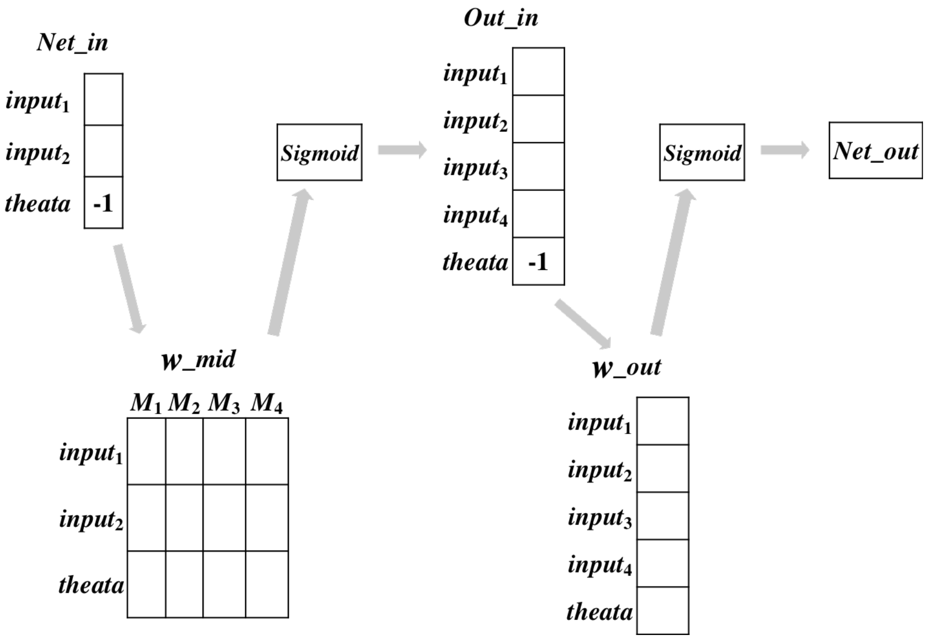
附录：BP神经网络自编代码

```
#中间层神经元输入和输出层神经元输入
Net_in = DataFrame(0.6,index=['input1','input2','theata'],columns=['a'])
Out_in =
 DataFrame(0,index=['input1','input2','input3','input4','theata'],columns=['a'])
Net_in.ix[2,0] = -1
Out_in.ix[4,0] = -1
```

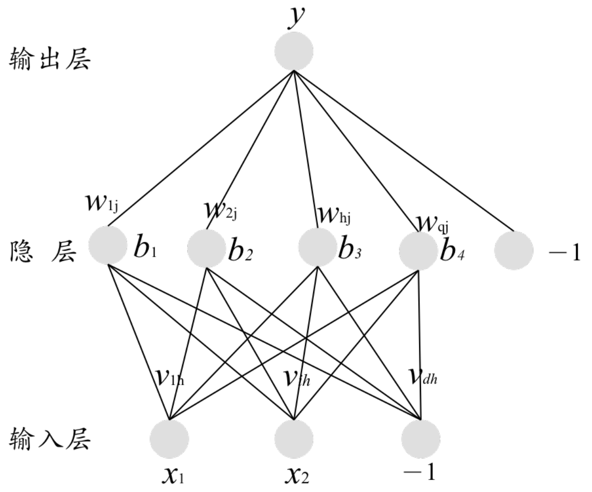


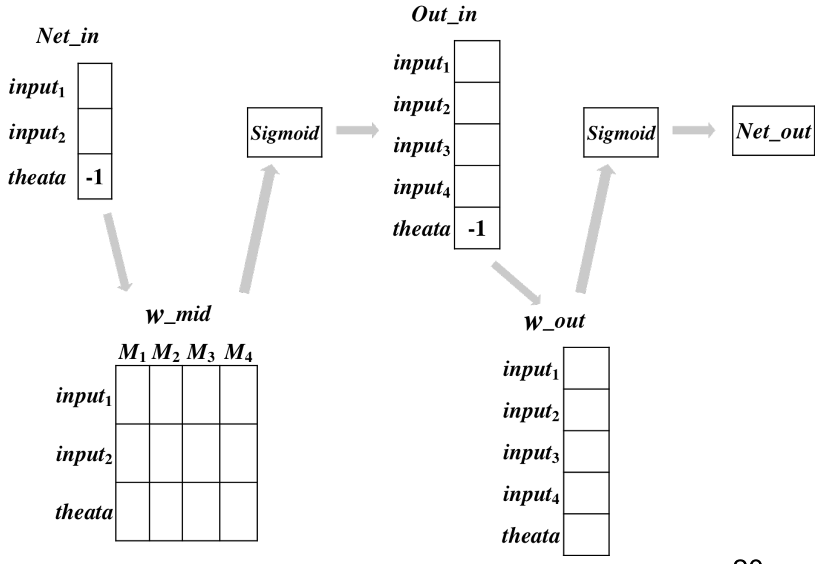


神经网络

附录：BP神经网络自编代码

```
#中间层和输出层神经元权值
W_mid=DataFrame(0.5,index=['input1','input2','theata'],
columns=['mid1','mid2','mid3','mid4'])
W_out=DataFrame(0.5,index=['input1','input2','input3','
input4','theata'],columns=['a'])
W_mid_delta=DataFrame(0,index=['input1','input2','the
ata'],columns=['mid1','mid2','mid3','mid4'])
W_out_delta=DataFrame(0,index=['input1','input2','inpu
t3','input4','theata'],columns=['a'])
```

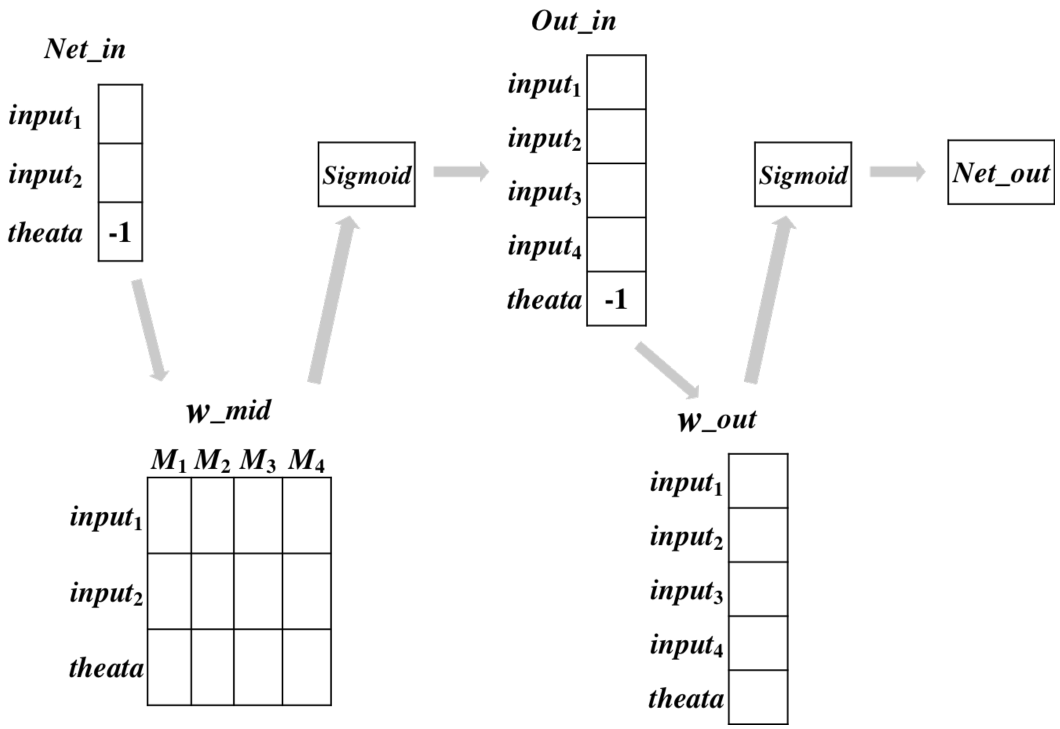




神经网络

附录：BP神经网络自编代码

```
#中间层的输出
for i in range(0,4):
    Out_in.ix[i,0] = sigmoid(sum(W_mid.ix[:,i]*Net_in.ix[:,0]))
#输出层的输出/网络输出
res = sigmoid(sum(Out_in.ix[:,0]*W_out.ix[:,0]))
error = abs(res-real)
```



神经网络

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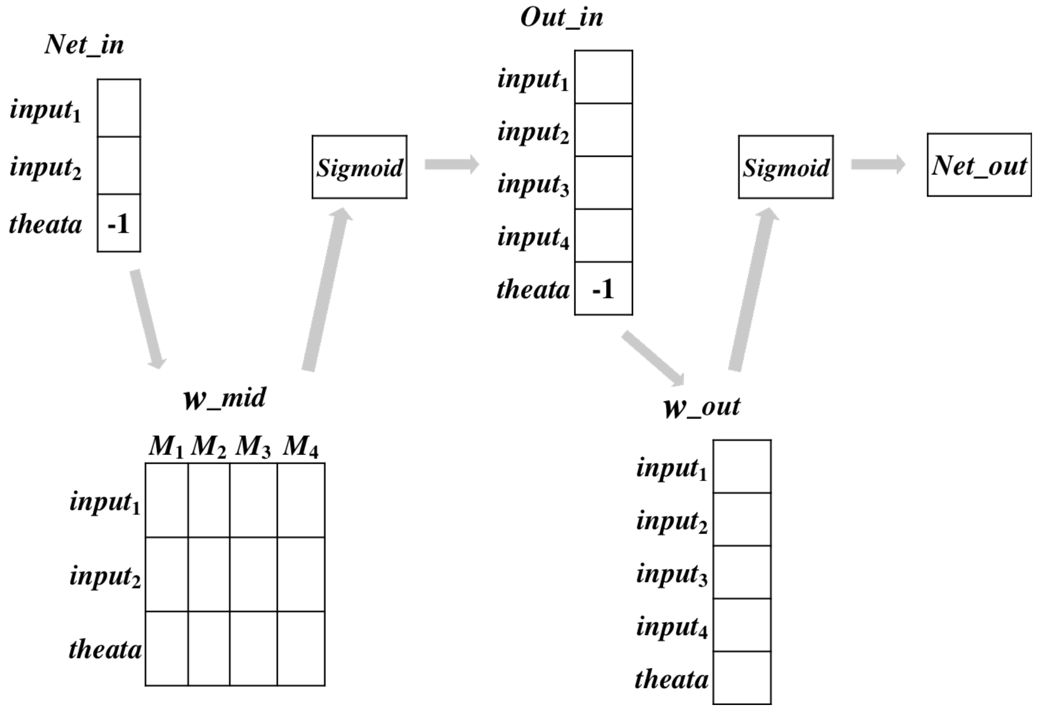


$$\Delta w_{hj} = \eta \hat{y}_j (1 - \hat{y}_j)(y_j - \hat{y}_j) b_h$$

$$\Delta \theta_j = -\eta \hat{y}_j (1 - \hat{y}_j)(y_j - \hat{y}_j)$$

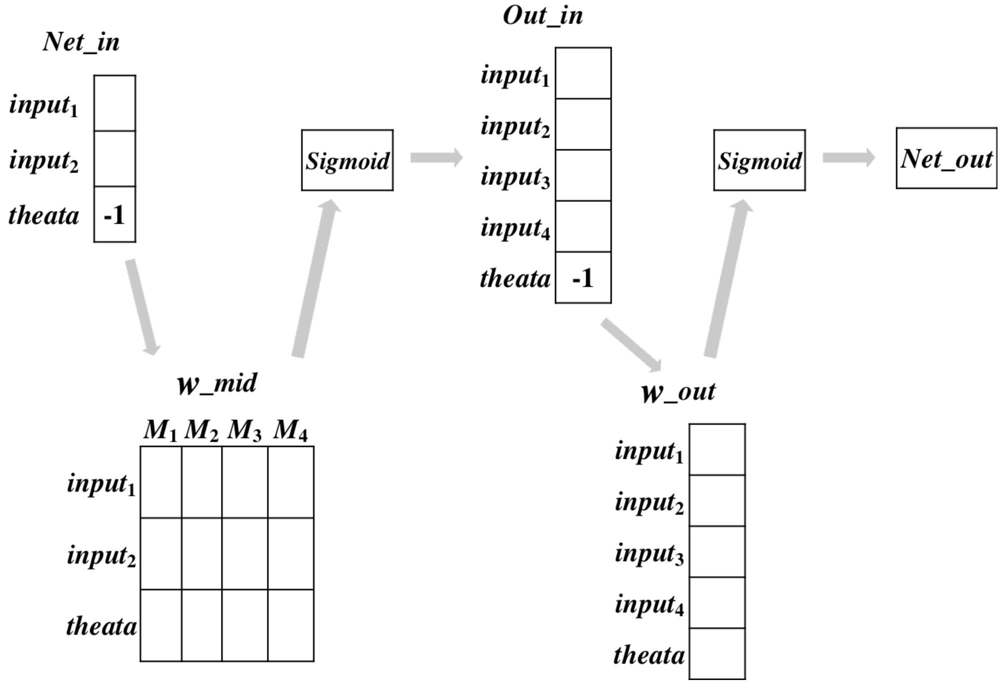
```
#输出层权值变化量
W_out_delta.ix[:,0] = yita*res*(1-res)*(real-res)*Out_in.ix[:,0]
W_out_delta.ix[4,0] = -(yita*res*(1-res)*(real-res))
W_out = W_out + W_out_delta #输出层权值更新
```

神经网络

附录：BP神经网络自编代码

$$\Delta v_{ih} = \eta b_h(1-b_h)\sum_{j=1}^{l} w_{hj} g_j x_i$$

$$\Delta \gamma_h = -\eta b_h(1-b_h)\sum_{j=1}^{l} w_{hj} g_j$$




```
#中间层权值变化量
for i in range(0,4):
    W_mid_delta.ix[:,i] = yita*Out_in.ix[i,0]*(1-Out_in.ix[i,0])*W_out.ix[i,0]*res*(1
-res)*(real-res)*Net_in.ix[:,0]
    W_mid_delta.ix[2,i] = -(yita*Out_in.ix[i,0]*(1-Out_in.ix[i,0])*W_out.ix[i,0]*res*(1
-res)*(real-res))
W_mid = W_mid + W_mid_delta #中间层权值更新
```

大数据成就未来

Thank you!

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