

CS 291A: Deep Learning for NLP

Word Embeddings

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Slides adapted from Y. V. Chen and R. Socher.

What to include in your proposal (Due in one week)

Motivation.

Significance of your problem.

Task definition.

Related work.

Proposed novel approaches.

Datasets and evaluation metrics.

Your plan.

What's the suggested tool?

Keras on Tensorflow for new application projects,
PyTorch for new model/algorithm projects.

A very strong baseline: Bidirectional LSTMs.

https://github.com/fchollet/keras/blob/master/examples/imdb_bidirectional_lstm.py

**To a first approximation,
the de facto consensus in NLP in 2017 is
that no matter what the task,
you throw a BiLSTM at it, with
attention if you need information flow**

Chris Manning (Stanford)

Word Meaning Representations

(How to represent the meaning of a word in a vector)

Knowledge-based representation

Corpus-based representation

- ✓ Atomic symbol
- ✓ Neighbors
 - High-dimensional sparse word vector
 - Low-dimensional dense word vector
 - Method 1 – dimension reduction
 - Method 2 – direct learning

Word Meaning Representations

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Atomic symbols: *one-hot* representation

Issues: difficult to compute the similarity
(i.e. comparing “car” and “motorcycle”)

Idea: words with similar meanings often have similar neighbors

Word Meaning Representations

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Window-based Co-occurrence Matrix

Example

- Window length=1
- Left or right context
- Corpus:

I love UCSB.
I love deep learning.
I enjoy learning.

similarity > 0

Counts	I	love	enjoy	UCSB	deep	learning
I	0	2	1	0	0	0
love	2	0	0	1	1	0
enjoy	1	0	0	0	0	1
UCSB	0	1	0	0	0	0
deep	0	1	0	0	0	1
learning	0	0	1	0	1	0

Issues:

- matrix size increases with vocabulary
- high dimensional
- sparsity → poor robustness

Idea: low dimensional
word vector

Word Meaning Representations

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Low-Dimensional Dense Word Vector

Method 1: dimension reduction on the matrix

Singular Value Decomposition (SVD) of co-occurrence matrix X

The diagram illustrates the Singular Value Decomposition (SVD) of a co-occurrence matrix X and its approximation \hat{X} .

Top Row (Full SVD):

- Matrix X (dimensions $n \times m$) is equal to the product of matrix U (dimensions $n \times r$), matrix S (dimensions $r \times r$), and matrix V^T (dimensions $r \times m$).
- Matrix U contains columns U_1, U_2, U_3, \dots .
- Matrix S is a diagonal matrix with singular values $S_1, S_2, S_3, \dots, S_r$ and zeros elsewhere.
- Matrix V^T contains rows V_1, V_2, V_3, \dots .

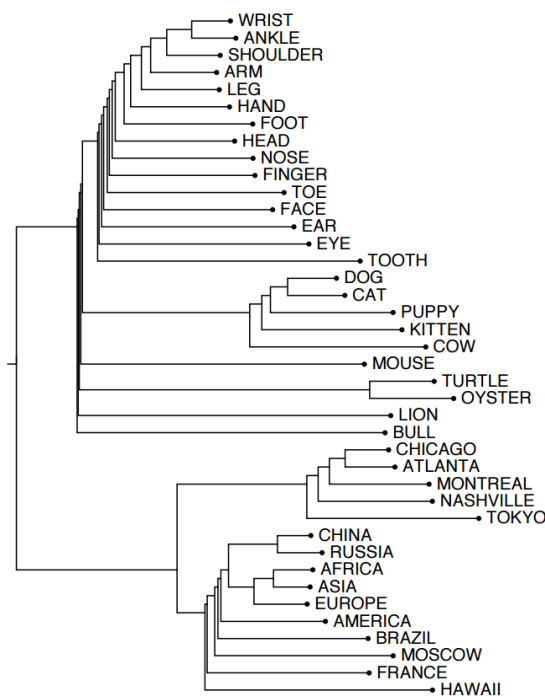
Bottom Row (Approximate SVD):

- Matrix \hat{X} (dimensions $n \times m$) is an approximation of X , indicated by a red arrow labeled "approximate".
- Matrix \hat{X} is equal to the product of matrix \hat{U} (dimensions $n \times k$), matrix \hat{S} (dimensions $k \times k$), and matrix \hat{V}^T (dimensions $k \times m$).
- Matrix \hat{U} contains columns U_1, U_2, U_3, \dots , with the first k columns highlighted in blue.
- Matrix \hat{S} is a diagonal matrix with singular values $S_1, S_2, S_3, \dots, S_k$ and zeros elsewhere.
- Matrix \hat{V}^T contains rows V_1, V_2, V_3, \dots , with the first k rows highlighted in blue.

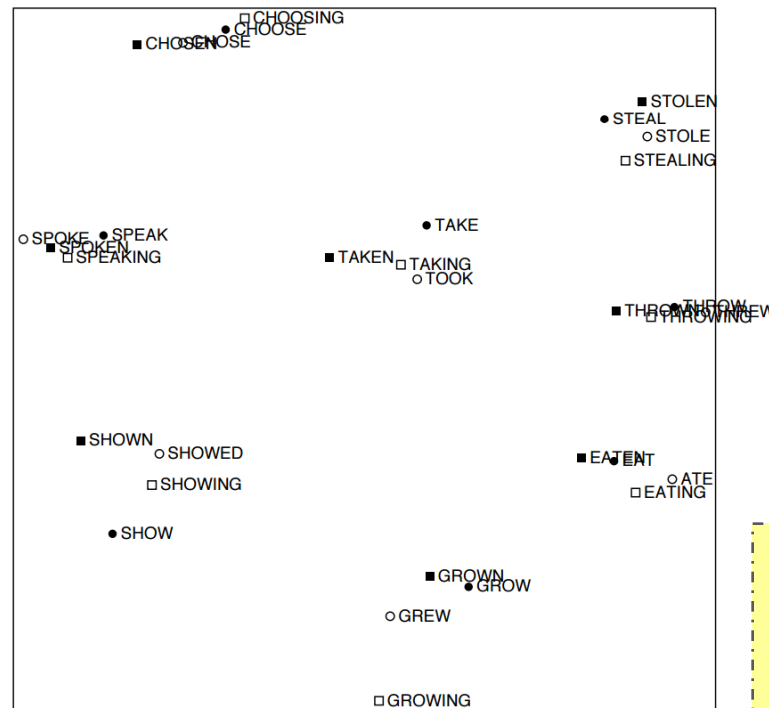
Low-Dimensional Dense Word Vector

Method 1: dimension reduction on the matrix

Singular Value Decomposition (SVD) of co-occurrence matrix X



semantic relations



syntactic relations

Issues:

- computationally expensive
- difficult to add new words

Idea: directly learn low-dimensional word vectors

Word Representation

Knowledge-based representation

Corpus-based representation

- ✓ Atomic symbol
- ✓ Neighbors
 - High-dimensional sparse word vector
 - Low-dimensional dense word vector
 - Method 1 – dimension reduction
 - Method 2 – direct learning → word embedding

Word Embedding

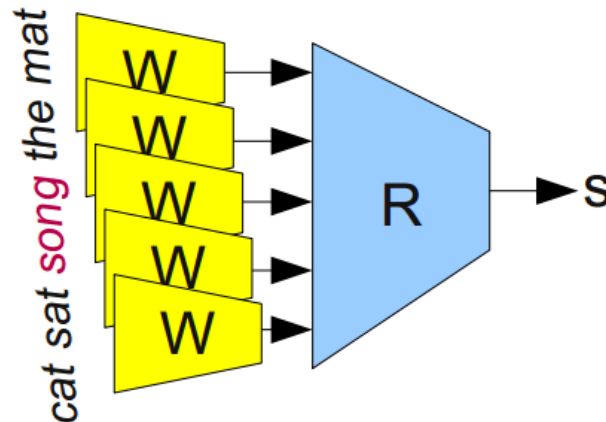
Method 2: directly learn low-dimensional word vectors

- Learning representations by back-propagation. (Rumelhart et al., 1986)
- A neural probabilistic language model (Bengio et al., 2003)
- NLP (almost) from Scratch (Collobert & Weston, 2008)
- Recent and most popular models: word2vec (Mikolov et al. 2013) and Glove (Pennington et al., 2014)

Word Embedding Benefit

Given an unlabeled training corpus, produce a vector for each word that encodes its semantic information. These vectors are useful because:

- ① semantic similarity between two words can be calculated as the cosine similarity between their corresponding word vectors
- ② word vectors as powerful features for various supervised NLP tasks since the vectors contain semantic information
- ③ propagate any information into them via neural networks and update during training



Word2Vec Skip-Gram

Mikolov et al., “Distributed representations of words and phrases and their compositionality,” in *NIPS*, 2013.

Mikolov et al., “Efficient estimation of word representations in vector space,” in *ICLR Workshop*, 2013.

Word2Vec – Skip-Gram Model

Learn word embeddings through a task: predict surrounding words of a target word.

Objective function: maximize the probability of any context word given the current center word

$w_1, w_2, \dots, w_{t-m}, \dots, w_{t-1}, \underbrace{w_t, w_{t+1}, \dots, w_{t+m}}_{\text{context window}}, \dots, w_{T-1}, w_T$

$$p(w_{O,1}, w_{O,2}, \dots, w_{O,C} \mid w_I) = \prod_{c=1}^C p(w_{O,c} \mid w_I)$$

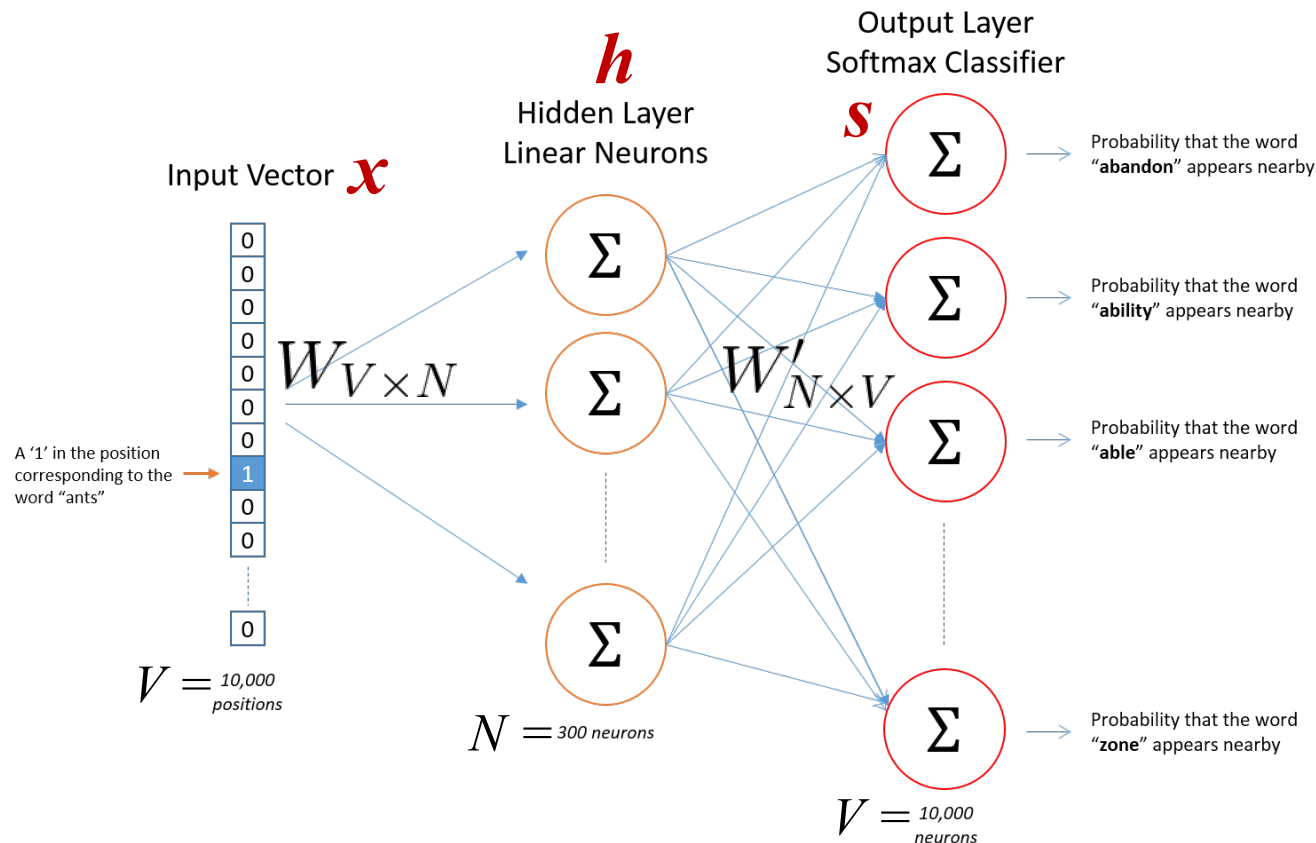
target word vector

$$C(\theta) = - \sum_{w_I} \sum_{c=1}^C \log p(w_{O,c} \mid w_I) \quad \underbrace{p(w_O \mid w_I)}_{\text{output}} = \frac{\exp(v_{w_O}'^T \underbrace{v_{w_I}}_{\text{target word}})}{\sum_j \exp(v_{w_j}'^T v_{w_I})}$$

Benefit: faster, easily incorporate a new sentence/document or add a word to vocab

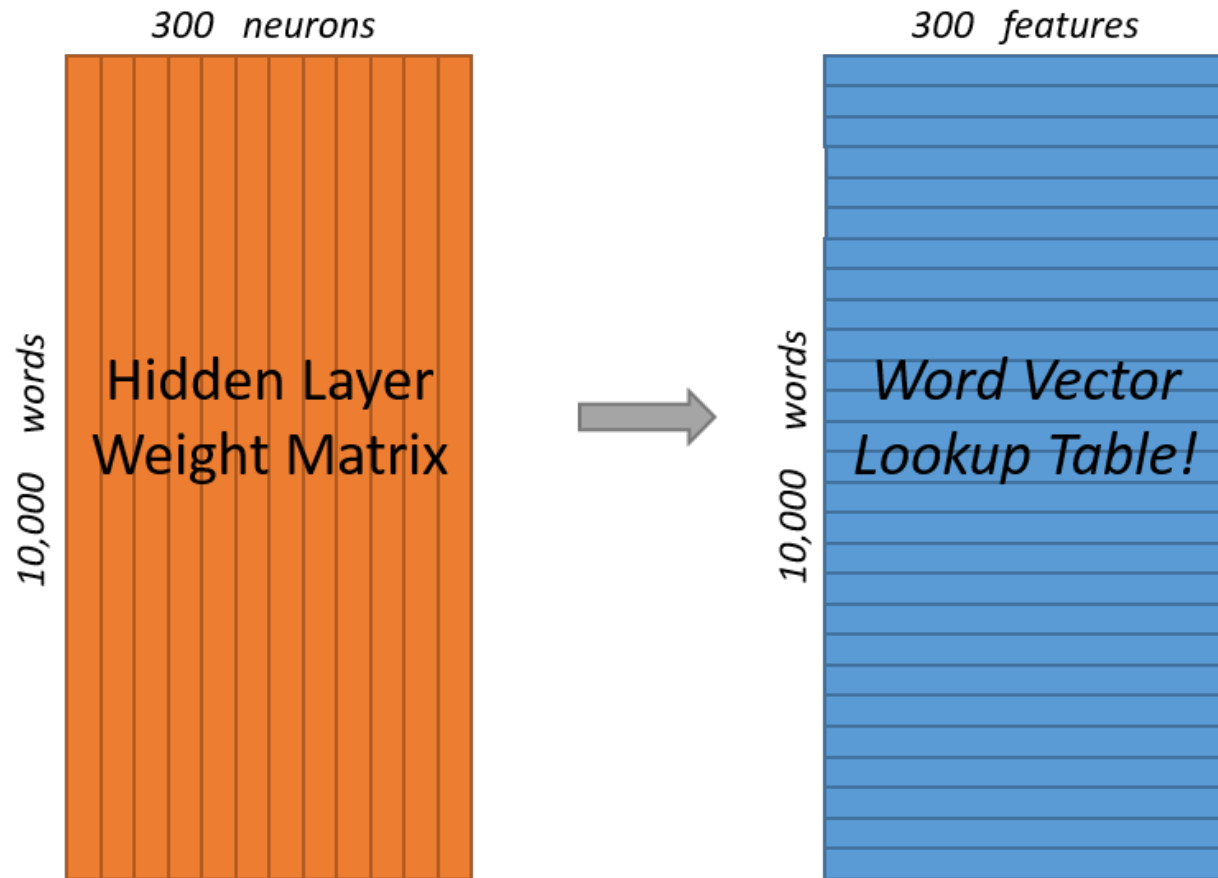
Word2Vec Skip-Gram Illustration

Goal: predict surrounding words of a target word.



Hidden Layer Weight Matrix
→ Word Embedding Matrix

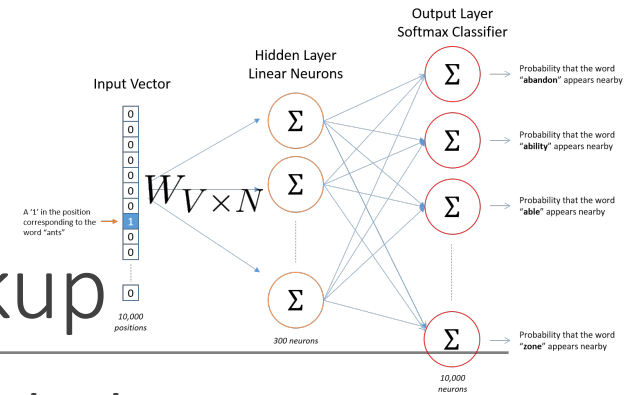
$$W_{V \times N}$$



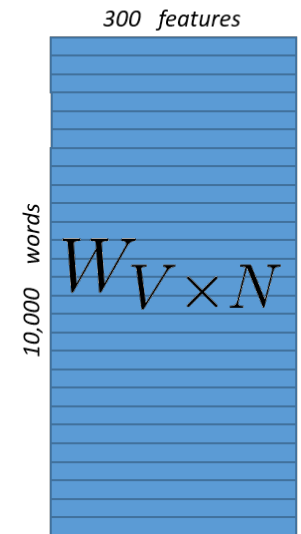
Input: word embedding lookup

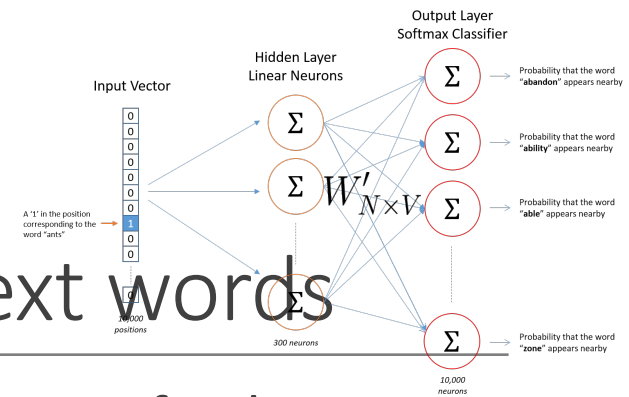
Hidden layer weight matrix = word vector lookup

$$h = x^T W = W_{(k, \cdot)} := v_{w_I}$$



$$[0 \quad 0 \quad 0 \quad 1 \quad 0] \times \begin{bmatrix} 17 & 24 & 1 \\ 23 & 5 & 7 \\ 4 & 6 & 13 \\ 10 & 12 & 19 \\ 11 & 18 & 25 \end{bmatrix} = [10 \quad 12 \quad 19]$$





Output: predicting the context words

Output layer weight matrix = weighted sum as final score

$$s_j = hv'w_j$$

$$p(w_j = w_{O,c} | w_I) = y_{jc} =$$

within the context window

Output weights for "car"

$$\frac{\exp(s_{jc})}{\sum_{j'=1}^V \exp(s_{j'})}$$

softmax

Word vector for "ants"



300 features

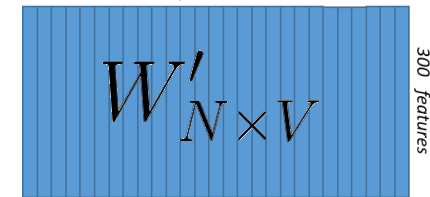
×

300 features

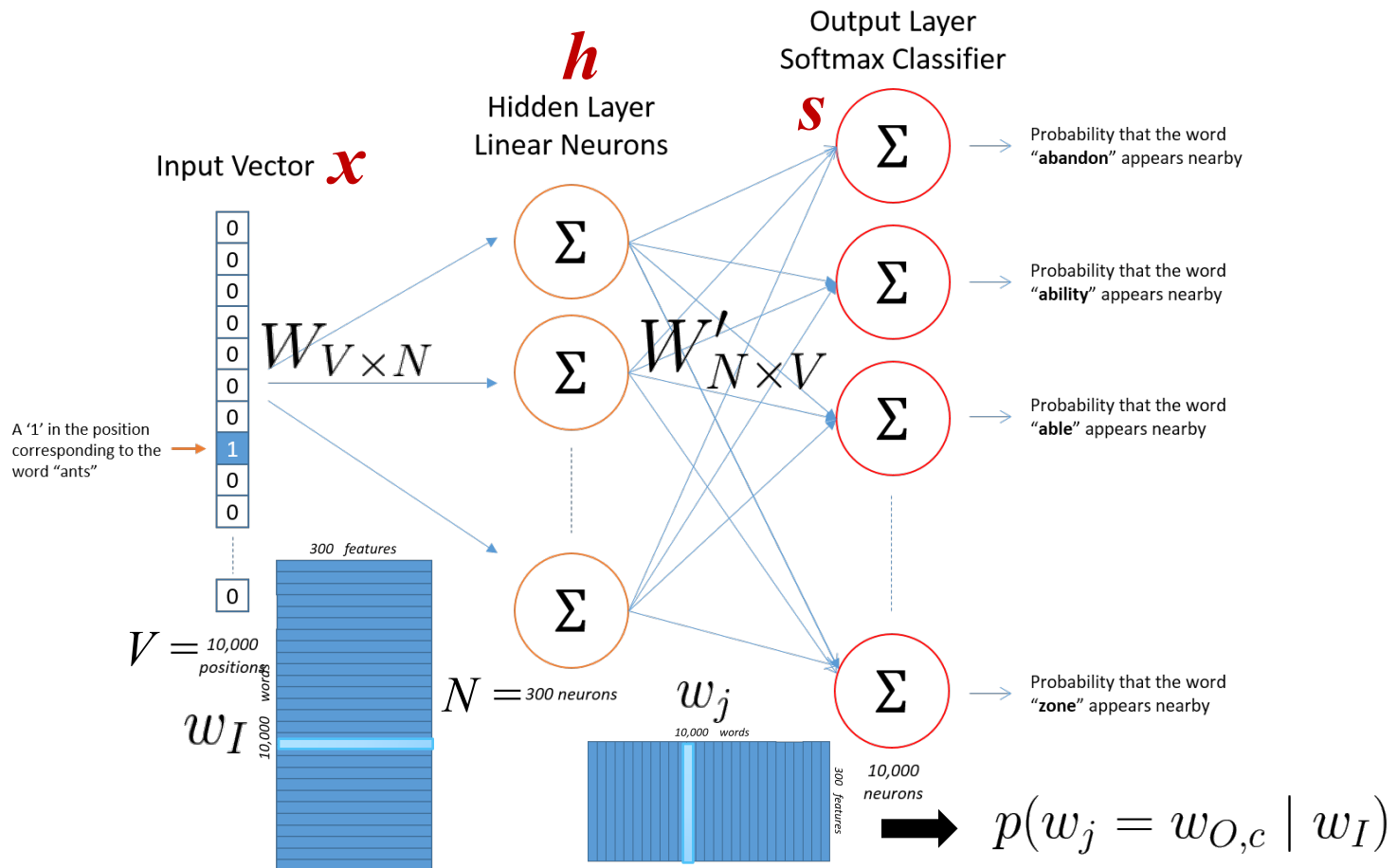


$$\frac{e^x}{\sum e^x}$$

= Probability that "car" shows up near "ants"



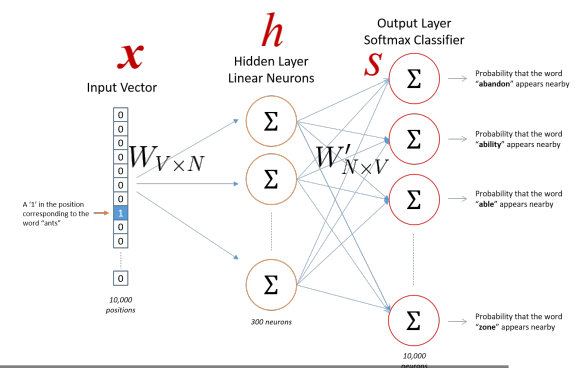
Word2Vec Skip-Gram Illustration



Negative Log-Likelihood

Given a target word (w_I)

$$\begin{aligned} C(\theta) &= -\log p(w_{O,1}, w_{O,2}, \dots, w_{O,C} \mid w_I) \\ &= -\log \prod_{c=1}^C \frac{\exp(s_{j_c})}{\sum_{j'=1}^V \exp(s_{j'})} \\ &= -\sum_{c=1}^C s_{j_c} + C \log \sum_{j'=1}^V \exp(s_{j'}) \end{aligned}$$



SGD Update for W'

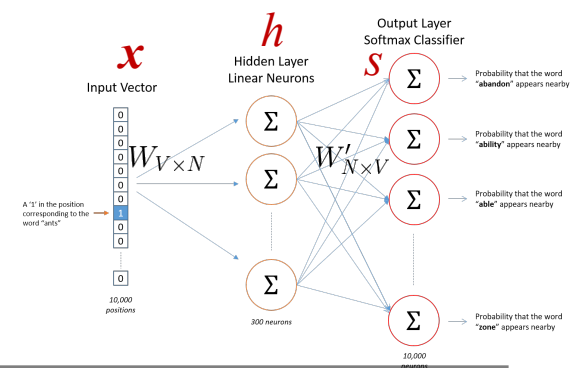
Given a target word (w_I)

$$\frac{\partial C(\theta)}{\partial w'_{ij}} = \sum_{c=1}^C \frac{\partial C(\theta)}{\partial s_{jc}} \frac{\partial s_{jc}}{\partial w'_{ij}} = \sum_{c=1}^C (y_{jc} - t_{jc}) \cdot h_i$$

$$s_j = v'_{w_j}^T \cdot h$$

$$\frac{\partial C(\theta)}{\partial s_{jc}} = y_{jc} - \underbrace{t_{jc}}_{=1, \text{ when } w_{jc} \text{ is within the context window}} := \underbrace{e_{jc}}_{=0, \text{ otherwise}} \text{ error term}$$

$$w'_{ij}^{(t+1)} = w'_{ij}^{(t)} - \eta \cdot \sum_{c=1}^C (y_{jc} - t_{jc}) \cdot h_i$$



SGD Update for W

$$\frac{\partial C(\theta)}{\partial w_{ki}} = \frac{\partial C(\theta)}{\partial h_i} \frac{\partial h_i}{\partial w_{ki}} = \sum_{j=1}^V \sum_{c=1}^C (y_{jc} - t_{jc}) \cdot w'_{ij} \cdot x_k$$

$h = x^T W$

$$\frac{\partial C(\theta)}{\partial h_i} = \sum_{j=1}^V \frac{\partial C(\theta)}{\partial s_j} \frac{\partial s_j}{\partial h_i} = \sum_{j=1}^V \sum_{c=1}^C (y_{jc} - t_{jc}) \cdot w'_{ij}$$

$s_j = v'_{w_j}{}^T \cdot h$

$$w_{ij}^{(t+1)} = w_{ij}^{(t)} - \eta \cdot \sum_{j=1}^V \sum_{c=1}^C (y_{jc} - t_{jc}) \cdot w'_{ij} \cdot x_j$$

SGD Update

$$w'_{ij}{}^{(t+1)} = w'_{ij}{}^{(t)} - \eta \cdot \sum_{c=1}^C (y_{jc} - t_{jc}) \cdot h_i$$

$$EI_j = \sum_{c=1}^C (y_{jc} - t_{jc})$$

$$v'_{w_j}{}^{(t+1)} = v'_{w_j}{}^{(t)} - \eta \cdot EI_j \cdot h$$

$$w_{ij}{}^{(t+1)} = w_{ij}{}^{(t)} - \eta \cdot \sum_{j=1}^V \sum_{c=1}^C (y_{jc} - t_{jc}) \cdot w'_{ij} \cdot x_j$$

$$v_{w_I}{}^{(t+1)} = v_{w_I}{}^{(t)} - \eta \cdot EH^T$$

$$EH_i = \sum_{j=1}^V EI_j \cdot w'_{ij} \cdot x_j$$

large vocabularies or large training corpora → expensive computations

limit the number of output vectors that must be updated per training instance
→ hierarchical softmax, sampling

Negative Sampling

Idea: only update a sample of output vectors

$$C(\theta) = -\log \sigma(v'_{w_O}{}^T v_{w_I}) + \sum_{w_j \in \mathcal{W}_{\text{neg}}} \log \sigma(v'_{w_j}{}^T v_{w_I})$$

$$v'_{w_j}{}^{(t+1)} = v'_{w_j}{}^{(t)} - \eta \cdot EI_j \cdot h$$

$$EI_j = \sigma(v'_{w_j}{}^T v_{w_I}) - t_j$$

$$v_{w_I}^{(t+1)} = v_{w_I}^{(t)} - \eta \cdot EH^T$$

$$EH = \sum_{w_j \in \{w_O\} \cup \mathcal{W}_{\text{neg}}} EI_j \cdot v'_{w_j}$$

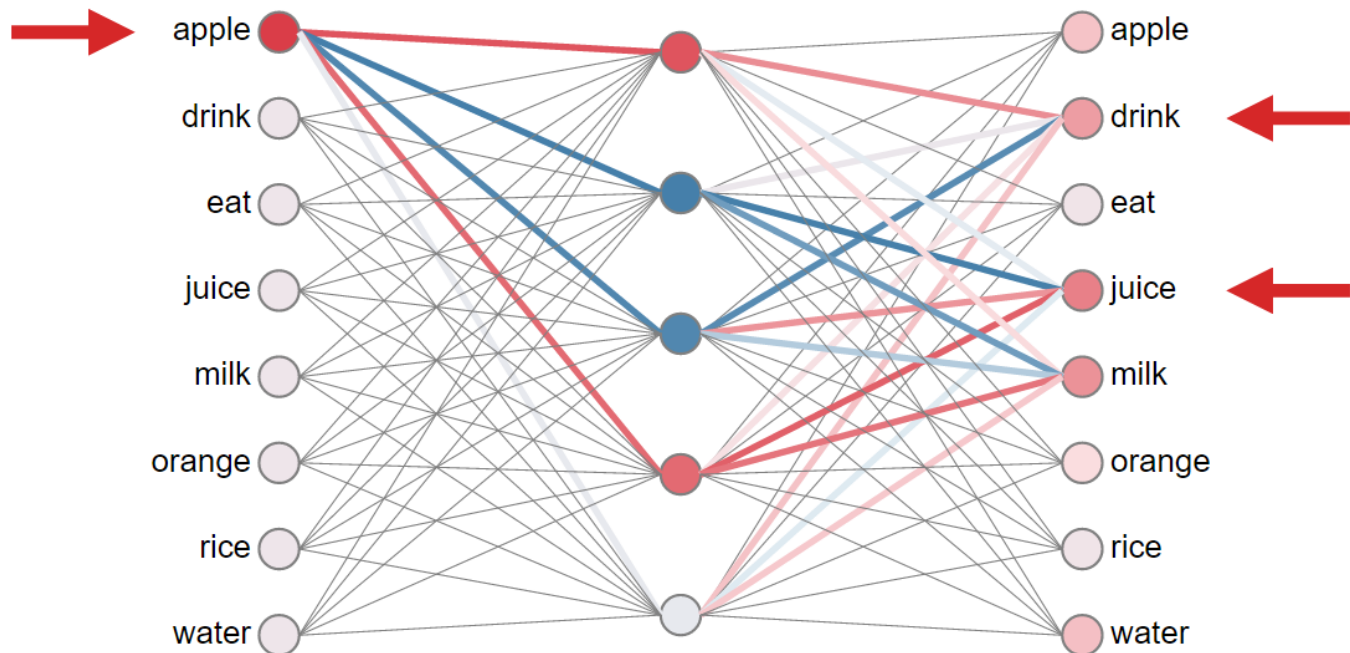
$$w_j \in \{w_O\} \cup \mathcal{W}_{\text{neg}}$$

Word2Vec Skip-Gram Visualization

<https://ronxin.github.io/wevi/>

Skip-gram training data:

apple | drink^juice,orange | eat^apple,rice | drink^juice,juice | drink^milk,
milk | drink^rice,water | drink^milk,juice | orange^apple,juice | apple^drink
,milk | rice^drink,drink | milk^water,drink | water^juice,drink | juice^water



Word2Vec Variants

Skip-gram: predicting surrounding words given the target word (Mikolov+, 2013)

better

$$p(w_{t-m}, \dots w_{t-1}, w_{t+1}, \dots, w_{t+m} \mid w_t)$$

CBOW (continuous bag-of-words): predicting the target word given the surrounding words (Mikolov+, 2013)

$$p(w_t \mid w_{t-m}, \dots w_{t-1}, w_{t+1}, \dots, w_{t+m})$$

LM (Language modeling): predicting the next words given the proceeding contexts (Mikolov+, 2013)

first

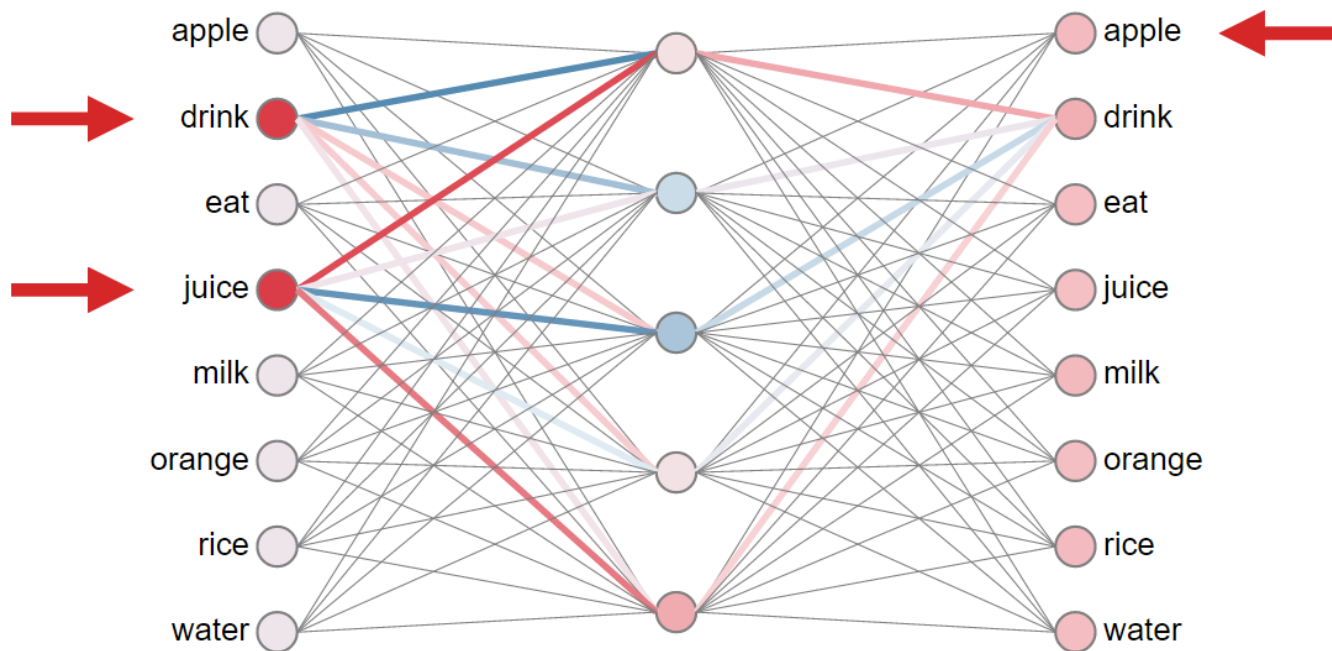
$$p(w_{t+1} \mid w_t)$$

Practice the derivation by yourself!!

Word2Vec CBOW

Goal: predicting the target word given the surrounding words

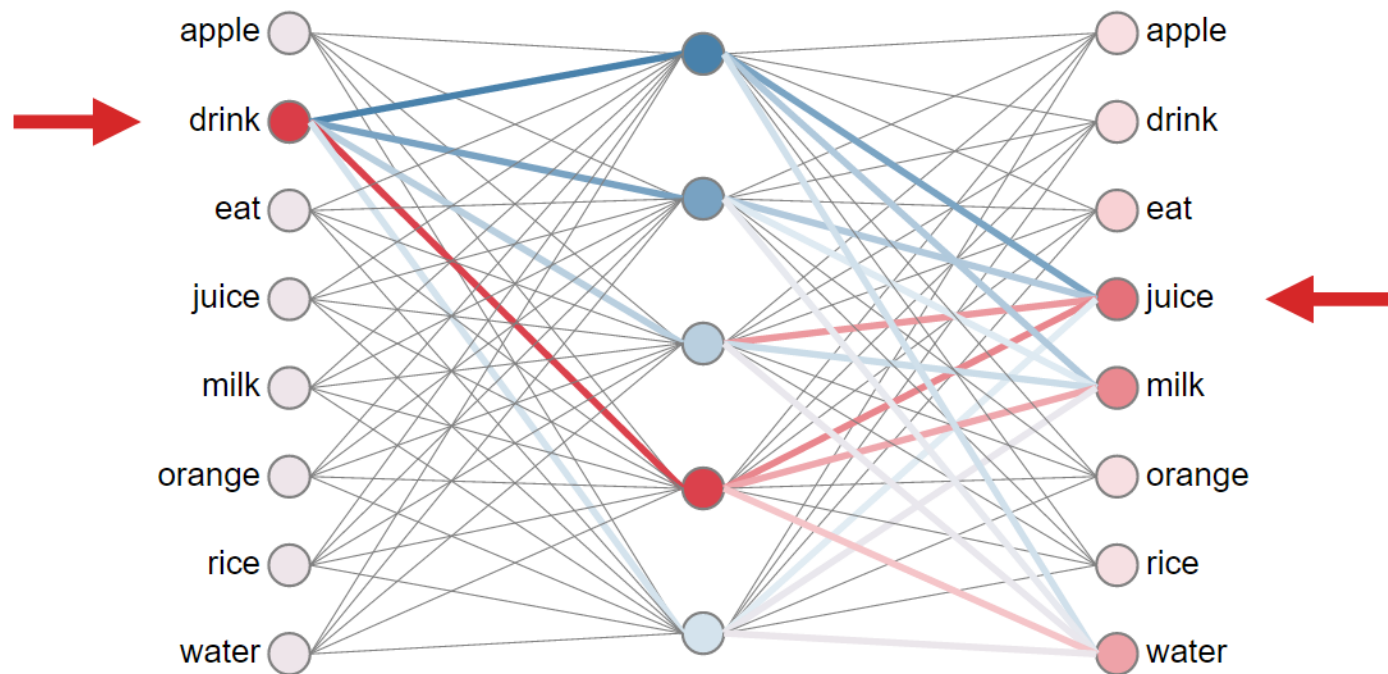
$$p(w_t \mid w_{t-m}, \dots, w_{t-1}, w_{t+1}, \dots, w_{t+m})$$



Word2Vec LM

Goal: predicting the next words given the proceeding contexts

$$p(w_{t+1} \mid w_t)$$



Comparison

Count-based

- Example
 - LSA, HAL (Lund & Burgess), COALS (Rohde et al), Hellinger-PCA (Lebret & Collobert)
- Pros
 - ✓ Fast training
 - ✓ Efficient usage of statistics
- Cons
 - ✓ Primarily used to capture word similarity
 - ✓ Disproportionate importance given to large counts

Direct prediction

- Example
 - NNLM, HLBL, RNN, Skipgram/CBOW, (Bengio et al; Collobert & Weston; Huang et al; Mnih & Hinton; Mikolov et al; Mnih & Kavukcuoglu)
- Pros
 - ✓ Generate improved performance on other tasks
 - ✓ Capture complex patterns beyond word similarity
- Cons
 - ✓ Benefits mainly from large corpus
 - ✓ Inefficient usage of statistics

Combining the benefits from both worlds → GloVe

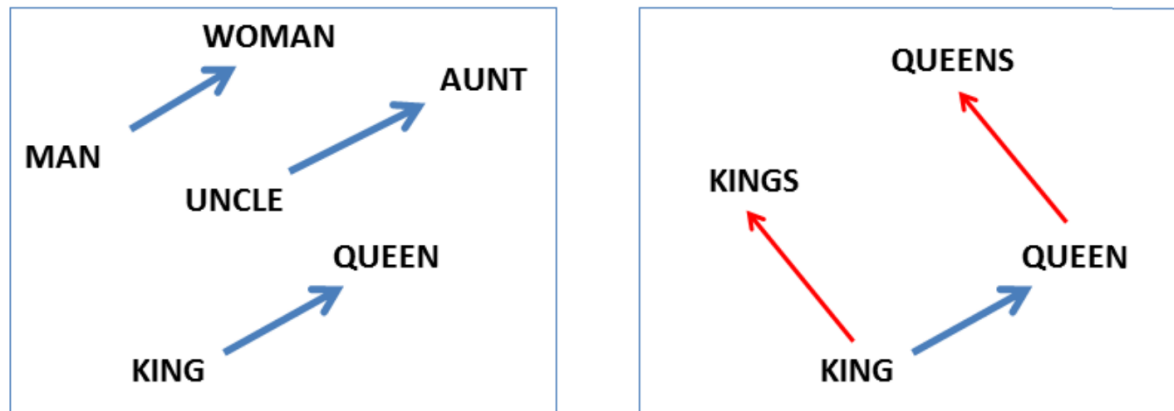
Word Vector Evaluation

Intrinsic Evaluation – Word Analogies

Word linear relationship $w_A : w_B = w_C : w_x$

$$x = \arg \max_x \frac{(v_{w_B} - v_{w_A} + v_{w_C})^T v_{w_x}}{\|v_{w_B} - v_{w_A} + v_{w_C}\|}$$

Syntactic and Semantic



Issue: what if the information is there but not linear

Intrinsic Evaluation – Word Analogies

Word linear relationship $w_A : w_B = w_C : w_x$

Syntactic and **Semantic** example questions [[link](#)]

city---in---state

Chicago : Illinois = Houston : Texas
Chicago : Illinois = Philadelphia : Pennsylvania
Chicago : Illinois = Phoenix : Arizona
Chicago : Illinois = Dallas : Texas
Chicago : Illinois = Jacksonville : Florida
Chicago : Illinois = Indianapolis : Indiana
Chicago : Illinois = Austin : Texas
Chicago : Illinois = Detroit : Michigan
Chicago : Illinois = Memphis : Tennessee
Chicago : Illinois = Boston : Massachusetts

capital---country

Abuja : Nigeria = Accra : Ghana
Abuja : Nigeria = Algiers : Algeria
Abuja : Nigeria = Amman : Jordan
Abuja : Nigeria = Ankara : Turkey
Abuja : Nigeria = Antananarivo : Madagascar
Abuja : Nigeria = Apia : Samoa
Abuja : Nigeria = Ashgabat : Turkmenistan
Abuja : Nigeria = Asmara : Eritrea
Abuja : Nigeria = Astana : Kazakhstan

Issue: different cities may have same name

Issue: can change with time

Intrinsic Evaluation – Word Analogies

Word linear relationship $w_A : w_B = w_C : w_x$

Syntactic and Semantic example questions [[link](#)]

superlative

bad : worst = big : biggest
bad : worst = bright : brightest
bad : worst = cold : coldest
bad : worst = cool : coolest
bad : worst = dark : darkest
bad : worst = easy : easiest
bad : worst = fast : fastest
bad : worst = good : best
bad : worst = great : greatest

past tense

dancing : danced = decreasing : decreased
dancing : danced = describing : described
dancing : danced = enhancing : enhanced
dancing : danced = falling : fell
dancing : danced = feeding : fed
dancing : danced = flying : flew
dancing : danced = generating : generated
dancing : danced = going : went
dancing : danced = hiding : hid
dancing : danced = hiding : hit

Intrinsic Evaluation – Word Correlation

Comparing word correlation with human-judged scores

Human-judged word correlation

Word 1	Word 2	Human-Judged Score
tiger	cat	7.35
tiger	tiger	10.00
book	paper	7.46
computer	internet	7.58
plane	car	5.77
professor	doctor	6.62
stock	phone	1.62

Ambiguity: synonym or same word with different POSs

Extrinsic Evaluation – Subsequent Task

Goal: use word vectors in neural net models built for subsequent tasks

Benefit

- Ability to also classify words accurately
 - Ex. countries cluster together a classifying location words should be possible with word vectors
- Incorporate any information into them other tasks
 - Ex. project sentiment into words to find most positive/negative words in corpus

Softmax & Cross-Entropy

Revisit Word Embedding Training

Goal: estimating vector representations s.t.

$$p(w_j = w_{O,c} \mid w_I) = y_{jc} = \frac{\exp(s_{jc})}{\sum_{j'=1}^V \exp(s_{j'})}$$

Softmax classification on x to obtain the probability for class y

- Definition

$$p(y \mid x) = \frac{\exp(W_y x)}{\sum_{c=1}^C \exp(W_c x)}$$

Softmax Classification

Softmax classification on x to obtain the probability for class y

- Definition

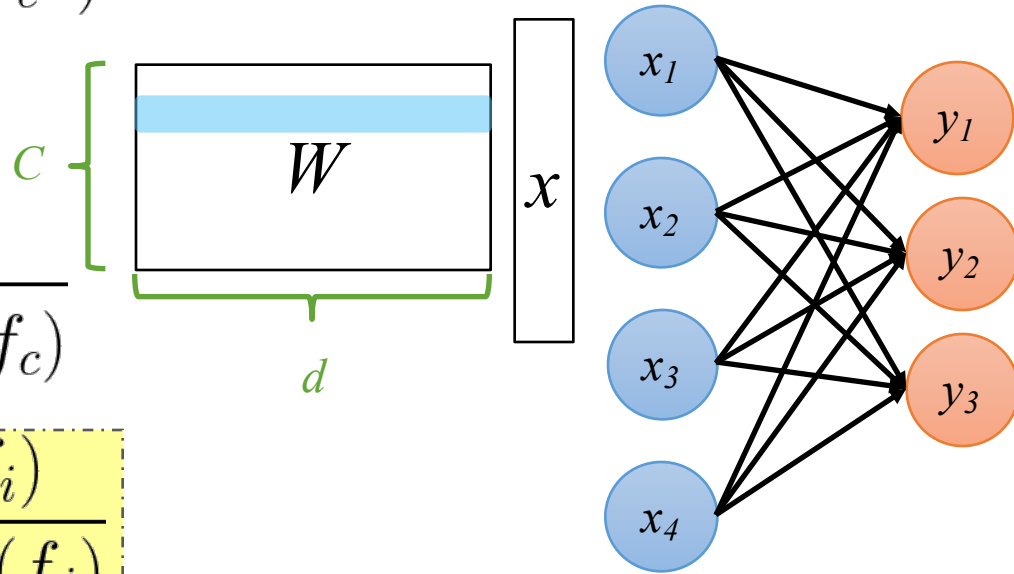
$$p(y \mid x) = \frac{\exp(W_y x)}{\sum_{c=1}^C \exp(W_c x)}$$

$W \in \mathbb{R}^{C \times d}$ usually $C > 2$
(multi-class classification)

$$W_y x = \sum_{i=1}^d W_{yi} x_i = f_y$$

$$p(y \mid x) = \frac{\exp(f_y)}{\sum_{c=1}^C \exp(f_c)}$$

$$\text{softmax}(f)_i = \frac{\exp(f_i)}{\sum_j \exp(f_j)}$$



Loss of Softmax

Objective function

$$O(\theta) = \text{softmax}(f)_i = \frac{\exp(f_i)}{\sum_j \exp(f_j)}$$

Loss function

$$C(\theta) = -\log \text{softmax}(f)_i = -f_i + \underbrace{\log \sum_j \exp(f_j)}_{\approx \max_j f_j}$$

- If the correct answer already has the largest input to the softmax, then the first term and the second term will roughly cancel
- the correct sample contributes little to the overall cost, which will be dominated by other examples not yet correctly classified

Softmax function always strongly penalizes the most active incorrect prediction

Cross Entropy Loss

Cross entropy of target and predicted probability distribution

- Definition

$$H(p, q) = - \sum_i p_i \log q_i$$

p : target one-hot vector
 q : predicted probability distribution

- Re-written as the entropy and Kullback-Leibler divergence

$$H(p, q) = H(p) + D_{KL}(p \parallel q) \quad D_{KL}(p \parallel q) = \sum_i p_i \log \frac{p_i}{q_i}$$

- KL divergence is not a distance but a non-symmetric measure of the difference between p and q p : target one-hot vector

cross entropy loss

$$D_{KL}(p \parallel q) = \log \frac{1}{q_i} = -\log q_i$$

loss for softmax

$$-\log \text{softmax}(f)_i = -\log \frac{\exp(f_i)}{\sum_j \exp(f_j)} = -\log q_i$$

cross entropy loss = loss for softmax

Concluding Remarks

Low dimensional word vector

- Word2vec: skip-gram and cbow
- GloVe: combining count-based and direct learning

Word vector evaluation

- Intrinsic: word analogy, word correlation
- Extrinsic: subsequent task

Softmax loss = cross-entropy loss

Research paper presentations

Conner : Efficient Non-parametric Estimation of Multiple Embeddings per Word in Vector Space, Neelakantan et al., EMNLP 2014

Sanjana : Glove: Global Vectors for Word Representation, J Pennington, R Socher, CD Manning - EMNLP, 2014

Wenhu : AutoExtend: Extending Word Embeddings to Embeddings for Synsets and Lexemes, Rothe and Schutze, ACL 2015

Research paper presentation rules

Presentations must be within 12mins (Hard Stop).

- Or an alarm will sound.

QA session (3mins): each discussant asks one question first, and open the floor to general audience if time permits.