Consumption Reallocation and the Natural Rate of Interest

Jung Hyun Kim

ECON281-Computational

June, 2024

1 Introduction

Understanding the decline in the natural rate of interest has long been a central focus of economic research. Several factors contributing to this downward trend include secular stagnation, characterized by low growth and investment (Summers, 2015; Eggertsson et al., 2019), increased demand and premiums for safe assets (Negro et al., 2017), and the intertemporal shifting channel of monetary policy (McKay and Wieland, 2021).

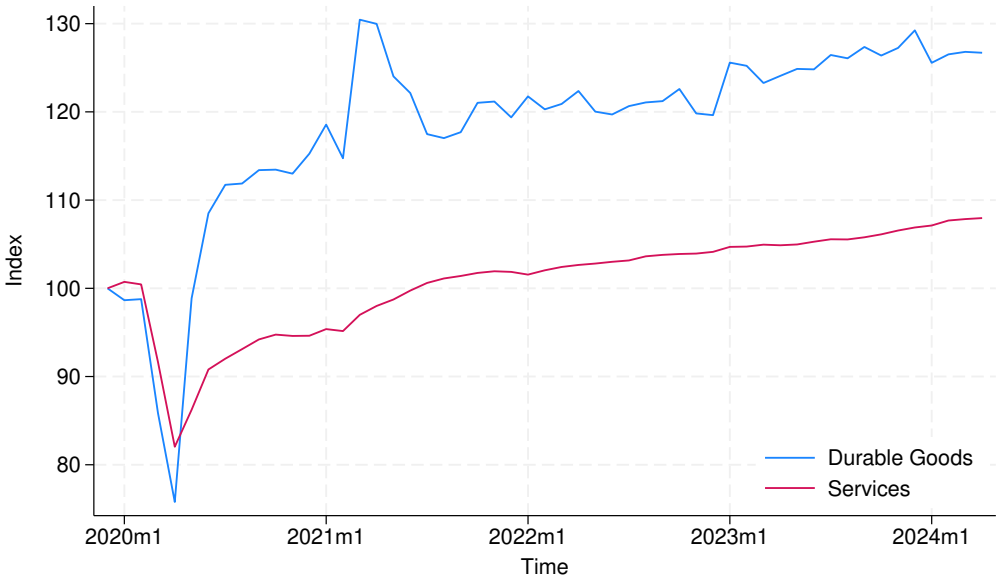
However, the post-COVID-19 era has marked a noticeable shift in this trend. The natural rate of interest in many advanced economies has started to increase (Benigno et al., 2024). At the same time, these economies also experienced a persistent shift from service expenditure to durable expenditure, primarily due to the shift to remote work (Mondragon and Wieland, 2022). As shown in Figure 1, consumption expenditure on durable goods increased sharply following the onset of the pandemic and has continued to grow. In contrast, consumption expenditure on services has recovered and grown more slowly.

This paper investigates the potential impacts of the permanent shift towards durable expenditure on the natural rate of interest. To this end, we build a two-sector RBC model with convex durable adjustment costs. We calibrate a permanent shock to the relative preference for durable goods to match the ratio of durable expenditure to service expenditure in the U.S., before and after the COVID-19 pandemic. We consider the transition path of the economy in response to this permanent shift towards durable goods. In a model with no adjustment cost in durable expenditure, a permanent shift in preferences towards durables causes a persistent increase in the real interest rate.[1] Higher relative demand for durable goods requires a reallocation of labor from the non-durable sector to the durable sector. This in turn requires non-durable consumption to fall to clear the goods market, which is induced by the increase in the real interest rate. With convex adjustment costs, however, the increase in the real interest rate is more muted. The adjustment cost induces households to smooth out the accumulation process of durable goods, reducing the initial increase in durable expenditure. This requires a smaller initial decrease in non-durable consumption and thus a smaller increase in the real interest rate.

The rest of the paper is structured as follows. Section 2 presents the baseline model. Section 3 analyzes the effect of the reallocation shock on the natural rate of interest. Section 4 concludes.

[1] Since there are no nominal rigidities in this economy, the real interest rate is the natural rate of interest.

Figure 1: Real Personal Consumption Expenditure (Base 100 = December 2019)



Notes: Real personal consumption expenditures on durable goods and services, normalized to a base value of 100 in December 2019. Source: FRED database.

2 A Two-sector RBC Model with Consumption Reallocation

The economy consists of two sectors, a durable good sector and a non-durable good sector.

2.1 Households

A representative household maximizes the present discounted value of lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \nu \frac{D_t^{1-\gamma}}{1-\gamma} + \chi \frac{{N_t}^{1+\psi}}{1+\psi} \right]$$

subject to the budget constraint:

$$C_t + P_t^x X_t + A_t = W_t N_t + (1 + r_{t-1}) A_{t-1} + \Phi_t^x + \Phi_t^c$$

and the law of motion for durable good:

$$D_t = (1-\delta) D_{t-1} + \phi \left(\frac{X_t}{D_{t-1}} \right) D_{t-1}$$

where β is the discount factor, C_t denotes the consumption of non-durable goods, D_t is the stock of durable goods, X_t is the durable expenditure, and N_t is household's labor supply. σ and γ are the inverses of the intertemporal elasticity of substitution for non-durable and durable consumption, respectively. ν determines the preference for the durable good, relative to the non-durable good. An increase in ν will capture a permanent shift in preferences

towards durable goods. χ is the labor disutility parameter. ψ is the inverse of the Frisch elasticity of labor supply. A_{t-1} is the holdings of assets at the beginning of time t that pay an interest $1 + r_{t-1}$. Φ_t^x and Φ_t^c are the profits from the durable and non-durable good producers, respectively. Lastly, $\phi\left(\frac{X_t}{D_{t-1}}\right) D_{t-1}$ captures the convex durable adjustment cost, where $\phi' > 0$, $\phi'' \leq 0$, $\phi'(\delta) = 1$, and $\phi(\delta) = \delta$.

2.2 Producers

A representative firm produces non-durable goods according to the production technology:

$$Y_t^c = Z_t N_t^c$$

Similarly, a representative durable good producer has output technology:

$$Y_t^x = Z_t N_t^x$$

Technology Z_t is common to all firms. Labor is perfectly mobile, so both producers will face a wage rate that is common across sectors. Subject to their technology, the non-durable good producer and the durable good producer maximize their profit :

$$\Phi_t^c = Y_t^c - W_t N_t^c$$
$$\Phi_t^x = P_t^x Y_t^x - W_t N_t^x$$

2.3 Market Clearing

The market clearing conditions for durable and non-durable goods are given by:

$$Y_t^x = X_t$$
$$Y_t^c = C_t$$

The labor market clearing condition is:

$$N_t^x + N_t^c = N_t$$

2.4 Solution Method and Calibration

After solving the equilibrium conditions, we log-linearize the model around the new steady state and solve for the transition path in response to a permanent shift in preferences towards durable goods. The sequence-space Jacobian method is used to compute the transition path.

Log-linearization For variable X, denote $\hat{x} \equiv \frac{X - X^{ss}}{X^{ss}}$. Denote the gross inflation rate of durable expenditure as $\Pi_t^x \equiv \frac{P_t^x}{P_{t-1}^x}$, so $\hat{\pi}_t^x$ is the (net) inflation rate of durable expenditure.

Also, denote the real shadow value of durable good as Q_t, so $\hat{q}_t$ is its log-linearized counterpart. $\eta \equiv -\frac{1}{\phi''(\delta)\delta}$ is the elasticity of the expenditure-stock ratio of the durable good with respect to its real shadow value. The log-linearized system of equilibrium conditions are given by:

Household block:

$$\begin{aligned}
\hat{c}_t &= \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma}\hat{r}_t \\
\hat{w}_t &= \psi \hat{n}_t + \sigma \hat{c}_t \\
\hat{q}_t &= \eta \left[\hat{x}_t - \hat{d}_{t-1}\right] \\
\hat{q}_t &= \beta \mathbb{E}_t \hat{q}_{t+1} + (1 - \beta(1-\delta)) \left[\sigma \hat{c}_t - \gamma \hat{d}_t - \hat{p}^x_{t+1}\right] + \mathbb{E}_t \hat{\pi}^x_{t+1} - \beta(1-\delta)\hat{r}_t \\
\hat{d}_t &= (1-\delta)\hat{d}_{t-1} + \delta \hat{x}_t
\end{aligned}$$

Firm block:[2]

$$\begin{aligned}
\hat{y}^x_t &= \hat{z}_t + \hat{n}^x_t \\
\hat{y}^c_t &= \hat{z}_t + \hat{n}^c_t \\
\hat{w}_t &= \hat{z}_t \\
\hat{p}^x_t &= 0
\end{aligned}$$

Market clearing:

$$\begin{aligned}
\hat{y}^x_t &= \hat{x}_t \\
\hat{y}^c_t &= \hat{c}_t \\
\hat{n}_t &= \frac{N^{x,ss}}{N^{ss}}\hat{n}^x_t + \frac{N^{c,ss}}{N^{ss}}\hat{n}^c_t
\end{aligned}$$

Calibration We calibrate the relative preference between durable and non-durable goods (ν) to match the ratio between durable and service expenditure in the U.S. The expenditure ratio ($\frac{X}{C}$) in 2019 was 0.154, and after the COVID-19 pandemic, this ratio has increased to 0.185 as of 2022 (U.S. Bureau of Economic Analysis, 2024). This implies an increase in the relative preference from $\nu = 0.242$ in 2019 to $\nu = 0.291$ in 2022. We use standard calibration values for the rest of the parameters. We set $\eta = 0$ to consider the case without convex adjustment cost first. Table 1 summarizes the baseline parameter values that are used to solve the transition path.

[2] $\hat{p}^x_t = 0$ since the relative price of durable good $P^x_t = 1$ in equilibrium. This is due to the constant returns to scale (CRS) assumption on the output technology and the assumption of perfectly mobile labor. The first-order condition for the non-durable good producer implies that the wage is equal to the technology. The first-order condition and the zero-profit condition for the durable good producer imply that $P^x_t = 1$.

Table 1: Baseline Calibration

Parameter	Value	Description
β	0.99	Discount factor
σ	1	Inverse EIS of non-durable consumption
γ	1	Inverse EIS of durable consumption
χ	1	Labor disutility parameter
ψ	1	Inverse of Frisch elasticity of labor supply
δ	0.017	Depreciation rate of durable stock
η	0	Elasticity of durable expenditure-stock ratio w.r.t. shadow value
ν_{2019}	0.242	Relative preference between durables and non-durables (2019)
ν_{2022}	0.291	Relative preference between durables and non-durables (2022)

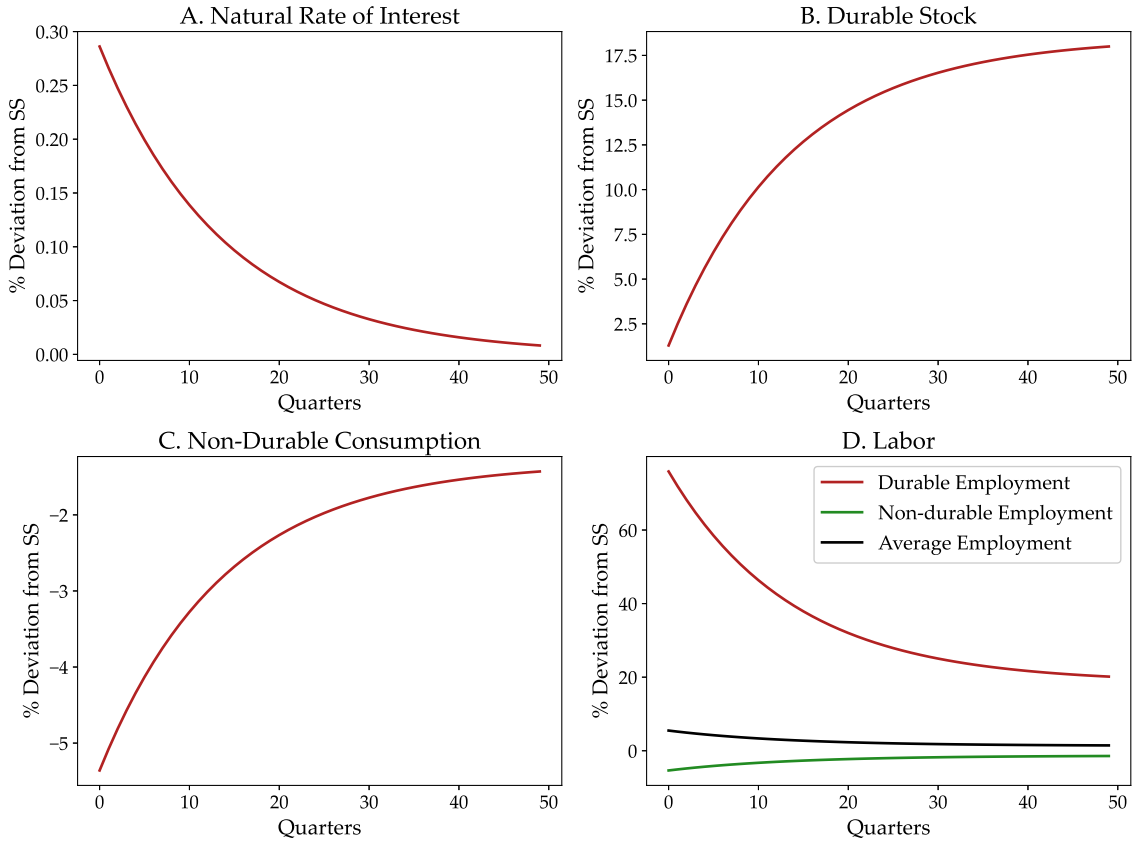
Note: The relative preference between durable and non-durable goods (ν) is calibrated to match the ratio between durable and service expenditure in the U.S. The expenditure ratio ($\frac{X}{C}$) in 2019 was 0.154, and after the COVID-19 pandemic, this ratio has increased to 0.185 as of 2022 (U.S. Bureau of Economic Analysis, 2024). This implies an increase in the relative preference from $\nu_{2019} = 0.242$ to $\nu_{2022} = 0.291$. Standard calibration values are used for the rest of the parameters.

3 Effect of Consumption Reallocation on the Natural Rate of Interest

Figure 2 plots the transition path in response to a permanent increase in durable good preference under an economy with no durable adjustment cost. The y-axis denotes the percent deviation from the old steady state. Panel A of Figure 2 shows that the permanent reallocation shock increases the natural rate of interest rate by 0.27%, and the natural rate of interest slowly returns to the old steady state. Panel B of Figure 2 shows that durable stock accumulates gradually and converges to the new steady state, which is 18% higher than the previous steady state. The increase in demand for durable goods requires more labor in the durable goods sector. As shown in Panel D of Figure 2, higher durable sector employment is supported by both an increase in aggregate employment and a reallocation of workers from the non-durable sector to the durable sector.[3] The disutility in labor prevents aggregate employment from increasing one-to-one with durable sector employment. Panel C of Figure 2 shows that non-durable consumption initially drops by 5.4%, and due to the household's consumption-smoothing motive, non-durable consumption slowly returns to the new steady state with a permanent decrease of 1.31% relative to the old steady state. The natural rate of interest rises and slowly returns to the new steady state to induce this persistent decrease in non-durable consumption.

[3]Figure A.1 in the Appendix plots the transition path under an economy with inelastic labor supply. Because households inelastically supply a unit of labor endowment in this economy, the increase in labor demand in the durable goods sector is solely supported by the shift of workers from the non-durable sector to the durable sector. As shown in Panels C and D, both consumption and employment in the non-durable sector decrease more than the consumption and employment in the economy with elastic labor supply.

Figure 2: Transition Path of Consumption Reallocation (No adjustment cost)

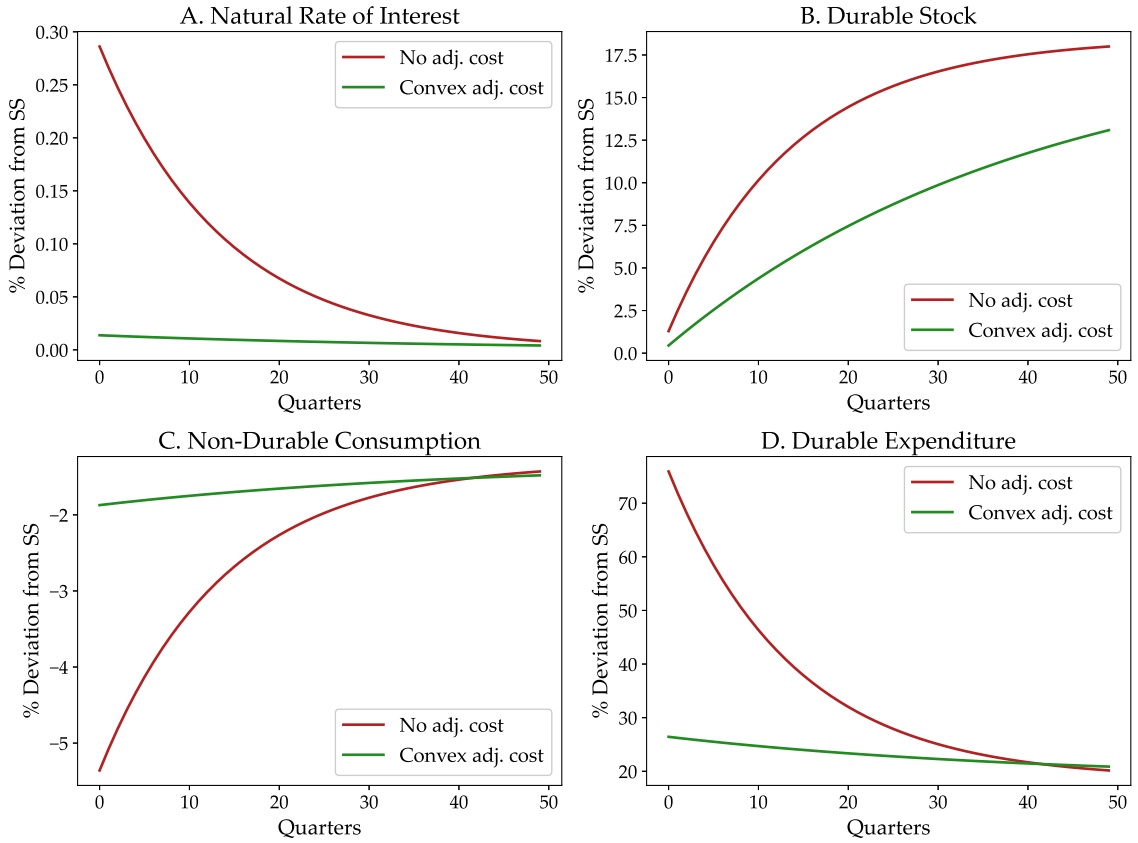


Notes. Transition path of the economy in response to a permanent increase in the preferences for durable goods (from $\nu = 0.242$ to $\nu = 0.291$). The y-axis denotes the percent deviation from the old steady state. Average employment is the weighted average between durable and non-durable employment, weighted by their respective steady-state levels.

Figure 3 compares the transition path between economies with ($\eta = 0.5$) and without ($\eta = 0$) convex durable adjustment cost. Panel B shows that durable stock accumulates more slowly in the case of convex adjustment costs. Although durable stock converges to the same steady state, the convex adjustment cost induces households to avoid abrupt changes in durable expenditure. Instead, households gradually smooth their durable spending over time, as shown in Panel D. This can be supported by a smaller drop in consumption during the initial phase of the transition and thus a smaller increase in the natural rate of interest.

The models with and without convex durable adjustment cost both show that a permanent increase in the share of durable expenditure causes a persistent increase in the natural rate of interest. However, the magnitude of the response of the real interest rate is very sensitive to the elasticity of durable expenditure-stock ratio with respect to its shadow price (η), which determines the convexity of the durable adjustment cost. Future work requires an empirically reasonable value of this elasticity.

Figure 3: Transition Path (No adjustment cost vs. Adjustment Cost)



Notes. Transition path of the economy in response to a permanent increase in the preferences for durable goods (from $\nu = 0.242$ to $\nu = 0.291$). The red line represents the economy with no adjustment cost in durable expenditure, and the green line represents the economy with a convex durable adjustment cost. The y-axis denotes the percent deviation from the old steady state.

4 Conclusion

In light of the recent reversal in the downward trend of the natural rate of interest in several advanced economies, this paper examines whether the shift towards durable expenditure contributed to this phenomenon. Using a two-sector RBC model, we show that a permanent increase in the preferences for durable goods causes a persistent rise in the natural rate of interest. Because aggregate labor supply is limited (either due to fixed labor or disutility in labor), a reallocation in the labor market occurs, shifting workers from the non-durable sector to the durable sector. As the aggregate supply of non-durable goods falls short of its aggregate demand, the real interest rate rises to clear the market for non-durable goods. Although models with and without convex adjustment cost both imply a persistent rise in the natural rate of interest, the magnitude of the transition path is extremely sensitive to the convexity of the durable adjustment cost.

Several features need to be included in the model for future work. First, the CRS assumption on the output technology needs to be changed to decreasing returns to scale

technology (DRS) to capture the increase in the relative price for the durable good. Mondragon and Wieland (2022) document that the shift to remote work during COVID-19 has increased the demand and price for housing. Figure A.2 also shows that the CPI inflation of durable goods was higher than that of services until the third quarter of 2022. A DRS technology in output will be able to generate an increase in the prices of durable goods. Second, calibrating an empirically plausible shadow price elasticity of durable expenditure-stock ratio and incorporating a fixed cost in durable expenditure are both essential to match the micro-level patterns of durable consumption. Lastly, including a monetary policy block is necessary to examine how the central bank's contractionary policy after COVID-19 may have further increased the persistence in the path of the natural rate of interest (McKay and Wieland, 2021).

References

Benigno, Gianluca, Boris Hofmann, Galo Nuño, and Damiano Sandri, "Quo vadis, r*? The natural rate of interest after the pandemic," *BIS Quarterly Review*, March 2024.

Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins, "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, January 2019, *11* (1), 1–48.

McKay, Alisdair and Johannes F. Wieland, "Lumpy Durable Consumption Demand and the Limited Ammunition of Monetary Policy," *Econometrica*, 2021, *89* (6), 2717–2749.

Mondragon, John A and Johannes Wieland, "Housing Demand and Remote Work," Working Paper 30041, National Bureau of Economic Research May 2022.

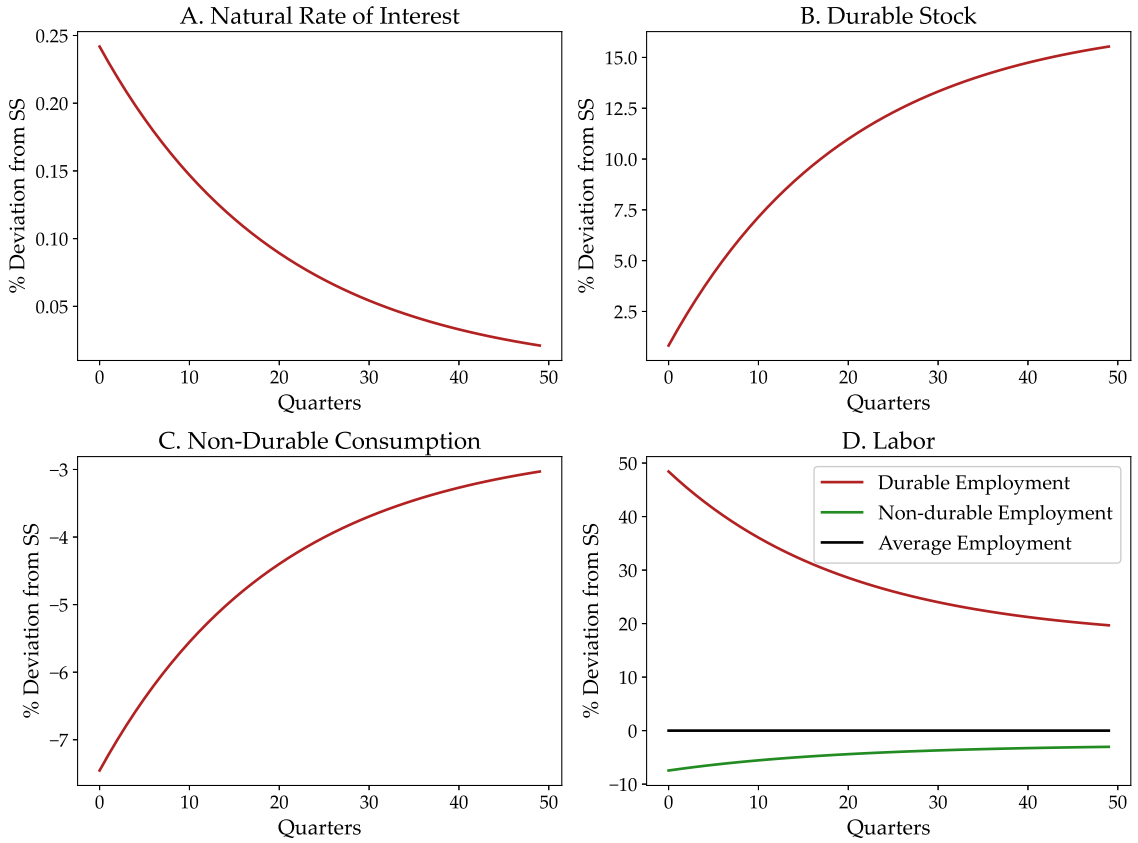
Negro, Marco Del, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti, "Safety, Liquidity, and the Natural Rate of Interest," *Brookings Papers on Economic Activity*, 2017, *2017*, 235–316.

Summers, Lawrence H., "Demand Side Secular Stagnation," *American Economic Review*, May 2015, *105* (5), 60–65.

U.S. Bureau of Economic Analysis, "Table 2.3.5U. Personal Consumption Expenditures by Major Type of Product and by Major Function," 2024. Accessed: Thursday, June 10, 2024.

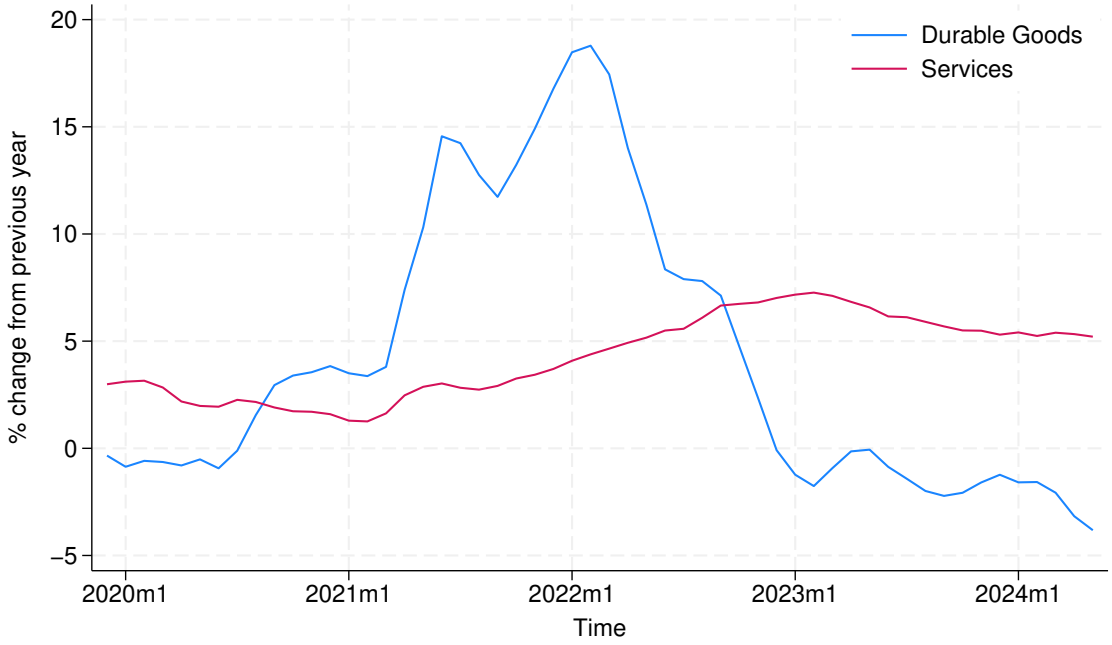
A Appendix

Figure A.1: Transition Path under Inelastic Labor Supply (No adjustment cost)



Notes. Transition path of the economy with no adjustment cost and inelastic labor supply in response to a permanent increase in the preferences for durable goods (from $\nu = 0.242$ to $\nu = 0.291$). The y-axis denotes the percent deviation from the old steady state. Average employment is the weighted average between durable and non-durable employment, weighted by their respective steady-state levels.

Figure A.2: Consumption Price Index (% Change from Previous Year)



Notes: CPI inflation of durable goods and services (less energy services) in U.S. city average. Source: FRED database.