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Time-variant Reliability Analysis of Ship Structure Subjected to Fatigue and Corrosion

Shan Wang[1,a], Xinghua Shi[1,b], Jian He[1,c]

[1]College of Civil Engineering, Harbin Engineering University, Harbin 150001, P.R. China.

[a]wangshan@hrbeu.edu.cn, [b]shixinghua1@hrbeu.edu.cn, [c]hejian@hrbeu.edu.cn,

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Abstract: Based on the characteristics of loading condition of ship structure in service and the degeneration of the strength under fatigue and corrosion, the random time-variant model of ship hull section modulus was set up using the theory of random process. Then the time-variant reliability was analyzed with the method of up-crossing analysis and parallel system reliability. This method could not only present successive decrease of reliability of the ship structure under fatigue and corrosion, but also avoid the complicated numerical integration. It could be seen that this method was easy to achieve. Furthermore, the time-variant reliability was compared with the annual instantaneous reliability through an example, which indicated that this method was more precise, and could provide some references to the decision-making of the reliability, maintenance and safeguard.

Introduction

Guedes Soares[1] used the up-crossing method to analyze the time-dependent reliability of the ship subjected to fatigue and corrosion, who solved the problem of the reliability changed with time continuously. But this method must come down to complicated numerical integration. In this paper, the random time-variant model of ship hull section modulus was firstly set up using the theory of random process, then using the parallel reliability method (in literature[2]) the reliability could be obtained through the up-crossing analysis, which avoided the numerical integration and was easy to be calculated.

1 Loads analysis

1.1 Stillwater bending moment

The mean value of stillwater bending moment could be 60% of the maximum value that the criterion allowed, and the coefficient of variance could take 0.15. The maximum stillwater bending moment $M_{s,0}$ could be written[3]:

$$M_{s,0} = \begin{cases} -0.065C_w L^2 B(C_B + 0.7) & \text{Sagging} \\ C_w L^2 B\left(0.1225 - 0.015C_B\right) & \text{Hogging} \end{cases} \tag{1}$$

Where L was the length, B was the width, C_B was the block coefficient, C_w was the wave coefficient which could be written:

$$C_w = \begin{cases} 10.75 - \left((300-L)/100\right)^{3/2}, 100 < L \le 300 \\ 10.75, \quad 300 < L \le 350 \\ 10.75 - \left((L-350)/150\right)^{3/2}, L > 350 \end{cases} \tag{2}$$

1.2 Wave bending moment

According to extremum analysis, the PDF of wave bending moment in the lifetime T was[4]:

$$F_{M_w} = \exp\left\{-\exp\left[-\alpha_w\left(M_w - \beta_w\right)\right]\right\}, \alpha_w = \frac{\ln\left(v_w T_0\right)}{M_{w,0}}, \beta_w = M_{w,0}\frac{\ln\left(v_w T\right)}{\ln\left(v_w T_0\right)} \tag{3}$$

Where v_w was the mean wave arrival rate, $M_{w,0}$ was the maxima wave bending moment, written as:

$$M_{w,0} = \begin{cases} -0.11C_w L^2 B\left(C_B + 0.7\right) & \text{Sagging} \\ 0.19C_w L^2 BC_B & \text{Hogging} \end{cases} \tag{4}$$

1.3 Combination of loads

To practical consideration, the combination factor φ_w was introduced, and the total longitudinal bending moment M could be written:

$$M = M_s + \varphi_w M_w \tag{5}$$

Where φ_w was the load reduction factor, which could be calculated using the method of literature [5]:

$$\varphi_w = \frac{0.83M_{w,T} - 0.17M_{s,T}}{M_{w,T}} \tag{6}$$

Where $M_{w,T}$ and $M_{s,T}$ were the maximum values of wave bending moment and stillwater bending moment in the period of T .

2 Time-variant model of ship section modulus

2.1 Fatigue model

The growth of cracks could be described using Paris-Erdogan equation:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^m \quad , \quad \Delta K = \Delta\sigma Y(a)\sqrt{\pi a} \tag{7}$$

Where a was the crack size, N was the number of cycles, ΔK was the stress range intensity factor, C and m were material coefficients, $\Delta\sigma$ was the stress range, $Y(a)$ was the geometry function.

If $Y(a) = Y$ was a constant and $N = \upsilon_0 t$, the fracture size could be written:

$$\begin{cases} a(t) = \left[a_0^{1-\frac{m}{2}} + \left(1 - \frac{m}{2}\right) C\Delta\sigma^m Y^m \pi^{\frac{m}{2}} \upsilon_0 t \right]^{\frac{1}{1-\frac{m}{2}}}, m \neq 2 \\ a(t) = a_0 \exp\left(CY^2 \Delta\sigma^2 \pi \upsilon_0 t\right), m = 2 \end{cases} \tag{8}$$

Where a_0 was the initial crack size, υ_0 was the mean rate of stress cycles, which was dependent on the wave, so $\upsilon_0 = \upsilon_w$.

The crack size was given by Eq. 8, which depended on parameters such as initial crack size, the stress range, the crack geometry, the material constants of the crack growth law and the number of cycles. Assume that these variables were uncorrelated with each other, the mean value and variance of the crack size could be written[1]:

$$E\left[a(t)\right] = \left[\left\{E\left[a_0\right]\right\}^{1-\frac{m}{2}} + \left(1 - \frac{m}{2}\right) E\left[C\right] E\left[\Delta\sigma^m\right] Y^m \pi^{\frac{m}{2}} \upsilon_0 t \right]^{\frac{1}{1-\frac{m}{2}}} \tag{9}$$

$$\begin{aligned} \sigma_{a(t)}^2 = & \left[\frac{\left\{E\left[a_0\right]\right\}^{-\frac{m}{2}} a(t)}{\left\{E\left[a_0\right]\right\}^{1-\frac{m}{2}} + E\left[C\right] E\left[\Delta\sigma^m\right] Y^m \pi^{\frac{m}{2}} \upsilon_0 t} \right]^2 \sigma_{a_0}^2 + \left[\frac{\left\{E\left[a_0\right]\right\}^{-\frac{m}{2}} a(t)}{\left\{E\left[a_0\right]\right\}^{1-\frac{m}{2}} + E\left[C\right] E\left[\Delta\sigma^m\right] Y^m \pi^{\frac{m}{2}} \upsilon_0 t} \right]^2 \sigma_C^2 \\ & + \left[\frac{\left\{E\left[a_0\right]\right\}^{-\frac{m}{2}} a(t)}{\left\{E\left[a_0\right]\right\}^{1-\frac{m}{2}} + E\left[C\right] E\left[\Delta\sigma^m\right] Y^m \pi^{\frac{m}{2}} \upsilon_0 t} \right]^2 \sigma_\Delta^2 \end{aligned} \tag{10}$$

2.2 Corrosion model

The general corrosion model was used:

$$r(t) = r_i (t - t_0) \tag{11}$$

Where $r(t)$ was the reduced thickness in the period of t , r_i was the year corrosion rate, t_0 was the coating lifetime that was assumed zero in this paper. According to the published literature, the corrosion rate could be assumed to be the same in the same field.

2.3 Time-variant reliability model of section modulus

Because of fatigue and corrosion, the effective section modulus degenerated. And the time-variant section modulus could be calculated as follows:

1. Disperse the midship section into stiffening elements and plate elements, which was seen in Fig. 1;

2. Calculate the effective areas and section inertia moment of all elements at any time;

3. Using the theory of beam section, calculate the section modulus.

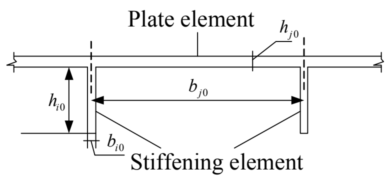

Fig. 1 Elements sketch map

The width of the stiffeners would decrease with time because of corrosion and height would decrease because of fatigue fracture:

$$A_i(t)=(b_{i0}-r_i t)(h_{i0}-a(t)),\ I_i(t)=\frac{(b_{i0}-r_i t)(h_{i0}-a(t))^3}{12} \quad (12)$$

Where $A_i(t)$ and $I_i(t)$ were the area and inertia moment of the i th stiffener, b_{i0} and h_{i0} were the initial width and height where the edge of wing was reduced to width, r_i was year corrosion rate.

Using the same method, the area and inertia of the plate element at t were obtained. Then the section modulus $W(t)$ could be derived.

In structure reliability, the PDF was needed, so according to [1] where the section modulus was modeled by a normal distribution, the mean value function and variance function could be written through the Taylor series of $W(t)$.

From the analysis above, when fatigue and corrosion were considered, the growth of fatigue crack and the reduction of element thickness were the function of time, so the section modulus was continuous function of time. The section modulus must be described by the random process $\{W(t), t\in[0,T]\}$, which had the following characteristics: 1. the mean value function decreased with time; 2. the variance function increased with time.

In order to confirm the autocorrelation coefficient function $\rho[W(t),W(t+\Delta t)]$, the section modulus was assumed to be an independent increments process, then the autocorrelation coefficient function could be written:

$$\mathrm{COV}[W(t),W(t+\Delta t)]=\mathrm{D}[W(t)]=\sigma_W^2(t) \quad (13)$$

Then $$\rho[W(t),W(t+\Delta t)]=\frac{\mathrm{COV}[W(t),W(t+\Delta t)]}{\sqrt{\mathrm{D}(W(t))}\cdot\sqrt{\mathrm{D}(W(t+\Delta t))}}=\frac{\sigma_W(t)}{\sigma_W(t+\Delta t)} \quad (14)$$

3 Reliability analysis

The limit state of hull girder subjected to fatigue and corrosion was:

$$G(t)=\sigma_y W(t)-M=\sigma_y W(t)-M_s-\varphi_w M_w \quad (15)$$

Where σ_y was the yield stress.

Because $G(t)$ was the random process, the up-crossing method was used to reliability analysis. According to [6], the boundary of $P_f(t_1,t_2)$ was:

$$\max_{t_1\le\tau\le t_2}(P_{f,i}(\tau))\le P_f(t_1,t_2)\le P_{f,i}(t_1)+E[N^+(t_1,t_2)] \quad (16)$$

Where $P_{f,i}(\tau)$ was instantaneous failure probability at τ, $E[N^+(t_1,t_2)]$ was number of the mean up crossing $E[N^+(t_1,t_2)]=\int_{t_1}^{t_2}v^+(\tau)d\tau$ (17)

The up-crossing rate $v^+(\tau)$ could be defined:

$$v^+(\tau)=\lim_{\Delta\tau\to 0,\Delta\tau>0}\frac{\mathrm{P}(\{G(\tau)>0\}\cap\{G(\tau+\Delta\tau)\}\le 0)}{\Delta\tau} \quad (18)$$

If the time increment $\Delta\tau$ was small in Eq. 18, which was dispersed using the finite-difference method:

$$v^+(\tau)=\frac{\mathrm{P}(\{G(\tau)>0\}\cap\{G(\tau+\Delta\tau)\}\le 0)}{\Delta\tau} \quad (19)$$

Where $G(\cdot)$ was the limit state function. The value of $\Delta\tau$ was very important. It had to be sufficiently small since it enters a finite-difference definition, but at most one crossing should occur during $\Delta\tau$.

Firstly, after having frozen τ and $\Delta\tau$, the random process $W(t)$ was divided into two random variables W_1 and W_2, whose correlation coefficient was $\rho(W(\tau),W(\tau+\Delta\tau))$. Then the limit state function was linearized at checking point using the second order method:

$$\begin{aligned} G(\tau)&=\overline{\alpha}^T(\tau)\overline{x}+\beta(\tau)\\ G(\tau+\Delta\tau)&=\overline{\alpha}^T(\tau+\Delta\tau)\overline{x}+\beta(\tau+\Delta\tau)\end{aligned} \quad (20)$$

Where $\overline{x}$ was the normalized sector, $\overline{\alpha}^T(\tau)$ and $\overline{\alpha}^T(\tau+\Delta\tau)$ were the unit sector, $\beta(\tau)$ and $\beta(\tau+\Delta\tau)$ were reliability index.

$$
\begin{aligned}
\mathrm{P}(\mathrm{Q}_1 \cap \mathrm{Q}_2) &= \mathrm{P}\left(\left(\bar{\alpha}^T(\tau)\bar{x} + \beta(\tau) > 0\right) \cap \left(\bar{\alpha}^T(\tau+\Delta\tau)\bar{x} + \beta(\tau+\Delta\tau) \le 0\right)\right) \\
&= \mathrm{P}\left(\left(-\bar{\alpha}^T(\tau)\bar{x} < \beta(\tau)\right) \cap \left(\bar{\alpha}^T(\tau+\Delta\tau)\bar{x} \le -\beta(\tau+\Delta\tau)\right)\right) \\
&= \Phi_2\left(\beta(\tau), -\beta(\tau+\Delta\tau), \rho_G\right)
\end{aligned} \tag{21}
$$

Where Q_1 and Q_2 were the event $G(\tau) > 0$ and $G(\tau+\Delta\tau) \le 0$, respectively. ρ_G was the relative coefficient of Q_1 and Q_2, $\rho_G = -\bar{\alpha}(\tau)\cdot\bar{\alpha}(\tau+\Delta\tau)$, $\Phi_2(\cdot)$ was two dimension standard normal function.

Then $P_r(t)$ and $\beta(t)$ were:

$$
P_r(t) = 1 - P_f(t), \beta(t) = -\Phi^{-1}\left(P_f(t)\right) \tag{22}
$$

4 Example

The method presented in this paper had been applied to the reliability analysis of a ship with a length of 130m, width of 19.2m, depth of 8.9m, and a block coefficient of 0.65, the age of 20 year, the mean arrival rate of wave of 16666.7/day. The yield stress σ_y was modeled by a normal distribution with mean of 315MPa and coefficient of variance of 0.06; The initial crack length obeyed normal distribution with mean of 1.5mm and coefficient of variance of 0.1; The fracture coefficient C was modeled as a log normal random variable with mean of 4.349×10^{-12} and coefficient of variance of 0.206; The fracture coefficient m and Y were constant, which were 3and 1, respectively. And mean value of year corrosion rate was different in different areas: 0.25 in deck, 0.2 in board side, 0.2 in bottom, the coefficient of variance was 0.1 in all elements.

The time-variant reliability and instantaneous reliability of the ship structure above using the program of FORTRAN90 that was edited by ourselves. Since the reliability of the structure that was in the beginning of service was very high, the failure probability was 0 at $t=0$. And assume that initial cracks happened in the beginning of service.

The time-variant failure probability of ship structure subjected to fatigue and corrosion was seen in Fig. 2. And the history of time-variant reliability and instantaneous index were seen in Fig. 3.

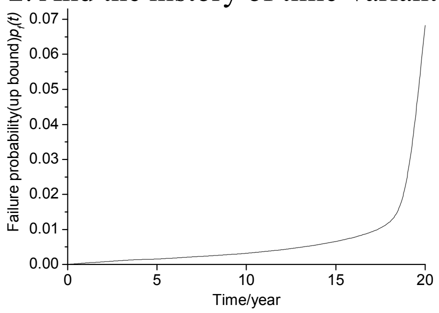

Fig. 2 Time-variant failure probability

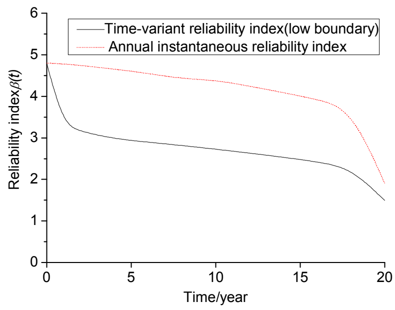

Fig. 3 Instantaneous and time-variant reliability index

From the example above, this method was accurate and easy to be achieved. Moreover, the analysis could indicate the influence of fatigue and corrosion continuously on the reliability.

References

[1] C.Guedes Soares,Y.Garbatov. Reliability of maintained ship hull girders subjected to corrosion and fatigue[J].Structural Safety,1998,20:201-219

[2] C.Andrieu-Renaud,B.Sudret,M.Lemaire. The PHI2 method: a way to compute time-variant reliability[J].Reliability engineering & system safety,2004,84:75-86

[3] IACS Requirement S11,Longitudinal Strength Standards,1995

[4] Guedes Soares C, Ivanov LD. Time-dependent reliability of the primary ship structure[J].Marine Structures,1989,26:59-71

[5] Hai-Hong Sun, Yong Bai. Time-variant reliability assessment of FPSO hull girders[J].Marine Structures,2003,16:219-253

[6] Shinozuka M. Probability of failure under random loading[J].Journal of Engineering Mechanics Division,1964,90(EM5):147-171

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