

A simplified approach in FAVAR estimation

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Abstract

In the field of empirical macroeconomics factor-augmented vector autoregressive (FAVAR) models have become a popular tool in explaining how economic variables interact over time. FAVAR is based upon a data-reduction step using factor estimation, which are then employed in a vector autoregressive model. This paper aims to study alternative methods regarding factor estimation. More precisely, we compare the generally used principal component method with the uncomplicated common correlated effect estimation. Results show low divergence between the two factor estimation methods employed, indicating interchangeability between the two estimation approaches.

Keywords: factor-augmented vector autoregression, principal component analysis, common correlated effects, impulse responses

1. Introduction

When making monetary policy decisions, central banks need to study hundreds of macroeconomic variables and how they change over time. Policymakers investigates how said variables interact with each other in order to make adequate policy decisions. This burdensome task was simplified by the work of Sims (1980), introducing a new methodology denoted as Vector Autoregression or VAR. The method employs a multivariate equation system in estimating time series autoregression, enabling researchers to study interactions between variables within the system. The VAR methodology has become an important tool in the central banks toolbox in studying effects of policy decisions upon macroeconomic variables.

Howbeit, a restriction in VAR modelling is the limitation of variables that can be taken into consideration to avoid the curse of dimensionality (Krolzig, 2003). Specifically, Stock and Watson (2002a) argues that VAR models commonly contain less than 10 variables in order to minimize loss of degrees of freedom. Thus, studying macroeconomic conditions using the relative sparse information contained in VAR models leads to uncertainty in the estimation as noted by Sims (1992). In his paper, Sims argues that the omission of information in the VAR leads to the so-called price puzzle. General economic theory implies that a rise in the interest rate should result in a decrease in price levels. However, when utilizing low-dimensional VAR models the price level has a tendency to increase when followed by an upward monetary policy shock, thus denoted as the price puzzle. Sims (1992) suggests that the underlying reason for this phenomenon is the model's inability to control for the informational edge held by the policy makers regarding future inflation levels.

Thus, the informational shortcomings in the VAR model could probably not be fully explained by individual observable variables, e.g. GDP or Industrial Production could not independently explain "Economic Activity". These limitations to the VAR methodology induced efforts to explore new methods explaining unobservable economic factors in time series data. Hence, factor estimation approaches in time series were introduced by Sargent and Sims (1997) along with Geweke (1997) to accommodate for a vast amount of cross sectional time series data without including a large number of variables in the model. The methodology was later popularised in Stock and Watson (1999) in which the authors forecasted inflation using factor estimation to account for more than hundreds of variables.

This method was further developed in Bernanke et al. (2005), introducing the factor-augmented vector autoregression model, FAVAR. In comparison to the VAR model, the introduced FAVAR model consisted of a two-step principal component estimation and enabled researchers to add variables and include unobservable and observable variables unitedly in the VAR model. Nowadays the FAVAR approach has become frequently applied when dealing with large datasets. The methodology has been applied in numerous studies, helping to estimate how dynamic systems behave over time. Some notable applications are Belke and Rees (2014) studying changes in international transmissions as a result of financial shocks. Dave, Dressler and Chang (2013) employed a FAVAR approach in studying bank lending. Gavin and Kliesen (2008) applied the methodology to study forecasting and argued that the models worked as best when studying longer horizons. Even though the FAVAR methodology has been widely applied in different studies, not many authors have considered the estimation of factors explicitly in the FAVAR methodology.

Bernanke et al. (2005) compared two factor estimation methods in FAVAR, both Principal Component ("PC") and a likelihood-based Gibbs-sampling-method. The PC-approach is based upon the two-stepped methodology facilitated in Stock and Watson (2002b). The other method, commonly denoted as Gibbs-sampling was introduced by Geman and Geman (1984), later discussed as an application to large dynamic factor models in Eliasz (2002). Kose, Otrok and Whiteman (2000, 2003) employed a version of the Gibbs-sampling studying international business cycles. Bernanke et al. (2005) concluded that the computationally more difficult Gibbs-sampling likelihood-based factor estimation approach did not yield significantly better results than the PC analysis approach in factor estimation. This conclusion could help explain the predominantly favouring of PC-approach in factor estimation throughout the immense literature on FAVAR.

Pesaran (2006) introduced a new framework for estimating factors in ordinary time series denoted as Common Correlation Effects or CCE. This computationally simple methodology is based upon utilizing grouped cross-sectional averages as factors. The usage of CCE to estimate factors in FAVAR was first introduced in Berggren (2017), where the author compared both the PC and the CCE estimation method. In the paper Berggren concluded that there were little differences between the methods but noted that it was difficult drawing definitive conclusions since results could be affected by the limited dataset employed.

Similarly to Berggren (2017), this paper will study the differences in factor estimation between PC and CCE in the FAVAR methodology. Howbeit, while Berggren used a somewhat limited Swedish dataset (N=38), we will study the differences using the same dataset as used in Bernanke et al. (2005). Furthermore, Berggren (2017) also included the Gibbs-sampling approach in the comparison. Although, since both Bernanke et al. (2005) and Berggren (2017) concluded that the Gibbs-sampling method did not outperform the PC-approach, in the spirit of simplification we have chosen to disregard the computationally difficult Gibbs-sampling approach in this paper.

This paper consists of five sections, the first is the introduction. Section 2 introduces the underlying econometric frameworks of FAVAR and CCE. In section 3 we describe the data employed in the paper. Section 4 presents the results found, and the paper is summed up in a discussion in section 5.

2. Econometric framework

Introduced by Bernanke et al. (2005), Factor-augmented vector autoregression, or FAVAR, is a method in which a data reduction technique is used to estimate factors that is later employed in a vector autoregression. We assume that Y_t , being a M x 1 vector of observable variables, have an effect on the economy in general. Furthermore, we let the information not captured by Y_t be a K x 1 vector of the unobserved factors, denoted F_t . As assumed in the framework developed by Bernanke et al. (2005) the joint dynamics of (F_t, Y_t) is thus:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \nu_t \tag{1}$$

Equation (1) will be referred to as the factor-augmented vector autoregression model, FAVAR

where $\Phi(L)$ is presented as the conformable lag polynomial with finite order d. v_t denotes the error term, with covariance matrix of Σ and a mean of zero. The unobserved factors used in the VAR model can be perceived as unobservable variables that are indicators of intangible conceptions such as economic activity. Thus, the calculated factors could be interpreted as unobservable elements that cannot be displayed by any observable macroeconomic series, but rather a number of recalculated and consolidated economic indicators. Being hard to explain using isolated variables, such factors could be approximately explained by combining an analysis of several related variables. We employ both a two-step principal component

estimation and a common correlated effect approach in estimating said underlying factors. Due to the fact that factors (F_t) are unobservable, Equation (1) cannot be estimated directly. As a result, we substitute F_t with the estimated form \widehat{F}_t . Nevertheless, we suppose that the large number of economic variables included in the time series can give us an interpretation of the factors using estimation methods. Therefore, we assume that there is a number of economic time series that can be denoted as a N x 1 vector X_t . Further, we assume that the number of time series N is "large" and concede possibility of being larger than the number of time periods T.

Bernanke et al. (2005) continue to make the assumption that the informational time series in X_t are related to the unobservable variables F_t and the observable variables Y_t as stated in the equation below:

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t \tag{2}$$

Where Λ^y is N × M, Λ^f is a N × K matrix of factor loadings, and e_t is the N × 1 vector of error terms, with mean zero and limited cross-correlation. For the estimations of the unobservable factors, \widehat{F}_t , Bernanke et al. (2005) compare two estimation methods. As stated above, the computationally more complex likelihood-based Gibbs-sampling technique did not necessarily yield better results, therefore we will limit this study to compare the two-step PC estimation popularised by Stock and Watson (2002b) to the CCE-approach. Performing two-step PC estimation, the variables are distinguished into two sets of groups; a "slow-moving" and a "fast-moving" group. The difference between these groups is that the slow-moving variables are assumed to be unaffected by contemporary shocks in Y_t . Fast-moving variables are contrarily instantaneously affected of a shock in interest rates. The first step of the two-step PC estimation is to identify the common factors extracted from Y_t and X_t . Turning to the second step, variables included in X_t are divided into slow-moving and fast-moving groups. The division is based upon economic theory, in accordance with Bernanke et al. (2005). Continuing by estimating the slow-moving factors \widehat{F}_t^S as a principal component of the slow-moving variables X_t^S .

$$\hat{C}_t = b_{F^s} \hat{F}_t^s + b_Y Y_t + e_t \tag{3}$$

Thenceforth we estimate the regression above, where \hat{F}_t can be found from the reorganised equation $\hat{C}_t - b_Y Y_t$. The purpose of regressing the factors in Equation (3) is to minimize eventual collinearity between interest rates and the fast-moving factors, since Y_t is thought to

account for information found in the fast-moving variables (Berggren, 2017). Bernanke et al. (2005) also discuss an alternative approach in extracting both a slow and fast-moving factor without using the regression, but since the authors decided that the regression yielded best results we employ the same approach for the sake of replicability. The second step of the PC-estimation is achieved through estimating FAVAR by substituting the unobservable factors, F_t with the estimated form, \hat{F}_t . Using this approach, more information can be incorporated in comparison to a standard VAR model.

The alternative factor estimation method used in this paper is the Common Correlated Effects ("CCE") method developed by Pesaran (2006), later applied in Berggren (2017) to study different approaches when estimating \widehat{F}_t . The CCE estimation is based upon extracting grouped cross-sectional averages and using these series as factors in the VAR, that is the average value for a large number of grouped time series for every period respectively. This method takes slow-moving variables into account and \widehat{F}_t^s is measured as the cross-sectional average in all the slow-moving variables. Likewise, the common factors \widehat{C}_t is estimated as the cross-sectional average of predetermined groupings of series in X_t . The advantages of CCE lays in the simplicity of calculating averages and it is not necessary to assess the number of unobserved factors. Having in mind the vast datasets often subject to analysis when using FAVAR, it is vital that factor estimation yields consistent estimations given different situations. Luckily, this has been proved in the work of Chudnik and Pesaran (2013a).

3. Data

The dataset used is replicated after the study in Bernanke et al. (2005), in turn an updated version of the data used by Stock and Watson (2002a). The dataset consists of 120 macroeconomic variables that span from January 1959 to August 2001 with 512 observations for each series, originally expressed in the form of both monthly and quarterly data. To deal with the inconsistency in the quarterly data, an Expectation Maximization algorithm is applied as featured in Bernanke and Boivin (2003) to conciliate for deficient observations in the variables that are of quarterly data. Time series has been transformed to induce approximate stationarity. Majority of the transformations has been based upon taking first differences of logarithms. The transformation process is the same employed in Bernanke et al. (2005) and is thoroughly explained in Appendix. Variables have been tested for unit roots and are all standardized with mean zero and unit variance.

Similarly to Bernanke et al. (2005), we assume the federal funds as the monetary policy mechanism. Accordingly, we assume that the federal funds rate is the only factor that is observable and included in Y_t . The observable variables that we aim to study is Industrial Production (IP), Unemployment (UNEMP), Personal Consumption (PCONS), Money stock M2 (FM2) and Consumer Price Index (CPI) which all have been generated from the FAVAR model as in Bernanke et al. (2005). Due to the fact that we treat and assort the federal funds rate as the monetary policy shock, a distinction of "slow-moving" and "fast-moving" variables have had to be made since we assume that all the underlying factors do not react to changes in monetary policy instantaneously. The separation of "slow-moving" and "fast-moving" variables is specified in the Appendix and is done in the same manner as in Bernanke et al. (2005).

When dividing the variables into different factors in CCE-estimation we have intuitively applied economic theory and taken slow-moving variables into account. Firstly we categorised variables in accordance with Bernanke et al. (2005). When constructing the first factor we included the categories containing slow-moving variables as denoted in the Appendix. Moreover, the second factor consists of the categories: *Housing starts sales, Real inventories, Orders and unfilled orders* and *Miscellaneous*. Lastly, the third and last factor consists of: *Stock prices, Exchange rates, Interest rates* and *Money and credit quantity aggregates*.

4. Results

In this section we present the results observed from our comparative analysis. We mainly focus both on explanatory power of variables and factors in the FAVAR, along with the outputs from impulse-response simulations. The impulses are defined as a one standard deviation shock in the Federal Funds Rate ("FFR"). Bernanke et al. (2005) discussed that the optimal number of lags in the FAVAR was found to be 13, although 7 lags produced approximately similar results. Thus, for the sake of similarity, we have chosen to employ 13 lags in our study. As stated previously, in the PC-based FAVAR our aim is to use the same method as in Bernanke et. al. (2005). Thus, we use three factors in the FAVAR.

Since our PC-estimated FAVAR model is a replication of the preferred model specified in Bernanke et al. (2005), we have replicated the results for the sake of validity. Full results of the

replication of Bernanke et al. (2005) can be found in the Appendix in the form of explanatory power and impulse response functions. When calculating confidence intervals for the impulse-responses for the PC-estimated factors we employ a 1000 iterations bootstrap method as introduced by Kilian (1998), also done in Bernanke et al. (2005). This method accounts for the uncertainty in both the factor estimation along with the uncertainty in the FAVAR estimate. However, considering that the aim of this paper is to compare factor estimation methods, we use a homogenous methodology for calculating confidence intervals in an effort to improve comparability. Thus, we have used a 2 standard deviation confidence interval for both factor estimation approaches in the impulse response functions presented below.

| Factor | Variable | R-Square |
|----------|---|----------|
| | | |
| Factor 1 | 34. LPGD - NONAG. PAYROLLS: GOODS-PRODUCING | 0.712 |
| | 32. LPNAG - NONAG. PAYROLLS: TOTAL | 0.697 |
| | 33. LP - NONAG PAYROLLS: TOTAL, PRIVATE | 0.686 |
| | 37. LPEM - NONAG PAYROLLS: MANUFACTURING | 0.679 |
| | 11. IPMFG - INDUSTRIAL PRODUCTION: MANUFACTURING | 0.634 |
| | | |
| Factor 2 | 102. PMCP - COMMODITY PRICES INDEX | 0.601 |
| | 48. PMEMP - EMPLOYMENT INDEX | 0.44 |
| | 61. PMNV - INVENTORIES INDEX | 0.402 |
| | 17. IPXMCA – CAP. UTIL RATE: MANUFACTURING, TOTAL | 0.390 |
| | 112. PUC - CPI: COMMODITIES | 0.384 |
| | | |
| Factor 3 | 30. LHU15 - UNEMPLOYMENT: 15 WKS+ | 0.673 |
| | 31. LHU26 - UNEMPLOYMENT: 15 TO 26 WKS | 0.636 |
| | 26. LHUR - UNEMPLOYMENT RATE: ALL WORKERS | 0.615 |
| | 29. LHU14 - UNEMPLOYMENT: 5 TO 14 WKS | 0.542 |
| | 27. LHU680 - UNEMPLOYMENT: AVERAGE | 0.492 |

Table 1, displays the degree of variance explained regressing the three PC-estimated factors independently using all the observable variables respectively. The table only displays the 5 highest values.

A common critique of PCA is the difficulty in qualitatively explaining which variables are best explained by which factors. For narrative purposes, we have employed a "quick and dirty" method in trying to give a basic explanation of what the different factors explain regressing estimated factors upon individual variables, results are shown in Table 1.

Berggren (2017) states that a conventional problem to deal with the implications of the prize puzzle is to introduce a variable coding for commodity prices in the VAR. Although this is

somewhat theoretically unjustified argues Hansen (2004) since the method lacks support in economic theory. Nonetheless, it is interesting that our second estimated PC-factor is explained to a large degree by just a commodity price index.

| | R- | Square |
|------------|-------|--------|
| | PC | CCE |
| IP | 0.466 | 0.492 |
| UNEMP | 0.993 | 0.993 |
| PCONS | 0.347 | 0.364 |
| FM2 | 0.736 | 0.743 |
| СРІ | 0.722 | 0.707 |
| FACTOR I | 0.770 | 0.888 |
| FACTOR II | 0.820 | 0.302 |
| FACTOR III | 0.879 | 0.700 |

Table 2, displays R-square values of equations for respective variables included in the FAVAR, with 3 estimated factors using PC respectively CCE. Factors are estimated through each separate estimation method, thus not directly comparable between models.

Examining the R-squared values for the PC-estimation and the CCE-estimation shown in Table 2, the results show no remarkable differences in the variance explained for the chosen variables included in FAVAR. Unemployment, displayed the highest R-square values in both estimation methods. It is prominent that Personal Consumption displayed the lowest R-square value in respective estimation methods. The notable difference between the outputs for the estimation methods is the factors. The retrieved R-square values for the factors in PC-estimated VAR indicates that each factor explains a large portion of the variability in the model. The R-square obtained in CCE-estimation for factor 1 yielded the largest value, while factor 2 yielded a substantially smaller value. This is probably due to assignation of variables in the CCE-estimation groups. As stated in Berggren (2017), a notable difference between PC and CCE R-square values could probably be explained to a large degree by the fact that PC explicitly estimates factors by minimizing the idiosyncratic variance, thus maximizing R-square in X_t . CCE on the other hand just calculates the average of X_t .

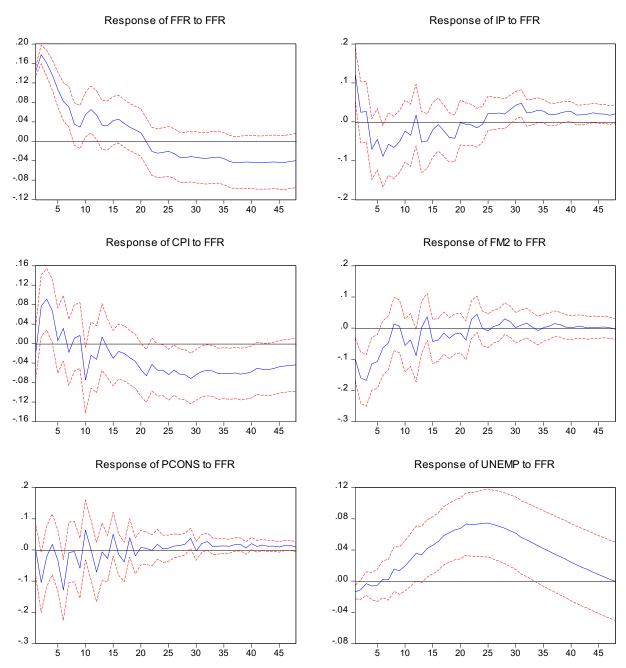


Figure 1, Impulse response functions for FAVAR estimations using a Principal Components-approach. The functions are presented along with a 2 standard deviations confidence interval. X-axis display months after initial shock.

Figure 1 shows the impulse responses to a one standard deviation shock in federal funds rate for the PC-estimated model. As seen in the graphs, the price puzzle is eliminated as CPI follows an upward trend as result of an increase in interest rates. Industrial production (IP) and Unemployment (UNEMP) follow a similar upward trend. Money supply (FM2) and Personal Consumption (PCONS) show signs of a decreasing response due to a shock in Federal Funds Rate. Findings are thus in line with economic theory. The effect upon the response variables tend to taper towards null as time progresses.

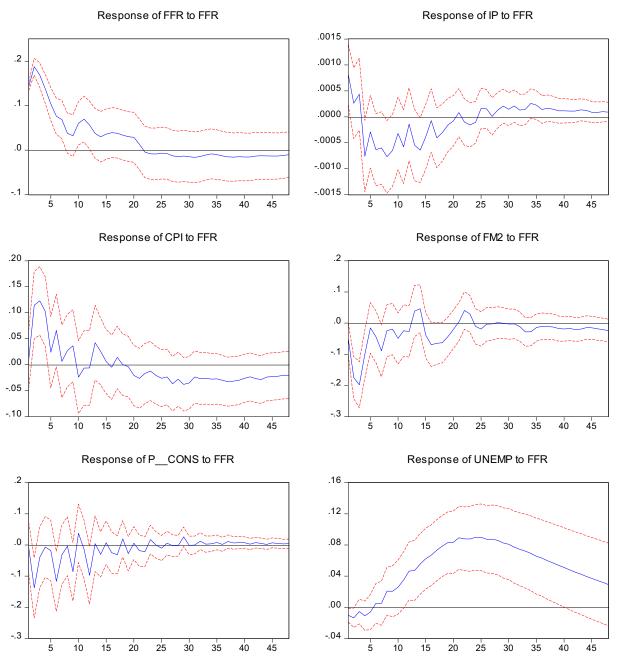


Figure 2, Impulse response functions for FAVAR estimations using a CCE-approach. The functions are presented along with a 2 standard deviations confidence interval. X-axis display months after initial shock.

The impulse response functions in Figure 2 are based upon the 3-factor CCE estimation approach. To the naked eye there are few differences to be found between the responses in Figure 1 and Figure 2. However, one notable although non-significant difference is the response in Industrial Production (IP) given a one standard-deviation shock in FFR. Contrary to what can be found in Figure 2, the responses derived from the PC-estimated model for Industrial Production increases at first. Although, over the long run responses for IP show similar results for both estimation methods.

The fact that both estimation methods show similar results indicates that FAVAR estimation in some regards could be simplified using CCE estimation. Although, the PC approach offers the advantage that the researcher can effortlessly extract more factors without imposing sample restrictions. Seeking to extract more factors in using CCE calls for the researcher to divide variables into sub-groups (Berggren, 2017). But as found in Bernanke et al. (2005) increasing the number of factors from 3 to 5 does not significantly improve the results.

5. Conclusion

The aim of this paper was to compare factor estimation methods in the FAVAR universe. Our first estimation method studied is the widely used two-step principal component approach introduced by Stock and Watson (2002b) and applied in Bernanke et al. (2005). The method used as comparison is the CCE factor estimation approach introduced by Pesaran (2006) and applied by Berggren (2017). All estimations are based upon US macroeconomic data spanning the period between 1959:1 and 2001:8, which is the same dataset as in Bernanke et al. (2005). In order to study differences in results for the respective factor estimation methodology we have compared the variation explained and impulse response functions. Studying the calculated impulse-responses, our results indicates that there are small differences between the methods.

Although our method is somewhat simplified due to manually grouping the variables, it can be seen as a disadvantage since it does not capture every possible combination. Our CCE approach was based upon comparing correlation coefficients along with some use of economic theory. This however can to be argued be somewhat arbitrarily, and further research in the field of Monte Carlo simulations in estimating the CCE factors might be fruitful. In comparison to CCE, the PCA-approach groups the variables automatically while the difficulty lies in interpreting the variables included in each factor.

In order to draw more decisive conclusions regarding which estimation method that yields better results, further research in the theoretical field is required. Another interesting aspect for further research might be adding weights to the CCE factors, although this requires the researcher to possess prior knowledge of which variables might explain factors better. Chudnik and Pesaran (2013b) introduced an updated methodology to the CCE where the estimator is consistent for a small time period, this might help further comparative studies using a more limited dataset.

Appendix

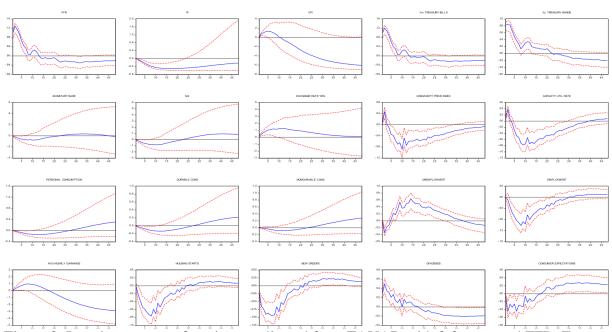


Figure 3, Impulse response functions replicated from FAVAR with 3 factors and FFR. Factors are estimated using principal components, and confidence intervals are calculated using 1000 iterations bootstrapping procedure. Replication from Bernanke et al. (2005).

| Variables | R-Square |
|-----------------------|----------|
| | |
| IP | 0.707 |
| CPI | 0.870 |
| 3m TREASURY BILLS | 0.975 |
| 5y TREASURY BONDS | 0.925 |
| MONETARY BASE | 0.104 |
| M2 | 0.052 |
| EXCHANGE RATE YEN | 0.025 |
| COMMODITY PRICE INDEX | 0.652 |
| CAPACITY UTIL RATE | 0.753 |
| PERSONAL CONSUMPTION | 0.108 |
| DURABLE CONS | 0.062 |
| NONDURABLE CONS | 0.062 |
| UNEMPLOYMENT | 0.817 |
| EMPLOYMENT | 0.707 |
| AVG HOURLY EARNINGS | 0.072 |
| HOUSING STARTS | 0.387 |
| NEW ORDERS | 0.624 |
| DIVIDENDS | 0.549 |
| CONSUMER EXPECTATIONS | 0.700 |
| | |

Table 3, The column "R-square" displays the fraction of the variance of the variable explained by the common factors (F_t, Y_t) . As replicated from Bernanke et al. (2005).

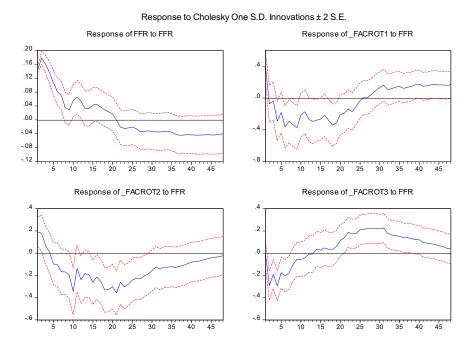


Figure 4, shows impulse responses for the factors employed in the PC estimated FAVAR. This shows how the factors are affected by the impulses in FFR and thus do contribute information in the multivariate regressions in the VAR.

| PC - FAVAR RESULTS | FFR | IP | CPI | FM2 | PCONS | UNEMP | _FACROT1 | _FACROT2 | _FACROT3 |
|---|-----------|----------|----------|----------|----------|---------|-----------|----------|----------|
| R-squared | 0.984 | 0.471 | 0.728 | 0.551 | 0.360 | 0.993 | 0.770 | 0.825 | 0.882 |
| Adj. R-squared | 0.979 | 0.308 | 0.645 | 0.413 | 0.163 | 0.990 | 0.699 | 0.772 | 0.845 |
| Sum sq. resids | 7.904 | 220.932 | 135.644 | 223.591 | 320.036 | 3.750 | 1781.856 | 837.880 | 615.888 |
| S.E. equation | 0.144 | 0.762 | 0.597 | 0.767 | 0.918 | 0.099 | 2.165 | 1.485 | 1.273 |
| F-statistic | 201.688 | 2.893 | 8.702 | 3.993 | 1.829 | 437.028 | 10.876 | 15.349 | 24.184 |
| Log likelihood | 325.029 | -504.258 | -382.791 | -507.237 | -596.532 | 510.682 | -1024.059 | -836.180 | -759.535 |
| Akaike AIC | -0.831 | 2.499 | 2.011 | 2.511 | 2.870 | -1.577 | 4.587 | 3.832 | 3.524 |
| Schwarz SC | 0.166 | 3.497 | 3.009 | 3.509 | 3.867 | -0.579 | 5.584 | 4.830 | 4.522 |
| Mean dependent | 0.025 | -0.012 | 0.022 | 0.018 | -0.001 | 0.010 | -0.086 | -0.012 | 0.049 |
| S.D. dependent | 1.002 | 0.917 | 1.002 | 1.001 | 1.003 | 1.011 | 3.949 | 3.107 | 3.235 |
| | | • | | | | | | | |
| Determinant resid covariance (dof adj.) | 1.710E-06 | | | | | | | | |
| Determinant resid covariance | 1.500E-07 | | | | | | | | |
| Log likelihood | -2447.168 | | | | | | | | |
| Akaike information criterion | 14.093 | | | | | | | | |
| Schwarz criterion | 23.072 | | | | | | | | |

Table 4 shows the PC-estimated FAVAR statistics from the Eviews output.

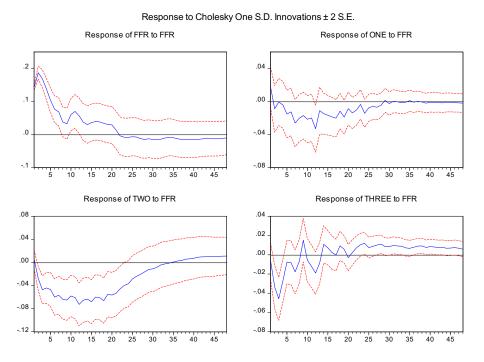


Figure 5, shows impulse responses for the factors employed in the CCE estimated FAVAR. Similarly as stated above, this shows how the factors are affected by the impulses in FFR and thus do contribute information in the multivariate regressions in VAR. Similarities between the IRF for respective factors in Figure 4 and Figure 5 are not as clear as for the observed variables. However, some similarities can still be seen to the naked eye, especially between the wide slow-moving factors (_facrot1 (PC) and One (CCE)). Factor interpretation for PC-model can be found in Table 1 above, factor ONE was a priori specified as the slow-moving factor when grouping the CCE-factors.

| CCE - FAVAR RESULTS | FFR | IP | CPI | FM2 | P_CONS | UNEMP | ONE | TWO | THREE |
|---|-----------|----------|----------|----------|----------|---------|--------|---------|---------|
| R-squared | 0.984 | 0.492 | 0.707 | 0.743 | 0.364 | 0.993 | 0.537 | 0.933 | 0.544 |
| Adj. R-squared | 0.980 | 0.336 | 0.617 | 0.664 | 0.168 | 0.991 | 0.394 | 0.912 | 0.404 |
| Sum sq. resids | 7.801 | 0.016 | 146.096 | 127.884 | 318.399 | 3.688 | 28.008 | 12.008 | 16.965 |
| S.E. equation | 0.143 | 0.007 | 0.620 | 0.580 | 0.915 | 0.099 | 0.271 | 0.178 | 0.211 |
| F-statistic | 204.388 | 3.150 | 7.847 | 9.384 | 1.855 | 444.495 | 3.764 | 45.010 | 3.880 |
| Log likelihood | 328.289 | 1867.404 | -401.274 | -368.121 | -595.255 | 514.870 | 10.016 | 220.904 | 134.848 |
| Akaike AIC | -0.845 | -7.026 | 2.085 | 1.952 | 2.864 | -1.594 | 0.434 | -0.413 | -0.068 |
| Schwarz SC | 0.153 | -6.028 | 3.083 | 2.950 | 3.862 | -0.596 | 1.431 | 0.584 | 0.930 |
| Mean dependent | 0.025 | 0.003 | 0.022 | 0.020 | -0.001 | 0.010 | 0.000 | -0.012 | 0.005 |
| S.D. dependent | 1.002 | 0.008 | 1.002 | 1.000 | 1.003 | 1.011 | 0.349 | 0.599 | 0.274 |
| Determinant resid covariance (dof adj.) | 1.800E-14 | - | | | | | | | |
| Determinant resid covariance | 1.570E-15 | | | | | | | | |
| Log likelihood | 2127.420 | | | | | | | | |
| Akaike information criterion | -4.279 | | | | | | | | |
| Schwarz criterion | 4.700 | | | | | | | | |

Table 5 shows the CCE-estimated FAVAR statistics from the Eviews output.

Data description

The data is replicated from Bernanke et al. (2005). The format and transformation code is equivalent to Stock and Watson (2002a). Transformation code is as followed:

- 1: no transformation
- 2: first difference
- 4: logarithm
- 5: first difference of logarithm
- *- implies that the variable is assumed to be "slow-moving"

1. Real output and income

```
IPP*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: PRODUCTS, TOTAL (1992=100,SA)
                                  INDUSTRIAL PRODUCTION: FINAL PRODUCTS (1992=100,SA)
   IPF*
                1959:01-2001:08
2.
3. IPC*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: CONSUMER GOODS (1992=100,SA)
4.
   IPCD*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: DURABLE CONS. GOODS (1992=100,SA)
   IPCN*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: NONDURABLE CONS. GOODS (1992=100,SA)
5.
6. IPE*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: BUSINESS EQUIPMENT (1992=100,SA)
   IPI*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: INTERMEDIATE PRODUCTS (1992=100,SA)
   IPM*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: MATERIALS (1992=100,SA)
8.
9.
   IPMD*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: DURABLE GOODS MATERIALS (1992=100,SA)
10.
   IPMND*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: NONDUR. GOODS MATERIALS (1992=100,SA)
                                  INDUSTRIAL PRODUCTION: MANUFACTURING (1992=100,SA)
11. IPMFG*
                1959:01-2001:08
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: DURABLE MANUFACTURING (1992=100,SA)
12. IPD*
   IPN*
13.
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: NONDUR. MANUFACTURING (1992=100,SA)
14. IPMIN*
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: MINING (1992=100,SA)
                1959:01-2001:08
                                  INDUSTRIAL PRODUCTION: UTILITIES (1992-=100,SA)
   IPUT*
15.
                                  INDUSTRIAL PRODUCTION: TOTAL INDEX (1992=100,SA)
   IP*
                1959:01-2001:08
16.
   IPXMCA*
                1959:01-2001:08
                                  CAPACITY UTIL RATE: MANUFAC., TOTAL(% OF CAPACITY, SA)(FRB)
17.
18.
   PMI*
                1959:01-2001:08
                                  PURCHASING MANAGERS' INDEX (SA)
                                  NAPM PRODUCTION INDEX (PERCENT)
   PMP*
                1959:01-2001:08
19.
                                  PERSONAL INCOME (CHAINED) (SERIÉS #52) (BIL 92$,SAAR)
20.
   GMPYQ*
                1959:01-2001:08
   GMYXPQ*
                1959:01-2001:08
                                  PERSONAL INC. LESS TRANS. PAYMENTS (CHAINED) (#51) (BIL 92$,SAAR)
```

2. Employment and hours

```
22. LHEL*
                1959:01-2001:08
                                 INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
   LHELX*
                1959:01-2001:08
                                 EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF
23
               1959:01-2001:08
24
   LHEM*
                                 CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
25. LHNAG*
                1959:01-2001:08
                                 CIVILIAN LABOR FORCE: EMPLOYED, NONAG.INDUSTRIES (THOUS.,SA)
                                 UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)
26. LHUR*
                1959:01-2001:08
27. LHU680*
               1959:01-2001:08
                                 UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
28. LHU5*
                1959:01-2001:08
                                 UNEMPLOY.BY DURATION: PERS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
   LHU14*
                1959:01-2001:08
                                 UNEMPLOY.BY DURATION: PERS UNEMPL.5 TO 14 WKS (THOUS.,SA)
30. LHU15*
               1959:01-2001:08
                                 UNEMPLOY.BY DURATION: PERS UNEMPL.15 WKS + (THOUS..SA)
                                 UNEMPLOY.BY DURATION: PERS UNEMPL.15 TO 26 WKS (THOUS.,SA)
31. LHU26*
               1959:01-2001:08
   LPNAG*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: TOTAL (THOUS.,SA)
32.
33. LP*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG PAYROLLS: TOTAL, PRIVATÉ (THOUS, SA)
   LPGD*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: GOODS-PRODUCING (THOUS.,SA)
34.
   LPMI*
35.
               1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: MINING (THOUS.,SA)
36. LPCC*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: CONTRACT CONSTRUC. (THOUS.,SA)
                                 EMPLOYEES ON NONAG. PAYROLLS: MANUFACTURING (THOUS.,SA)
37.
   LPEM*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: DURABLE GOODS (THOUS.,SA)
38 LPED*
               1959:01-2001:08
39. LPEN*
               1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: NONDURABLE GOODS (THOUS.,SA)
40. LPSP*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: SERVICE-PRODUCING (THOUS.,SA)
41. LPTU*
               1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: TRANS. & PUBLIC UTIL. (THOUS., SA)
                1959:01-2001:08
42. LPT*
                                 EMPLOYEES ON NONAG. PAYROLLS: WHOLESALE & RETAIL (THOUS.,SA)
   LPFR*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: FINANCE, INS. & REAL EST (THOUS., SA)
43.
44. LPS*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: SERVICES (THOUS.,SA)
   LPGOV*
                1959:01-2001:08
                                 EMPLOYEES ON NONAG. PAYROLLS: GOVERNMENT (THOUS.,SA)
45.
46.
   LPHMR*
               1959:01-2001:08
                                 AVG. WEEKLY HRS. OF PRODUCTION WKRS.: MANUFACTURING (SA)
47. LPMOSA*
                1959:01-2001:08
                                 AVG. WEEKLY HRS. OF PROD. WKRS.: MFG., OVERTIME HRS. (SA)
48. PMEMP*
                1959:01-2001:08
                                 NAPM EMPLOYMENT INDEX (PERCENT)
```

3. Consumption

```
    49. GMCQ*
    50. GMCDQ*
    51. GMCNQ*
    52. GMCSQ*
    53. GMCSQ*
    54. GMCSQ*
    55. PERSONAL CONSUMPTION EXPEND (CHAINED) - TOT. DUR. (BIL 92$,SAAR)
    55. PERSONAL CONSUMPTION EXPEND (CHAINED) - NONDUR. (BIL 92$,SAAR)
    56. PERSONAL CONSUMPTION EXPEND (CHAINED) - NONDUR. (BIL 92$,SAAR)
    57. PERSONAL CONSUMPTION EXPEND (CHAINED) - SERVICES (BIL 92$,SAAR)
    58. PERSONAL CONSUMPTION EXPEND (CHAINED) - SERVICES (BIL 92$,SAAR)
```

4. Housing starts and sales

| 54. | HSFR | 1959:01-2001:08 | 4 | HOUSING STARTS:NONFARM(1947-58);TOT.(1959-)(THOUS.,SA |
|-----|-------|-----------------|---|--|
| 55. | HSNE | 1959:01-2001:08 | 4 | HOUSING STARTS:NORTHEAST (THOUS.U.)S.A. |
| 56. | HSMW | 1959:01-2001:08 | 4 | HOUSING STARTS:MIDWEST(THOUS.U.)S.A. |
| 57. | HSSOU | 1959:01-2001:08 | 4 | HOUSING STARTS:SOUTH (THOUS.U.)S.A. |
| 58. | HSWST | 1959:01-2001:08 | 4 | HOUSING STARTS:WEST (THOUS.U.)S.A. |
| 59. | HSBR | 1959:01-2001:08 | 4 | HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING (THOUS.,SAAR) |
| 60. | HMOB | 1959:01-2001:08 | 4 | MOBILE HOMES: MANUFACTURERS' SHIPMENTS (THOUS OF UNITS SAAR) |

5. Real inventories, orders and unfilled orders

| 61. | PMNV | 1959:01-2001:08 | 1 | NAPM INVENTORIES INDEX (PERCENT) |
|-----|--------|-----------------|---|--|
| 62. | PMNO | 1959:01-2001:08 | 1 | NAPM NEW ORDERS INDEX (PERCENT) |
| 63. | PMDEL | 1959:01-2001:08 | 1 | NAPM VENDOR DELIVERIES INDEX (PERCENT) |
| 64. | MOCMQ | 1959:01-2001:08 | 5 | NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1992 \$ (BCI) |
| 65. | MSONDQ | 1959:01-2001:08 | 5 | NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1992 DOLLARS (BCI) |

6. Stock prices

| 66. | FSNCOM | 1959:01-2001:08 | 5 | NYSE COMMON STOCK PRICE INDEX: COMPOSITE (12/31/65=50) |
|-----|--------|-----------------|---|--|
| 67. | FSPCOM | 1959:01-2001:08 | 5 | S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10) |
| 68. | FSPIN | 1959:01-2001:08 | 5 | S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10) |
| 69. | FSPCAP | 1959:01-2001:08 | 5 | S&P'S COMMON STOCK PRICE INDEX: CAPITAL GOODS (1941-43=10) |
| 70. | FSPUT | 1959:01-2001:08 | 5 | S&P'S COMMON STOCK PRICE INDEX: UTILITIES (1941-43=10) |
| 71. | FSDXP | 1959:01-2001:08 | 1 | S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM) |
| 72. | FSPXE | 1959:01-2001:08 | 1 | S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%.NSA) |

7. Exchange rates

| 73. | EXRSW | 1959:01-2001:08 | 5 | FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.\$) 74 |
|-----|--------|-----------------|---|--|
| 74. | EXRJAN | 1959:01-2001:08 | 5 | FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.\$) |
| 75. | EXRUK | 1959:01-2001:08 | 5 | FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND) |
| 76. | EXRCAN | 1959:01-2001:08 | 5 | FOREIGN EXCHANGE RATE: CANADA (CANADIAN \$ PER U.S.\$) |

8. Interest rates

```
1959:01-2001:08 1 INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)
1959:01-2001:08 1 INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NS
77. FYFF
78. FYGM3
                                     INTEREST RATE: U.S.TREASURY BILLS, SEC MKT, 3-MO. (% PER ANN, NSA
                 1959:01-2001:08 1 INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
79. FYGM6
                 1959:01-2001:08 1
1959:01-2001:08 1
80. FYGT1
                                     INTEREST RATE: U.S.TREASURY CONST MATUR. ,1-YR.(% PER ANN,NSA)
                                     INTEREST RATE: U.S.TREASURY CONST MATUR., 5-YR.(% PER ANN,NSA)
81. FYGT5
82. FYGT10
                 1959:01-2001:08 1
                                     INTEREST RATE: U.S.TREASURY CONST MATUR., 10-YR. (% PER ANN, NSA)
                 1959:01-2001:08 1
1959:01-2001:08 1
                                     BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
83. FYAAAC
                                     BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
                 1959:01-2001:08
84. FYBAAC
85. SFYGM3
                 1959:01-2001:08 1
                                     Spread FYGM3 - FYFF
                                     Spread FYGM6 - FYFF
86. SFYGM6
                 1959:01-2001:08
                                     Spread FYGT1 - FYFF
87. SFYGT1
                 1959:01-2001:08 1
                 1959:01-2001:08
   SFYGT5
                                     Spread FYGT5 - FYFF
89.
    SFYGT10
                 1959:01-2001:08
                                     Spread FYGT10 - FYFF
                                     Spread FYAAAC - FYFF
90. SFYAAAC
                 1959:01-2001:08 1
                 1959:01-2001:08 1 Spread FYBAAC - FYFF
91. SFYBAAC
```

9. Money and credit quantity aggregates

| 92. | FM1 | 1959:01-2001:08 | 5 | MONEY STOCK: M1 (BIL\$,SA) |
|------|--------|-----------------|---|--|
| 93. | FM2 | 1959:01-2001:08 | 5 | MONEY STOCK:M2 (BIL\$, SA) |
| 94. | FM3 | 1959:01-2001:08 | 5 | MONEY STOCK: M3 (BIL\$,SA) |
| 95. | FM2DQ | 1959:01-2001:08 | 5 | MONEY SUPPLY - M2 IN 1992 DOLLARS (BCI) |
| 96. | FMFBA | 1959:01-2001:08 | 5 | MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL\$,SA) |
| 97. | FMRRA | 1959:01-2001:08 | 5 | DEPOSITORY INST RESERVES:TOTAL, ADJ FOR RES. REQ CHGS(MIL\$, SA) |
| 98. | FMRNBA | 1959:01-2001:08 | 5 | DEPOSITORY INST RESERVES:NONBOR. ,ADJ RES REQ CHGS(MIL\$,SA) 99 |
| 99. | FCLNQ | 1959:01-2001:08 | 5 | COMMERCIAL & INDUST. LOANS OUSTANDING IN 1992 DOLLARS (BCI) |
| 100. | FCLBMC | 1959:01-2001:08 | 1 | WKLY RP LG COM. BANKS: NET CHANGE COM & IND. LOANS(BIL\$,SAAR) |
| 101. | CCINRV | 1959:01-2001:08 | 5 | CONSUMER CREDIT OUTSTANDING NONREVOLVING G19 |

10. Price indexes

102. PMCP 1959:01-2001:08 1 NAPM COMMODITY PRICES INDEX (PERCENT)

```
103. PWFSA*
                   1959:01-2001:08 5 PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
                                         PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
PRODUCER PRICE INDEX:INTERMED MAT.SUP & COMPONENTS(82=100,SA)
104. PWFCSA*
                   1959:01-2001:08
                   1959:01-2001:08
105. PWIMSA*
106. PWCMSA*
                   1959:01-2001:08
                                         PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
                                         INDEX OF SENSITIVE MATERIALS PRICES (1990=100)(BCI-99A)
107. PSM99Q*
                   1959:01-2001:08
108. PUNEW*
                                        CPI-U: ALL ITEMS (82-84=100,SA)
CPI-U: APPAREL & UPKEEP (82-84=100,SA)
                   1959:01-2001:08
109. PU83*
                   1959:01-2001:08
110. PU84*
                   1959:01-2001:08
                                        CPI-U: TRANSPORTATION (82-84=100,SA)
                                        CPI-U: MEDICAL CARE (82-84=100,SA)
111. PU85*
                   1959:01-2001:08
112. PUC*
                   1959:01-2001:08
                                        CPI-U: COMMODITIES (82-84=100,SA)
113. PUCD*
                   1959:01-2001:08
                                         CPI-U: DURABLES (82-84=100,SA)
114. PUS*
                   1959:01-2001:08
                                        CPI-U: SERVICES (82-84=100,SA)
115. PUXF*
                                     5 CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
5 CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
                   1959:01-2001:08
116. PUXHS*
                   1959:01-2001:08
117. PUXM*
                   1959:01-2001:08
                                        CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA)
```

11. Average hourly earnings

| 118. | LEHCC* | 1959:01-2001:08 | 5 | AVG HR EARNINGS OF CONSTR WKRS: CONSTRUCTION (\$,SA) |
|------|--------|-----------------|---|--|
| 119. | LEHM* | 1959:01-2001:08 | 5 | AVG HR EARNINGS OF PROD WKRS: MANUFACTURING (\$,SA) |

12. Miscellaneous

120. HHSNTN 1959:01-2001:08 1 U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)

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