

Project 2: Measuring Planet Mass, Radius, and Density

Team 6

Motivation

Looking at exoplanets, many questions may arise about their properties. By parameterizing exoplanets by radius, mass, and density, it is possible to categorize them within the realms of rocky, Neptunian, and Jovial worlds. This categorization can aid humanity in the endless search for life and further understand the distribution of exoplanets and what this distribution may implicate. This project will derive planetary parameters such as the radius, mass, and density of an exoplanet using radial velocity and photometric light curve data. The parameters will be compared to existing exoplanets and categorized according to the mass-radius relation from Chen and Kipping (2016). The main goal of this project is to strengthen understanding of planet mass and radius measurement, uncertainty quantification, and the usage of NEA and other tools to collect and analyze exoplanet data.

Methods

By selecting the exoplanet system GJ 436, the following methods were used to calculate the planet's mass, radius, density, and their respective uncertainties. The photometric light curve data is from Demory et al. (2007) and the radial velocity data is used from Butler et al. (2006).

To determine planet radius, transit depth is used via the following equation:

$$(1) \delta = \left(\frac{R_p}{R_s}\right)^2$$

Rearranging Eq (1), the planetary radius can be found using

$$(2) R_p = \sqrt{\delta} * R_s$$

GJ 436b's star has a radius of $R_s = 0.417 \pm 0.0075 R_{Sun}$ (Rosenthal et al. 2021).

GJ 436b's mass was determined using the amplitude of the radial velocity model under the assumption that $M_p \ll M_s$ by the following equation,

$$(3) M_p = \left(\frac{P}{2\pi G}\right)^{\frac{1}{3}} \frac{M_s^{2/3} K}{\sin i}$$

The stellar mass and $M_p \sin i$ quantity from Rosenthal et al. (2021) are used.

Finally, the planetary density was calculated using the previously found parameters through the following equation,

$$(4) \rho_p = \frac{3M_p}{4\pi R_p^3}$$

Models for the transit photometric light curve and radial velocity were developed through the parameter output from EXOFAST, an integrated tool part of the NASA Exoplanet Archive. The photometric light curve and radial velocity were plugged into this tool to receive the parameters necessary for creating the models. The photometric light curve model demonstrates the drop in flux, δ , as a function of time. The radial velocity model was developed using the following equation:

$$(5) RV = \gamma + K \cos\left(2\pi \frac{t}{P} + \frac{e \sin \omega}{e \cos \omega}\right)$$

The uncertainty of the planetary radius and mass was found utilizing the EXOFAST tool. A Markov Chain Monte Carlo (MCMC) simulation is run based on the photometric light curve and radial velocity curve data. The uncertainty is extracted from the parameter output of the simulation and applied to the planetary parameter measurements.

Results

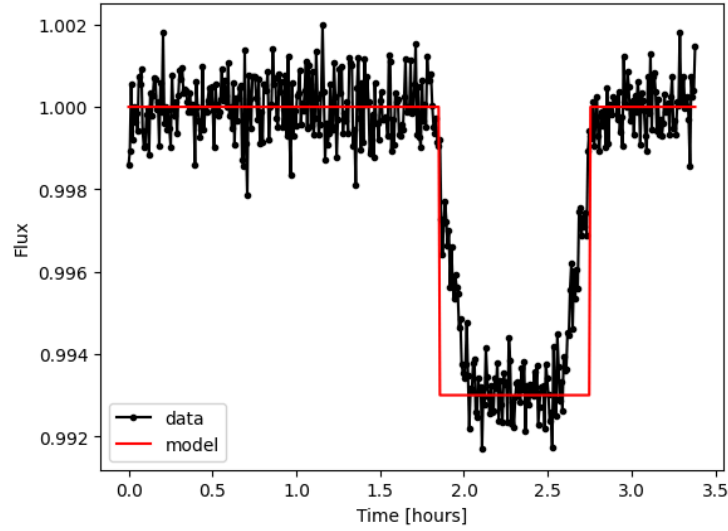


Figure 1: Flux vs. Time: Comparison between Data and Model

Figure 1 shows the plotted photometric data from Demory et al. (2007) compared to the generated model. The model has a flux drop of $\delta = 0.007$, which is used to calculate the planetary radius given the stellar radius $R_s = 0.5035R_{sun}$.

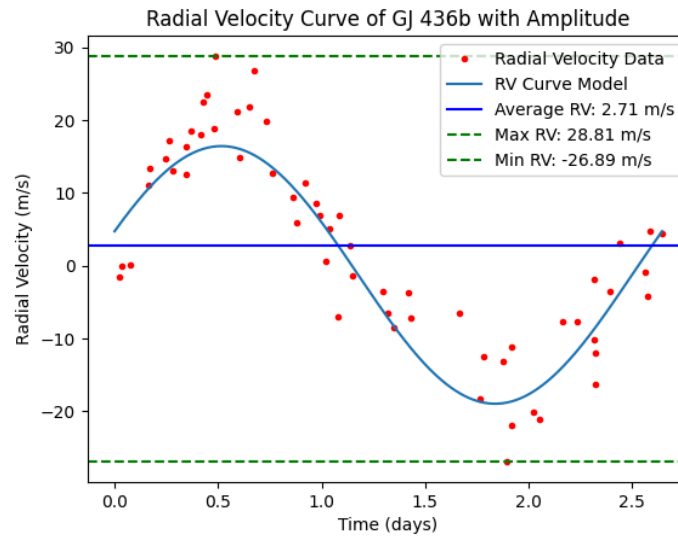


Figure 2: Radial Velocity vs. Time: Comparison between Data and Model

Figure 2 shows the radial velocity data, marked in red, from Butler et al. (2006) compared to the radial velocity model, in blue, extracted from EXOFAST and plugged into Eq. (5). The minimum and maximum radial velocity extracted from the data as well as the average is marked on the graph to show how error can arise.

Table 1: Planetary Parameters and Uncertainty

Parameter	Value	Error
Radius	$3.821R_{Earth}$	± 0.347
Mass	$22.22M_{Earth}$	± 2.34
Density	$1.92 \frac{g}{cm^3}$	± 0.28

The calculations from Eq. (2), Eq. (3), and Eq. (4) based on the transit and radial velocity models are shown in Table 1. The uncertainty extracted from EXOFAST is shown in the error column. The error can be an explanation as to why there may be a discrepancy between the model line and the data points in both Figures 1 and 2.

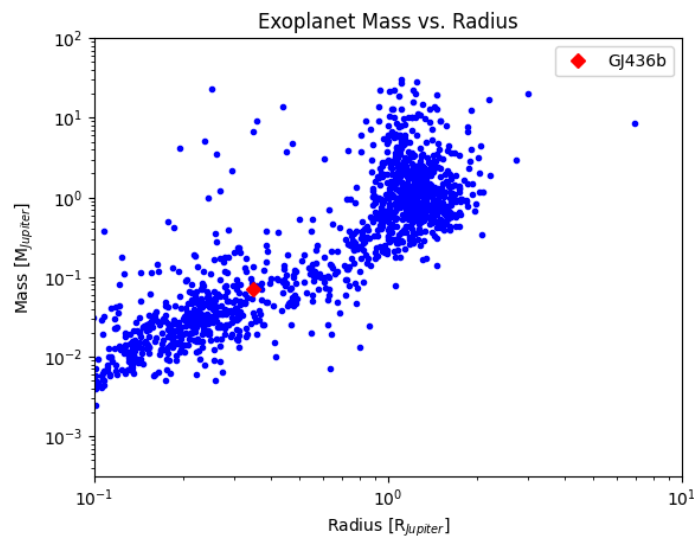


Figure 3: GJ 436b Comparison to Other Exoplanets

In Figure 3, GJ 436b is plotted on Mass vs. Radius as a red diamond against over 4000 exoplanets in the NASA Exoplanet Archive. Looking at the mass-radius relation from Chen & Kipping (2016), GJ 436b fits into the Neptunian category, almost directly on the model line. GJ 436b is an average planet within its class and fits within the realm of possibility.

Conclusion:

The planet GJ 436b is an exoplanet that fits well within the Neptunian world category. Through this project, it was demonstrated that model fitting through photometric light curve and radial velocity data is a sound way of parameterizing exoplanets within a reasonable margin of error, as demonstrated in Table 1 with GJ 436b.

References:

Chen, Jingjing & Kipping, David (2016). Probabilistic Forecasting of the Masses and Radii of Other Worlds. *The Astrophysical Journal*.

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Rosenthal, Lee J. et al. (2021). The California Legacy Survey. I. A Catalog of 178 Planets from Precision Radial Velocity Monitoring of 719 Stars over Three Decades. *The*

Astrophysical Journal. <https://iopscience.iop.org/article/10.3847/1538-4365/abe23c/pdf>

Demory, Drake, et al. (2007). Spitzer Transit and Secondary Eclipse Photometry of GJ 436b. *The*

Astrophysical Journal. <https://ui.adsabs.harvard.edu/abs/2007ApJ...667L.199D/abstract>

Butler, R. P. et al. (2006). Catalog of Nearby Exoplanets. *The Astrophysical Journal*.

<https://ui.adsabs.harvard.edu/abs/2006ApJ...646..505B/abstract>

AI Usage Disclosure:

In this project, the team utilized Google Gemini (an advanced AI model), which is offered as a feature in Colab, in hopes of more efficiently debugging our code and further supplementing it.

Google Presentation link:

https://docs.google.com/presentation/d/1vML-mFsNdYeP7HeLGpLjPu3dtzXm2L3s_hn0tsWGzEo/edit?usp=sharing