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# CS32: Introduction to Computer Science II

## **Discussion Week 8**

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- Homework 4 is due 11PM next Tuesday, May 28. (@A@)
- How do you feel about midterm 2?

- Sorting (cont'd)
- Tree (Part 1)
- Homework 4 Guidelines

Most important algorithm ever!

Methods:

- Selection sort
- Bubble sort
- Insertion sort
- Merge sort
- Quick sort

Focus on:

1. Steps for each sorting algorithm
2. Runtime complexity for worst cases, best cases and average cases
3. Space complexity
4. How about additional assumptions, such as the array is “almost sorted” / “reversed” arrays

# Sorting

## Selection sort

### Steps:



**Idea:** Find the smallest item in the unsorted portion and place it in the front.

### Runtime complexity:

Average:  $O(n^2)$

Worst:  $O(n^2)$

Best:  $O(n^2)$

**Space complexity:**  $O(1)$

# Sorting

## Insertion sort

### Steps:



**Idea:** Pick one from the unsorted part and place it in the right position.

### Runtime complexity:

Average:  $O(n^2)$

Worst:  $O(n^2)$

Best:  $O(n)$

**Space complexity:**  $O(1)$

# Sorting

## Bubble sort

### Steps:

4	3	1	5	2
3	4	1	5	2
3	1	4	5	2
3	1	4	2	5
1	3	2	4	5
1	2	3	4	5

**Idea:** Well, just “bubble” as its name

### Runtime complexity:

Average:  $O(n^2)$

Worst:  $O(n^2)$

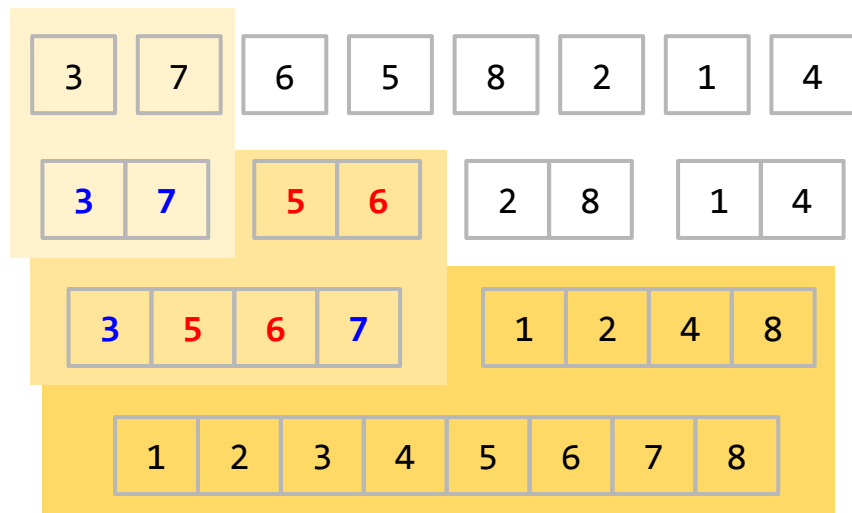
Best:  $O(n)$

**Space complexity:**  $O(1)$

# Sorting

## Merge sort

### Steps:



**Idea:** Divide and conquer

**Runtime complexity:**

Average:  $O(n \log n)$

Worst:  $O(n \log n)$

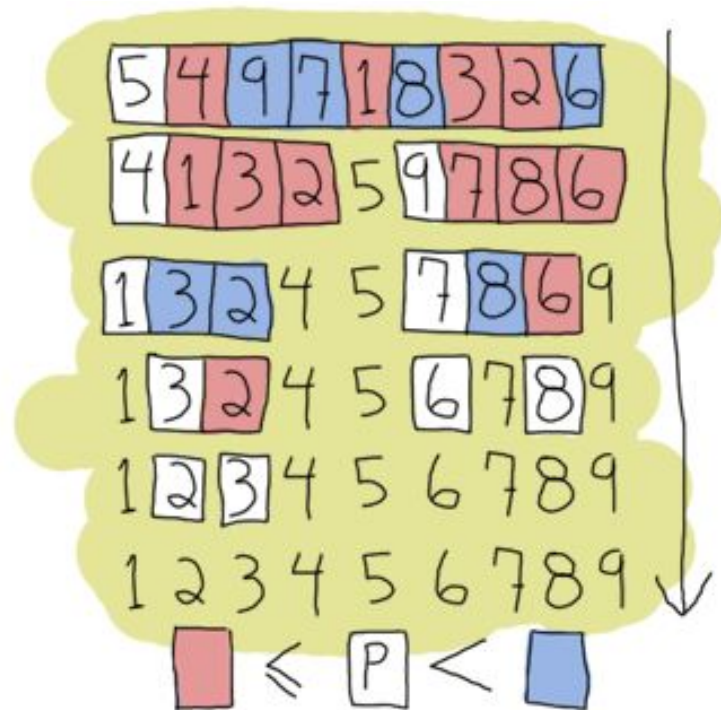
Best:  $O(n \log n)$

**Space complexity:**  $O(n)$



# Sorting

## Quicksort



**Idea:** Set a pivot. Numbers less than pivot are placed to front while other to end.

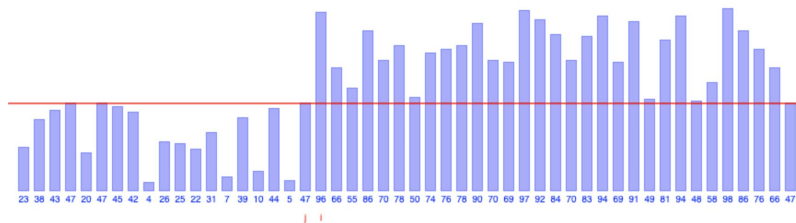
**Runtime complexity:**

Average:  $O(n \log n)$

Worst:  $O(n^2)$

Best:  $O(n \log n)$

**Space complexity:**  $O(\log n)$

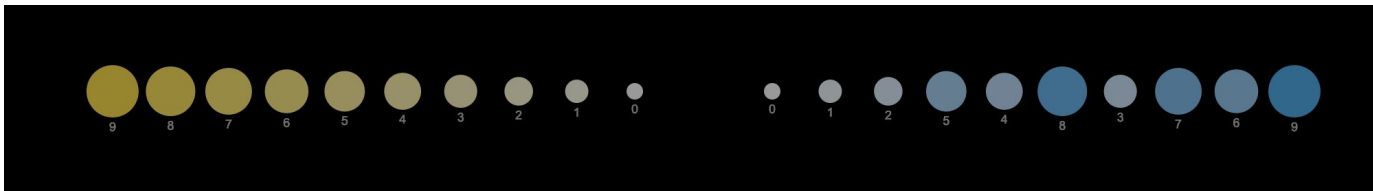


# Sorting

## Other methods and complexity?

- $O(n \log n)$  is faster than  $O(n^2)$  → Merge sort is more efficient than selection, insertion and bubble sort in runtime.
- $O(n \log n)$  is best average complexity that a general sorting algorithm can achieve.
- With more information about the data provided, you can sometimes sort things almost linearly.

Question: What is the complexity of these sorting algorithms if you know the array is **reversed**? What if the array is **almost already sorted**?

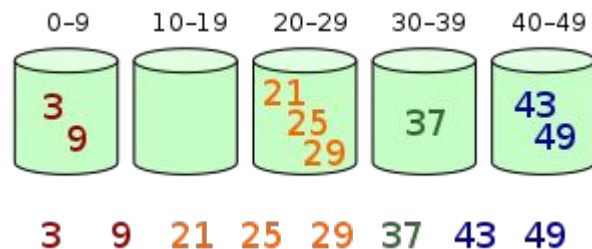


# Sorting

## Other methods and complexity?

There are many other sorting methods:

- Shell sort (shell 1959, Knuth 1973, Ciura 2001)
- Quicksort 3-way
- Heap sort
- Bucket sort



# Sorting

## Why sorting is important?

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Sorting is the most important and basic algorithm. Many other real-world problems are somewhat based on sorting, including:

Sorting Algorithms Animations: <https://www.toptal.com/developers/sorting-algorithms>

Other good demos:

<https://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html>

<http://sorting.at/>

## Variant sorting problems - Example

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Question: How about get the *K-th* largest numbers in one array?

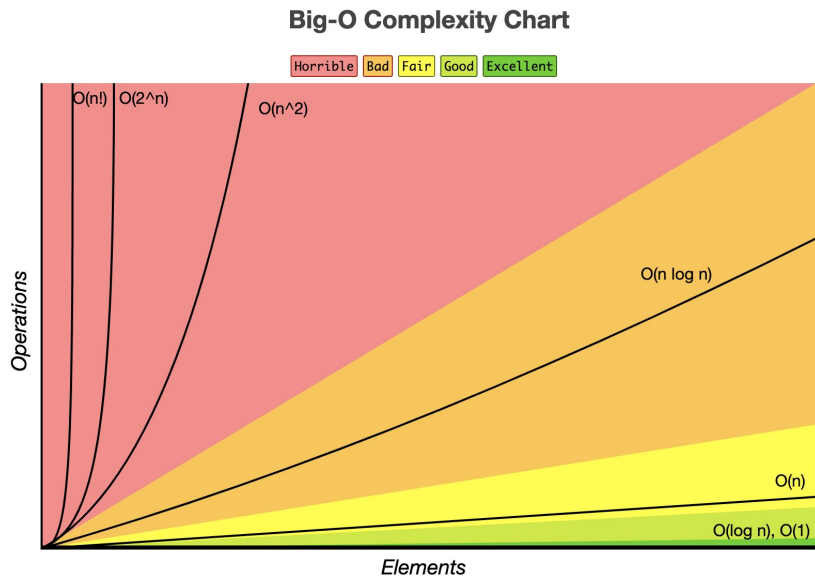
[Leetcode question #215](#)

Hint:

1. How to find the k-th largest numbers by merge sort and quicksort (or other sort methods)? What are the average and worst complexity?
2. What data structures is good to use?

# Big-O Notation

## Big-O Complexity Chart



## Array Sorting Algorithms

Algorithm	Time Complexity			Space Complexity
	Best	Average	Worst	Worst
<u>Quicksort</u>	$\Omega(n \log(n))$	$\theta(n \log(n))$	$O(n^2)$	$O(\log(n))$
<u>Mergesort</u>	$\Omega(n \log(n))$	$\theta(n \log(n))$	$O(n \log(n))$	$O(n)$
<u>Timsort</u>	$\Omega(n)$	$\theta(n \log(n))$	$O(n \log(n))$	$O(n)$
<u>Heapsort</u>	$\Omega(n \log(n))$	$\theta(n \log(n))$	$O(n \log(n))$	$O(1)$
<u>Bubble Sort</u>	$\Omega(n)$	$\theta(n^2)$	$O(n^2)$	$O(1)$
<u>Insertion Sort</u>	$\Omega(n)$	$\theta(n^2)$	$O(n^2)$	$O(1)$
<u>Selection Sort</u>	$\Omega(n^2)$	$\theta(n^2)$	$O(n^2)$	$O(1)$
<u>Tree Sort</u>	$\Omega(n \log(n))$	$\theta(n \log(n))$	$O(n^2)$	$O(n)$
<u>Shell Sort</u>	$\Omega(n \log(n))$	$\theta(n(\log(n))^2)$	$O(n(\log(n))^2)$	$O(1)$
<u>Bucket Sort</u>	$\Omega(n+k)$	$\theta(n+k)$	$O(n^2)$	$O(n)$
<u>Radix Sort</u>	$\Omega(nk)$	$\theta(nk)$	$O(nk)$	$O(n+k)$
<u>Counting Sort</u>	$\Omega(n+k)$	$\theta(n+k)$	$O(n+k)$	$O(k)$
<u>Cubesort</u>	$\Omega(n)$	$\theta(n \log(n))$	$O(n \log(n))$	$O(n)$

# Tree

## Definition

- Terms: Node/edge, root node, leaf node, parent and child node, subtree, levels (height/depth).
- Features: No loop, no shared children
- Question: How many edges should there be in a tree with  $n$  nodes?
- Binary tree: no node has more than two children.
- Question: How many nodes can a binary tree of height  $h$  have? → Full binary tree

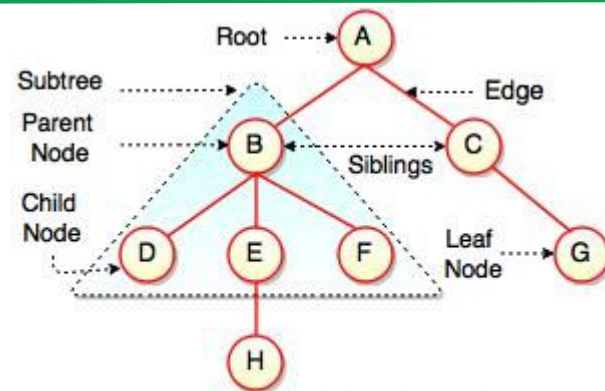
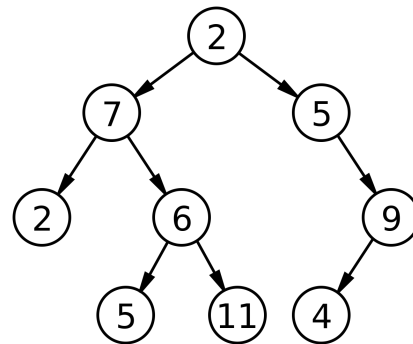


Fig. Structure of Tree



# Tree

## As a data structure

- Tree is a useful data structure!
- Basic functions: insert, remove, search, **traverse**
- How to traverse a tree?

```
struct Node{  
    ItemType val;  
    Node* leftChild;  
    Node* rightChild;  
} // a simple node
```

```
Class Tree{  
public:  
    // ???  
private:  
    // ???  
}
```

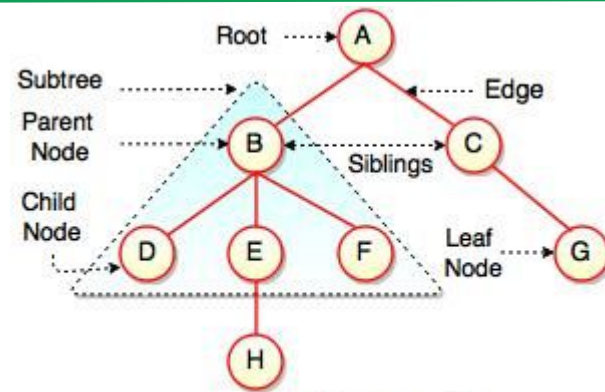
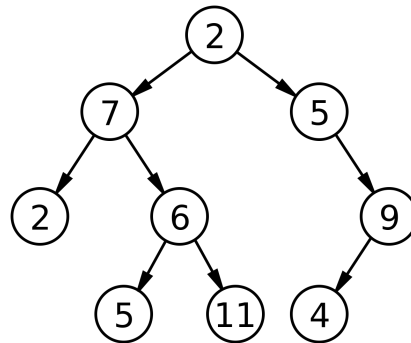


Fig. Structure of Tree





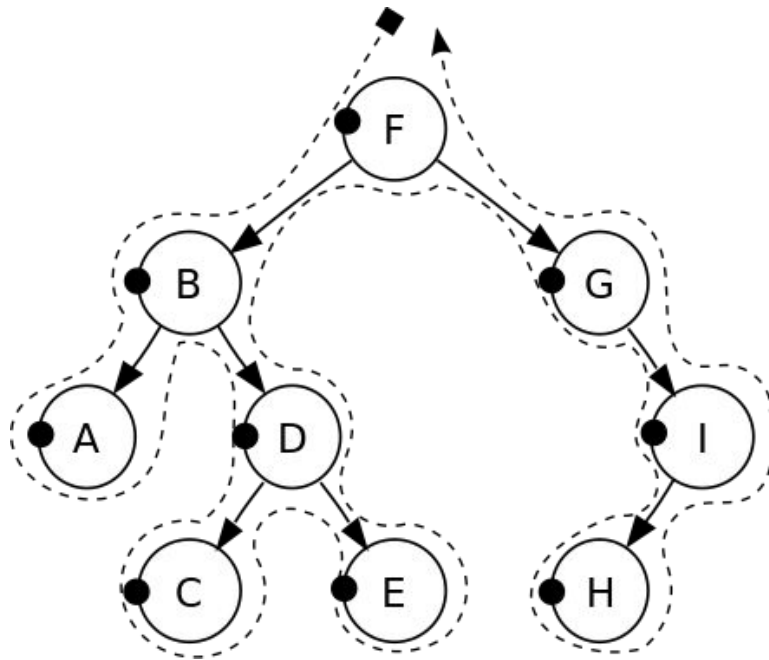
# Tree Traversal: Pre-order

## Three methods of tree traversal

```
void preorder(const Node* node)
{
    if (node == nullptr) return;
    cout << node->val << ",";
    preorder(node->left);
    preorder(node->right);
}
```

Pre-order output:

F, B, A, D, C, E, G, I, H



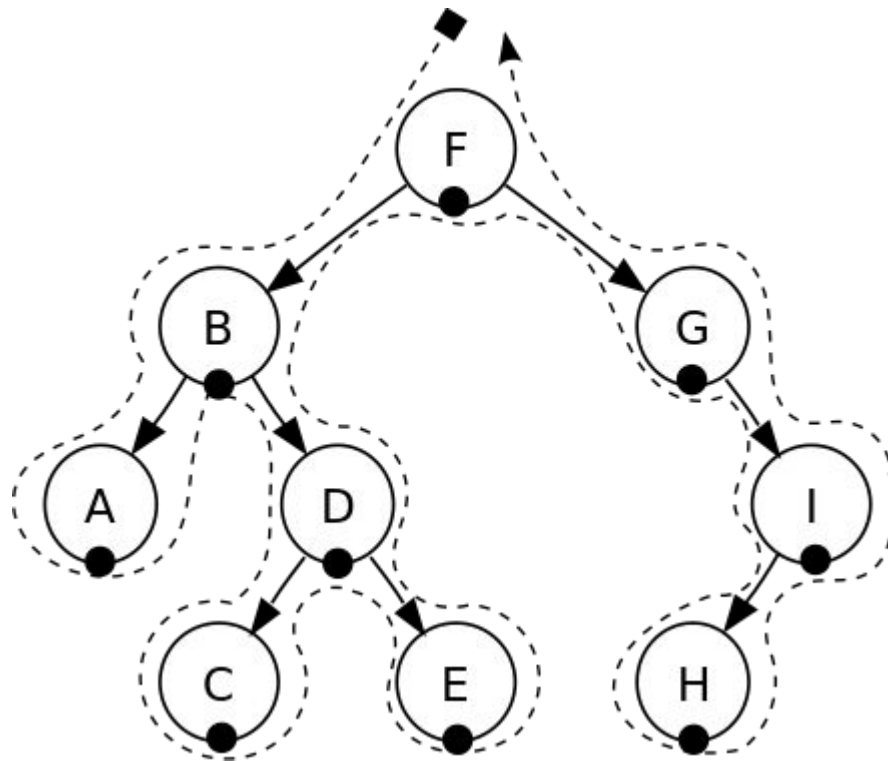
# Tree Traversal: In-order

## Three methods of tree traversal

```
void inorder(const Node* node)
{
    if (node == nullptr) return;
    inorder(node->left);
    cout << node->val << ",";
    inorder(node->right);
}
```

In-order output:

A, B, C, D, E, F, G, H, I



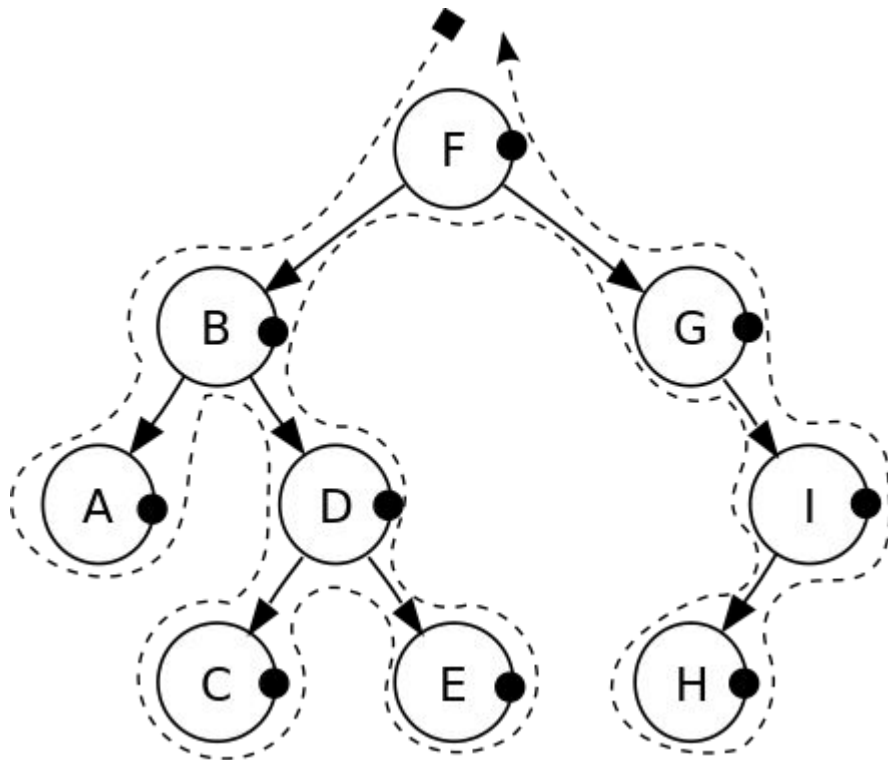
# Tree Traversal: Post-order

## Three methods of tree traversal

```
void postorder(const Node* node)
{
    if (node == nullptr) return;
    postorder(node->left);
    postorder(node->right);
    cout << node->val << ",";
}
```

Post-order output:

A, C, E, D, B, H, I, G, F



# Tree Traversal: Compare

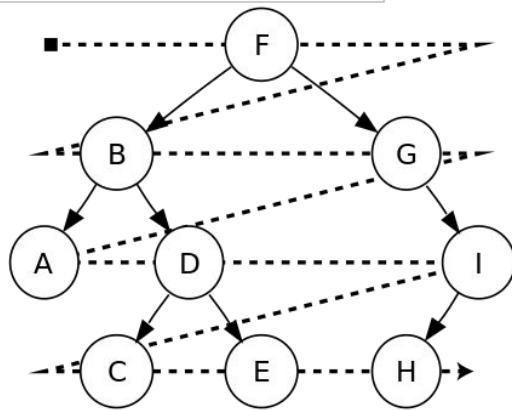
```
void preorder(const Node* node)
{
    if (node == nullptr) return;
    cout << node->val << ",";
    preorder(node->left);
    preorder(node->right);
}
```

```
void postorder(const Node* node)
{
    if (node == nullptr) return;
    postorder(node->left);
    postorder(node->right);
    cout << node->val << ",";
}
```

```
void inorder(const Node* node)
{
    if (node == nullptr) return;
    inorder(node->left);
    cout << node->val << ",";
    inorder(node->right);
}
```

//Other ways?

**Level-order or say  
breadth-first search!**

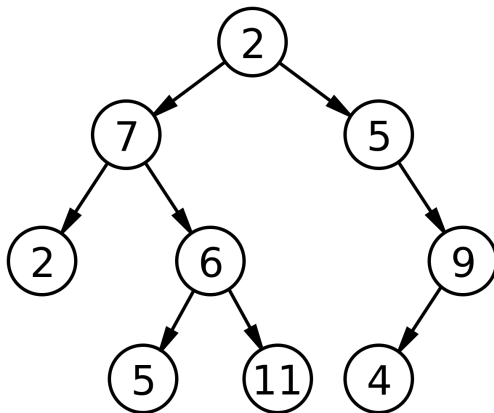


# Tree

## Recursion in trees

- It is easy and natural to apply recursion on trees!
- Pre-order / in-order / post-order are all recursive methods to traverse a tree.
- Question: How to calculate a height of a given tree?

```
int getTreeHeight(const Node* node)
{
    if (node == nullptr) return 0;
    int leftHeight = getTreeHeight(node->left);
    int rightHeight = getTreeHeight(node->right);
    if (leftHeight > rightHeight)
        return leftHeight + 1;
    else
        return rightHeight + 1;
}
```



# Tree

## Conclusions and applications

- Non-linear data structures unlike arrays or lists.
- Present hierarchy
- Optimized (like balanced) tree and variants can improve efficiency of search, sort and other typical problems.
- Other tree variants:
  - B tree / B+ tree
  - Red-black tree
  - K-D Tree
  - Suffix Tree

```
file system
-----
      /      <-- root
     / \
    /   \
   ...  home
        / \
       /   \
      /     \
     /       \
    /         \
   ...        ugrad   course
               /      |      \
              /       |       \
             /        |        \
            /         |         \
           /          |          \
          /           |           \
         /            |            \
        /             |             \
       /              |              \
      /               |               \
     /                |                \
    /                 |                 \
   ...              cs31 cs32 cs35L
```

# Next week about tree

- Binary Search Tree
- Heap
- More interesting questions ...

6 5 3 1 8 7 2 4

# Hints for Homework 4 - P1

```
template <typename T>
class S
{
    ...
    struct N
    {
        ...
    };
    N* f();
    ...
};
```

```
template <typename T>
typename S<T>::N* S<T>::f()    // OK
{
    ...
}
```

## Note & Reminders:

1. Only need to change (or add) a few lines if you decide to use template **<typename ItemType>**
2. Put **template <typename T>** ahead of every function you declare outside the class statement.



# Hints for Homework 4 - P2

```
#include "Sequence.h" // class template from
problem 1
class Coord
{
public:
    Coord(int r, int c) : m_r(r), m_c(c) {}
    Coord() : m_r(0), m_c(0) {}
    double r() const { return m_r; }
    double c() const { return m_c; }
private:
    double m_r;
    double m_c;
};
int main()
{
    Sequence<int> si;
    si.insert(50); // OK
    Sequence<Coord> sc;
    sc.insert(0, Coord(50,20)); // OK
    sc.insert(Coord(40,10)); // error!
}
```

## Note & Reminders:

1. Check the 1-argument `insert()` function and think about the operation which leads to error for `Coord` but not for `int`.

# Hints for Homework 4 - P3, P4

```
// Problem 4
```

```
void listAll(const Domain* d, string path)
// two-parameter overload
{
    // You will write this code.
}
```

```
void listAll(const Domain* d)
// one-parameter overload
{
    if (d != nullptr)
        listAll(d, d->label());
}
```

## Note & Reminders:

1. (P3) Implement functions with `remove()` in STL.
2. (P4) Necessity for two-parameter overload of `listAll()` function. → What is the purpose of string `path` parameter?
3. (P4) If the container is `const`, you must use a `const_iterator` to traverse an STL container.

# Hints for Homework 4 - P5, 6, 7

## Note & Reminders:

1. Only the highest order will be considered as your final answer.
2. Analysis implementation of **every** function in **every** step.



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# Break Time! (5 minutes)

## Q & A

- Exercise problems from **Worksheet 7** (see “LA worksheet” tab in CS32 website). Answers will be posted next week.
- Questions for today:
  - TBA

# Question X

Content



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# Thank you!

## Q & A