# CS 32 Worksheet 7

This worksheet is entirely **optional**, and meant for extra practice. Some problems will be more challenging than others and are designed to have you apply your knowledge beyond the examples presented in lecture, discussion or projects. All exams will be done on paper, so it is in your best interest to practice these problems by hand and not rely on a compiler.

Solutions are written in red. The solutions for **programming** problems are not absolute, it is okay if your code looks different; this is just one way to solve the specific problem.

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## Concepts

Algorithm Analysis, Trees

1. Consider this function that returns whether or not an integer is a prime number:

```
bool isPrime(int n) {
  if (n < 2 || n % 2 == 0) return false;
  if (n == 2) return true;
  for (int i = 3; (i * i) <= n; i += 2) {
    if (n % i == 0) return false;
  }
  return true;
}
```

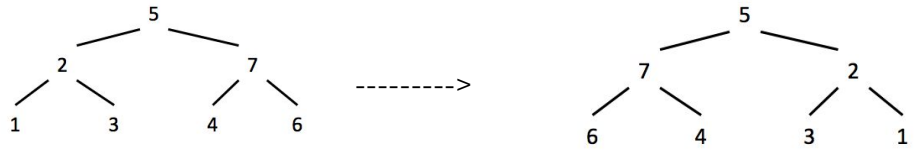
   What is its time complexity?

   O($\sqrt{n}$). The function's loop only runs while i <= $\sqrt{n}$, and the square root is not the same as a constant multiple of n, so we include it in our Big-O analysis.

2. Write a function that returns whether or not an integer value *n* is contained in a binary tree (that might or might not be a binary search tree). That is, it should traverse the entire tree and return true if a *Node* with the value *n* is found, and false if no such *Node* is found. (Hint: recursion is the easiest way to do this.)

```
class Node {
  int val;
  Node* left;
  Node* right;
};

bool treeContains(const Node* head, int n);

bool treeContains(const Node* head, int n) {
  // Base case
  if (head == nullptr) {
    return false;
  } else if (head->val == n) {
    return true;
  } else {
    // Check all children
    return treeContains(head->left, n) ||
           treeContains(head->right, n);
  }
}
```

3. Write a function that takes a pointer to the root of a binary tree and recursively reverses the tree. Example:

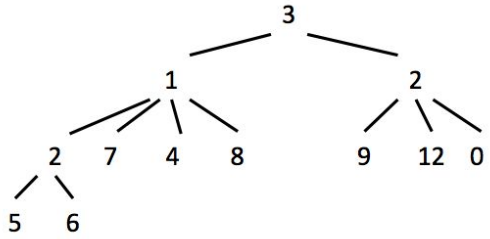



Use the following Node definition and header function to get started.

```
Node {
   int val;
   Node* left;
   Node* right;
};

void reverse(Node* root) {
    if (root != nullptr) {
        Node* temp = root->left;
        root->left = root->right;
        root->right = temp;
```

```
        reverse(root->left);
        reverse(root->right);
    }
}
```

4. Write a function that takes a pointer to a tree and counts the number of leaves in the tree. In other words, the function should return the number of nodes that do not have any children. Note that this is not a binary tree. Also, it would probably help to use recursion. Example:



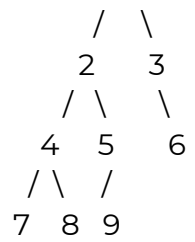

This tree has 8 leaves: 5 6 7 4 8 9 12 0

Use the following Node definition and header function to get started.

```
Node {
   int val;
   vector<Node*> children;
}

int countLeaves(Node* root) {
     if (root == nullptr) // no leaves from this node
          return 0;
     if (root->children.size() == 0) // this node is a leaf
          return 1;

     int count = 0;
     for (int i = 0; i < root->children.size(); i++)
     {
          count += countLeaves(root->children[i]);
     }
     return count;
}
```

5. Write a function that does a level-order traversal of a binary tree and prints out the nodes at each level with a new line between each level.

1 ←- root



Function declaration: `void levelOrder(Node* root)`.
If the root from the tree above was passed as the parameter, levelOrder should print:

1
2 3
4 5 6
7 8 9

Then analyze the time complexity of your algorithm.

```
/* Function to print level order traversal a tree*/
void printLevelOrder(Node* root)
{
    int h = height(root);
    int i;
    for (int i = 1; i <= h; i++)
        printGivenLevel(root, i);
}

/* Print nodes at a given level */
/* It visits no more than 2^(level-1) nodes of the tree */
void printGivenLevel(Node* t, int level)
{
    if (root == nullptr)
        return;
    if (level == 1)
        cout << t->data << " ";
    else if (level > 1)
    {
        printGivenLevel(t->left, level-1);
        printGivenLevel(t->right, level-1);
    }
}

/* Compute the "height" of a tree -- the number of
```

```
    nodes along the longest path from the root node
    down to the farthest leaf node.*/
int height(Node* t)
{
    if (node == nullptr)
        return 0;
    else
    {
        /* compute the height of each subtree */
        int lheight = height(t->left);
        int rheight = height(t->right);

        /* use the larger one */
        if (lheight > rheight)
            return lheight+1);
        else
            return rheight+1;
    }
}
```

2^(1-1) + 2^(2-1) + 2^(3-1) + … + 2^((log n) - 1) ~ 2^(log n) = n. This algorithm has a time complexity of O(n).

6. Implement a level-order traversal of a tree with two queues this time (if you haven't already done so). Analyze the time complexity.
   Hint: We can insert the first level in first queue and print it and while popping from the first queue insert its left and right nodes into the second queue. Now start printing the second queue and before popping insert its left and right child nodes into the first queue. Continue this process till both the queues become empty.

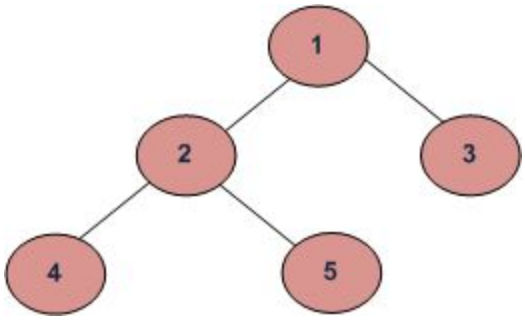
```
#include <iostream>
#include <queue>
using namespace std;

// Iterative method to do level order traversal line by line
void printLevelOrder(Node *root)
{
    // Base Case
    if (root == nullptr)  return;

    // Create an empty queue for level order traversal
```

```
    queue<node*> q;

    // Enqueue Root and initialize height
    q.push(root);

    while ( ! q.empty())
    {
        // Dequeue all nodes of current level and Enqueue all
        // nodes of next level
        int nodeCount = q.size();
        while (nodeCount > 0)
        {
            Node* node = q.front();
            cout << node->data << " ";
            q.pop();
            if (node->left != nullptr)
                q.push(node->left);
            if (node->right != nullptr)
                q.push(node->right);
            nodeCount--;
        }
        cout << endl;
    }
}
```

The time complexity of this algorithm is O(n)

7. Implement all of the following depth-first-search (DFS) traversals of a binary tree, and write the time complexity of each:

   Example Tree:




   a. Inorder Traversal: [4, 2, 5, 1, 3]
   b. Preorder Traversal: [1, 2, 4, 5, 3]
   c. Postorder Traversal: [4, 5, 2, 3, 1]

```
struct Node {
      int val;
      Node* left, right;
};

// We can approach these problems both recursively and
// iteratively. We will use the recursive solutions here,
// for which we will need helper functions.

// Helper void function with reference to vector
void inorder(Node* root, vector<int>& nodes) {
      if (root == nullptr) return;
      inorder(root->left, nodes);
      nodes.push_back(root->val);
      inorder(root->right, nodes);
}
// Use helper
vector<int> inorderTraversal(Node* root) {
      vector<int> nodes;
      inorder(root, nodes);
      return nodes;
}
// Helper void function with reference to vector
void preorder(Node* root, vector<int>& nodes) {
      if(root == nullptr) return;
      nodes.push_back(root->val);
      preorder(root->left, nodes);
      preorder(root->right, nodes);
}
// Use helper
vector<int> preorderTraversal(Node* root) {
      vector<int> nodes;
      preorder(root, nodes);
      return nodes;
}
// Helper void function with reference to vector
void postorder(Node* root, vector<int>& nodes) {
      if(root == nullptr) return;
      preorder(root->left, nodes);
      preorder(root->right, nodes);
      nodes.push_back(root->val);
}
// Use helper
```

```
vector<int> postorderTraversal(Node* root) {
      vector<int> nodes;
      postorder(root, nodes);
      return nodes;
}
```

8. Given a binary tree, return all root-to-leaf paths. What is the time complexity of this function. For example, given the following binary tree:

```
  1
/  \
2   3
\
 5
```

All root-to-leaf paths are:
["1->2->5", "1->3"]

```
struct Node {
      int val;
      Node* left, right;
};

// We will use a helper function to keep a track of the current
// path

void rootToLeaf(Node* root, vector<std::string>& paths,
                                                std::string curr)
{
      if (root->left == nullptr  &&  root->right == nullptr) {
            paths.add(curr);
            return;
      }
      if (root->left != nullptr) {
            rootToLeaf(root->left, paths, curr + "->" +
                                 std::to_string(root->left->val));
      }
      if (root->right != nullptr) {
            rootToLeaf(root->right, paths, curr + "->" +

std::to_string(root->right->val));
      }
}
// Use helper
```

```
vector<std::string> rootToLeaf(Node* root) {
      vector<std::string> paths;
      if (root != nullptr)
            rootToLeaf(root, paths, std::to_string(root->val));
      return paths;
}
```

9. What is the time complexity of the following code?

```
int randomSum(int n) {
  int sum = 0;
  for(int i = 0; i < n; i++) { 1^2 + 2^2 + … + (n-1)^2 is
O(n^3)
    for(int j = 0; j < i; j++) { O(i^2)
      if(rand() % 2 == 1) {
        sum += 1;
      }
      for(int k = 0; k < j*i; k += j) { O(i)
        if(rand() % 2 == 2) {
          sum += 1;
        }
      }
    }
  }
  return sum;
}
```

O(n^3), note it doesn't matter that "if(rand() %2 == 2)" is always false, rand is still run each time

10. Write two functions. The first function, tree2Vec, turns a binary tree of positive integers into an vector. The second function, vec2Tree, turns a vector of integers into a binary tree. The key to these functions is that the first function must transform the tree such that the second function can reverse it. So,

`tree2Vec(vec2Tree(`*vec*`)) == vec.`

```
Node* vec2Tree(vector<int> v);
vector<int> tree2Vec(Node* root);
```

*Hint: Think about how to encode the parent-child relationship of a tree inside of a vector.
*Hint 2: Note the vector length doesn't need to equal the number of nodes

```
bool hasNonNull(vector<Node*>& v) {
  for (int i = 0; i < v.size(); i++) {
    if (v[i] != nullptr) {
      return true;
    }
  }
  return false;
}

vector<int> tree2Vec(Node* root) {
  vector<int> myV;
  vector<Node*> curLevel;
  vector<Node*> nextLevel;
  curLevel.push_back(root);
  myV.push_back(root->value);
  while(hasNonNull(curLevel)) {
    // cout << "New Iteration" << endl;
    // printVector(curLevel);
    for(int i = 0; i < curLevel.size(); i++) {
      // printVector(nextLevel);
      Node* curNode = curLevel[i];
      if(curNode == nullptr) {
        myV.push_back(-1);
        myV.push_back(-1);
        nextLevel.push_back(nullptr);
        nextLevel.push_back(nullptr);
        continue;
      }

      nextLevel.push_back(curNode->left);
      if(curNode->left == nullptr){
        myV.push_back(-1);
      } else {
        myV.push_back(curNode->left->value);
      }

      nextLevel.push_back(curNode->right);
      if(curNode->right == nullptr) {
        myV.push_back(-1);
      } else {
        myV.push_back(curNode->right->value);
      }
```

```
    }

    curLevel = nextLevel;
    nextLevel.clear();
  }

  return myV;
}

Node* vec2TreeHelper(vector<int> v, int i, Node* nodeI) {
  int leftIndex = i*2+1;
  int rightIndex = i*2+2;
  int leftValue = v[leftIndex];
  int rightValue = v[rightIndex];

  if(leftValue != -1) {
    nodeI->left = new Node;
    nodeI->left->value = leftValue;
    vec2TreeHelper(v, leftIndex, nodeI->left);
  } else {
    nodeI->left = nullptr;
  }

  if(rightValue != -1) {
    nodeI->right = new Node;
    nodeI->right->value = rightValue;
    vec2TreeHelper(v, rightIndex, nodeI->right);
  } else {
    nodeI->right = nullptr;
  }

  return nodeI;
}

Node* vec2Tree(vector<int> v) {
  Node* n = new Node;
  n->value = v[0];
  // n->left = nullptr;
  // n->right = nullptr;
  return vec2TreeHelper(v, 0, n);
}
```

11. Find the time complexity of the following function

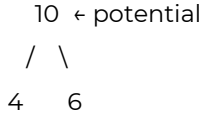
```
int obfuscate(int a, int b) {
      vector<int> v;
      set<int> s;
      for (int i = 0; i < a; i++) {
            v.push_back(i);
            s.insert(i);
      }
      v.clear();

      int total = 0;
      if (!s.empty()) {
            for (int x = a; x < b; x++) {
                  for (int y = b; y > 0; y--) {
                        total += (x + y);
                  }
            }
      }
      return v.size() + s.size() + total;
}
```

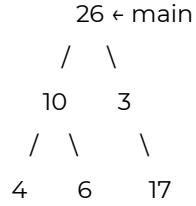
O(a log a + b^2)

12. Write the following recursive function:

```
bool isSubtree(Node* main, Node* potential);

struct Node {
      int val;
      Node* left, right;
};
```

The function should return true if the binary tree with the root, `potential`, is a subtree of the tree with root, `main`. You may use any helper functions necessary. A subtree of a tree `main` is a tree `potential` that contains a node in `main` and all of its descendants in `main`.



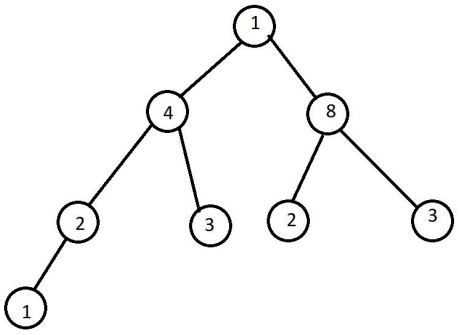



`isSubtree(main, potential)` would return true.

```
// helper function
bool identical(Node* r1, Node* r2)
{
	// Base case
	if (r1 == nullptr && r2 == nullptr)
		return true;

	if (r1 == nullptr || r2 == nullptr)
		return false;


	// Check if the data of both roots is same
	// and data of left and right subtrees are also same
	return   (r1->val   ==   r2->val   &&   identical(r1->left,
r2->left) && identical(r1->right, r2->right) );
}

bool isSubtree(Node* main, Node* potential)
{
	// Base cases
	if (potential == nullptr)
		return true;

	if (main == nullptr)
		return false;

	if (identical(main, potential))
		return true;

	return     isSubtree(main->left, potential) ||
		   isSubtree(main->right, potential);
}
```

13. Write a function that finds the maximum depth of a binary tree. A tree with only one node has a depth of 0; let's decree that an empty tree has a depth of -1.

```
struct Node {
	int val;
	Node* left, right;
};

int maxDepth(Node *root);
```




This tree has a maximum depth of 3.

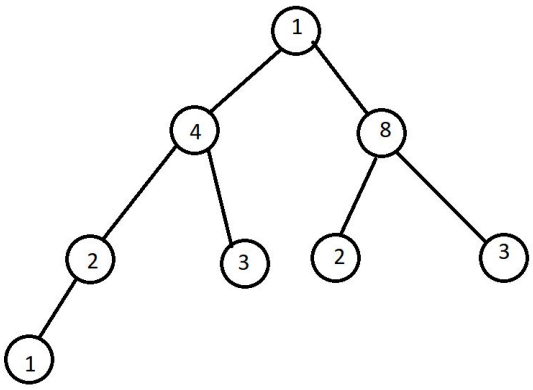
```
int maxDepth(Node* root) {
    if (root == nullptr)
        return -1;
    // computer depth of each subtree
    int lDepth = maxDepth(root->left);
    int rDepth = maxDepth(root->right);

    // return the max of the two
    if (lDepth > rDepth)
        return lDepth+1;
    else
        return rDepth+1;
}
```

14. Implement the following function, `getGreatestPath`.

```
int getGreatestPath(Node* head);

struct Node {
     int val;
     Node* left, right;
};
```

The value of a path in a binary tree is defined as the sum of all the values of the nodes within that path. This function takes a pointer to the head of a binary tree, and it finds the value of the path from the head to a leaf with the greatest value.

Ex: The following tree has a greatest path value of 12 (1->8->3).




```
int getGreatestPath(Node* head) {
  if (head == nullptr)
    return 0;
  int leftMax = getGreatestPath(head->left);
  int rightMax = getGreatestPath(head->right);

  if (leftMax > rightMax)
    return head->val + leftMax;
  else
    return head->val + rightMax;
}
```

15. Here are the elements of an array after each of the first few passes of a sorting algorithm discussed in class. Which sorting algorithm is it?

3 7 4 9 5 2 6 1

**3** 7 4 9 5 2 6 1

3 **7** 4 9 5 2 6 1

3 **4** 7 9 5 2 6 1

3 4 7 **9** 5 2 6 1

3 4 **5** 7 9 2 6 1

**2** 3 4 5 7 9 6 1

2 3 4 5 **6** 7 9 1

**1** 2 3 4 5 6 7 9

a. bubble sort
b. insertion sort
c. quicksort with the pivot always being chosen as the first element

d. quicksort with the pivot always being chosen as the last element

Notice that the section to the left of the underlined digit is in sorted order. One of the most prominent insertion sort characteristics is having a section that is already in sorted order and continue to put the current element in the right place.

16. Given the following vectors of integers and sorting algorithms, write down what the vector will look like after 3 iterations or steps and whether it has been perfectly sorted.

    a. `{45, 3, 21, 6, 8, 10, 12, 15}` insertion sort
    b. `{5, 1, 2, 4, 8}` bubble sort
    c. `{-4, 19, 8, 2, -44, 3, 1, 0}` quicksort (where pivot is always the last element)

```
a. {3 6 21 45 8 10 12 15}     not perfectly sorted
b. {1, 2, 4, 5, 8}            perfect sorted, the algorithm
   finishes before three iterations
c. 1st iteration -> {-4, -44, 0, 2, 19, 3, 1, 8}
      -4 and -44 might be in a different order
      2, 19, 3, 1, 8 might be in a different order
   2nd iteration, if starting from the order shown after the
      1st iteration, and left part is sorted before right
      part -> {-44, -4, 0, 2, 19, 3, 1, 8}
   3rd iteration, if starting from the order shown after the
      2nd iteration, and left part is sorted before right
      part -> {-44, -4, 0, 2, 1, 3, 8, 19}
      2, 1, 3 might be in a different order
   Not perfectly sorted
```

17. Given an array of *n* integers, where each integer is guaranteed to be between 1 and 100 (inclusive) and duplicates are allowed, write a function to sort the array in O(*n*) time.
    (Hint: the key to getting a sort faster than O(*n log n*) is to avoid directly comparing elements of the array!) (MV)

```
void sort(int a[], int n);

void sort(int a[], int n) {
  int counts[100] = {};  // Count occurrences of each integer.
```

```
  for (int i = 0; i < n; i++)
    counts[a[i] - 1]++;

  // Add that many of each integer to the array in order.
  int j = 0;
  for (int i = 0; i < 100; i++)
    for (; counts[i] > 0; counts[i]--)
      a[j++] = i + 1;
}
```