Probablistic Robotics: Nonparametric Filters (chpater 4) Exercise Solutions

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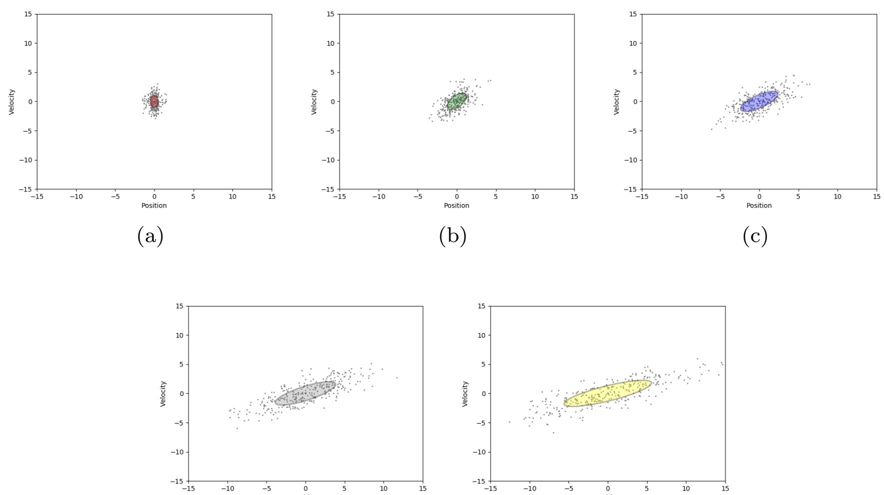
Problem 1.a



(a) (b) (c) (d) (e)

Figure 1: From (a) - (e) are the $t = 1$ to $t = 5$ beliefs of the prediction steps of the Kalman filter shown in ellipses and the histogram filter shown in point.

In order to implement the histogram filter, we need to discretize the state space. In this problem, the state space is represented as the x coordinate and the velocity, $\mathbf{x}_t = [x_t, \dot{x}_t]^T$. To discretize the state space, we need to introduce some notations listed below.

1. x_t represents the state of the robot at time t.
2. $x_{i,t}$ represents the value of the i^{th} dimension of the robot state at time t.

k	x	$\dot{x}$
0	$[0, \Delta_0)$	$[0, \Delta_1)$
1	$[0, \Delta_0)$	$[\Delta_1, 2\Delta_1)$
...	...	...
$g_1 - 1$	$[0, \Delta_0)$	$[(g_1 - 1)\Delta_1, g_1\Delta_1)$
g_1	$[\Delta_0, 2\Delta_0)$	$[0, \Delta_1)$
...	...	...
$g_0g_1 - 1$	$[(g_0 - 1)\Delta_0, g_0\Delta_0)$	$[(g_1 - 1)\Delta_1, g_1\Delta_1)$

Table 1: The table for illustrating the discretization. Each row represents one state region. The first column (k) is the state region index. The last two columns describe the regions (intervals) of the discretization. The total number of discrete states is g_0g_1, where g_0, g_1 represent the number of decomposition in x and $\dot{x}$.

3. k is the discrete state index.
4. $s_{k,t}$ describes a convex region of the state space. The union of all the regions is the entire state space.
5. $s_{k,t}^i$ is the range of the i^{th} dimension of the state in the region $s_{k,t}$.

For each dimension of the state space, the number of discretized grids is also defined as $g_i, i \in [0, I-1]$, where I is the number of dimensions. Thus, the resolution of the i^{th} dimension can be defined as

$$\Delta_i = \frac{\max(x_i) - \min(x_i)}{g_i}, \tag{1}$$

where max and min are the maximum and minimum values of the i^{th} dimension. The last dimension is discretized the first. In this case, the velocity is discretized first and then the x position. This can be represented using a table as shown in Table 1.

Eq. 2 shows the function to map from the continuous state value to the discrete.

$$k = \pi(x_t) = \sum_{k=0}^{K} \frac{x_{K-k,t} - \min(x_{K-k,t})}{\Delta_{K-k}} \prod_{i=0}^{k-1}(g_{K-i,t}), \tag{2}$$

where K is the total number of regions (discrete states) and $\min(x_{K-i}, t)$ is the maximum value of the i^{th} dimension.

Eq. 3 is the mapping function which maps from the discrete state region $s_{k,t}$ to the continuous value. After mapping, the value of each dimension of the continuous value is the mid-point value of the corresponding dimension in the region $s_{k,t}$.

$$x_t = f(s_{k,t}) = \min(s_{k,t}) + \frac{\max(s_{k,t}) - \min(s_{k,t})}{2}, \tag{3}$$

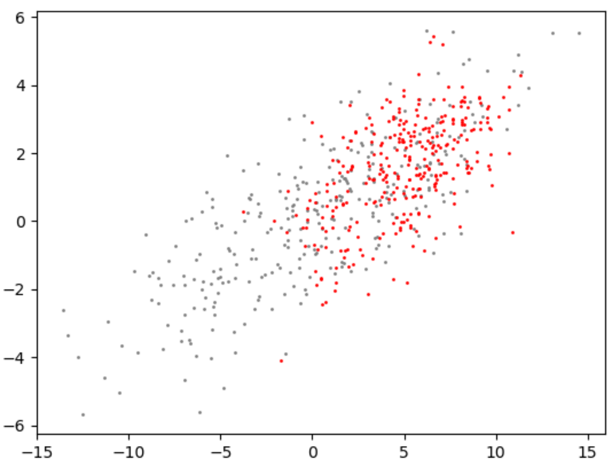


Figure 2: The comparison between the belief distribution before (in gray) and after (in red) of the measurement update step of the histogram filter.

where $\max(s_{k,t}), \min(s_{k,t})$ are two vectors represent the maximum and minimum values of each dimension in the region $s_{k,t}$.

Eq. 4 is the prediction step of the histogram filter for the region $s_{k,t}$.

$$p_{k,t} = \sum_{i}^{K} P(x_t = f(s_k)|u_t, x_{t-1} = f(s_i))p_{i,t-1}, \tag{4}$$

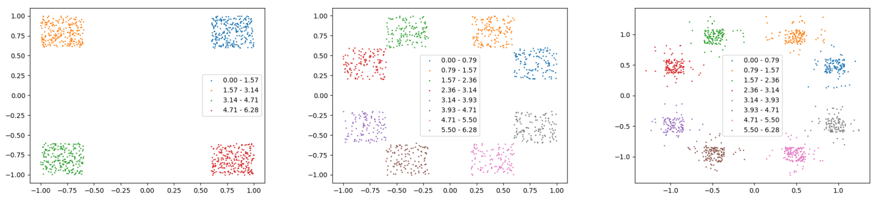
where $p_{k,t}$ and $p_{i,t-1}$ are the belief for the k^{th} state region at time t and i^{th} state region at time $t-1$. Note that we ignore the time step in f because we assume the state region does not change over time.

In this question, the range of the state is set to $x \in [-15, 15], \dot{x} \in [-15, 15]$ and the number of grids at each dimension is set to 29. Fig. 1 compares the the prediction step results of the Kalman filter and the histogram filter. From the figures, we can see that the histogram filter shares the same trend of the Kalman filter which is the optimal estimation when the motion model is a linear function.

Problem 1.b

Fig. 2 shows belief posterior after the measurement update step. The red and gray dots are the samples from the posterior distribution before and after the measurement update, respectively. As we can see, the red dots are more centered around the measurement $x = 8$. Note that different runs should have different posterior since the measurement is random sampled.

Problem 2.a



(a) 9-9-4 (b) 9-9-8 (c) 15-15-8

Figure 3: The three figures show the prediction step result in different discretization of the state space. The labels show different heading ranges. The ranges for the three state dimensions are $x \in [-1.8, 1.8]$, $y \in [-1.8, 1.8]$, and $\theta \in [0, 2\pi]$. The number under figures show the discretization along the three dimensions.

The motion model for the nonlinear dynamics can be expressed in Eq. 5.

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \\ \theta_{t+1} \end{bmatrix} = \begin{bmatrix} x_t + \cos\theta_t \\ y_t + \sin\theta_t \\ \theta_t \end{bmatrix}. \tag{5}$$

To discretize the state space, we specify the range of the three state dimensions as $x \in [-1.8, 1.8]$, $y \in [-1.8, 1.8]$, and $\theta \in [0, 2\pi]$. The initial mean and covariance are given by

$$\mu_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \Sigma_0 = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 10000 \end{bmatrix}. \tag{6}$$

The initial belief is given by

$$p_{k,0} = \eta \mathcal{N}(x = f(k); \mu_0, \Sigma_0), \tag{7}$$

where the normalizer $\eta = \frac{1}{\sum_{k=0}^{K} \mathcal{N}(x=f(k); \mu_0, \Sigma_0)}$.

Fig 3 shows the posterior after the prediction step under different discretizations. Compared to the EKF, the histogram filter gives a better result, but the result highly depends on the discretization. The finer grids used in the heading θ, the more areas in $x - y$ coordinates the histogram filter will cover.

Problem 2.b

Eq. 8 shows the measurement model in this problem

$$z_t = Cx_t + \sigma_t, \tag{8}$$

where $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ and $\sigma_t \sim \mathcal{N}(\sigma_t; 0, 0.01)$. Fig 4 shows the sampled posterior belief after the measurement update, where the measurement is $x = 1.0214$.

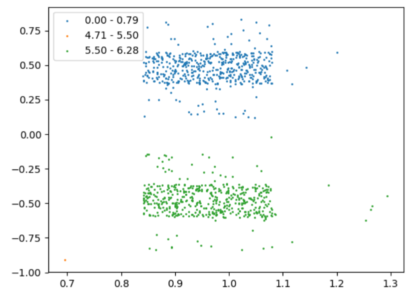
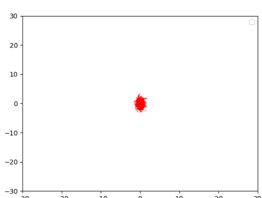
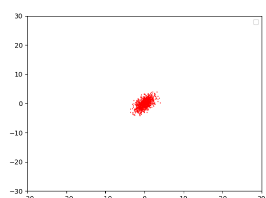


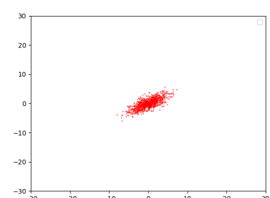
Figure 4: The distribution of the belief posterior after seeing the measurement at $x = 1.0214$.



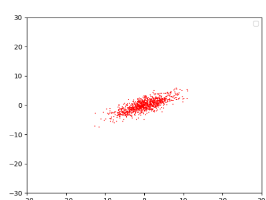
(a) t=0



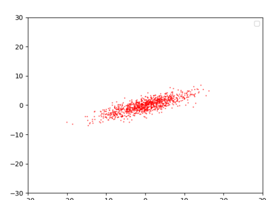
(b) t=1



(c) t=2



(d) t=3



(e) t=4

Figure 5: The five figures represent 1000 particles in the particle filter's posterior belief before the measurement update step from time $t = 0$ to $t = 4$.

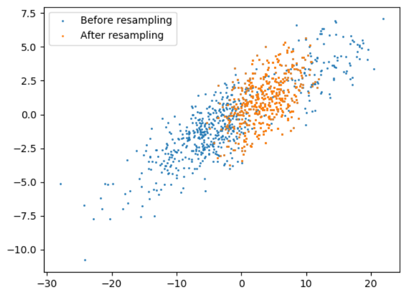


Figure 6: The comparison between the belief after the resampling step (orange) and the sampling step (blue). The measurement is $x = 4.43$.

The $15-15-8$ discretization scheme is used in this question.

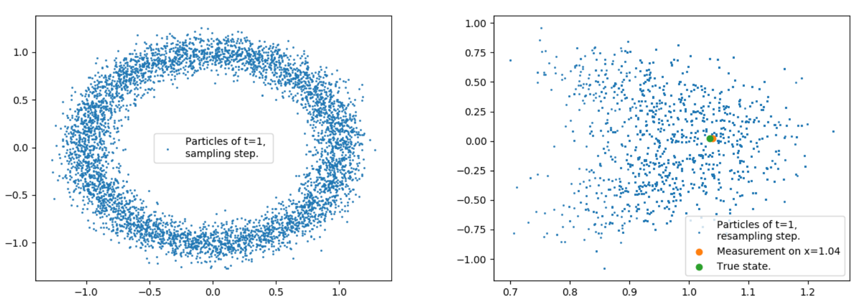
Problem 4

In this question, the motion model and measurement model are the same as in the Problem.1. Here, 1000 number of particles are used to approximate the belief distribution. The prediction (sampling) step result are shown in Fig. 5, for the linear motion model, the result of the prediction step is similar to the ones in the Kalman filter and the histogram filter.

Fig. 6 shows the posterior belief of the resampling step of the particle filter. The particles are more compact and center around the measurement, $x = 4.43$.

Problem 5

The settings in this question are the same as Problem 2 and the results are shown in Fig. 7. (Not finished yet...)



(a) (b)

Figure 7: (a) The particles at $t = 1$ after taking the control $d = 1$. (b) The particles at $t = 1$ after seeing the measurement $= 1$.