Advertising on the Web

CS246: Mining Massive Datasets
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http://cs246.stanford.edu

Infinite data: Online Algorithms

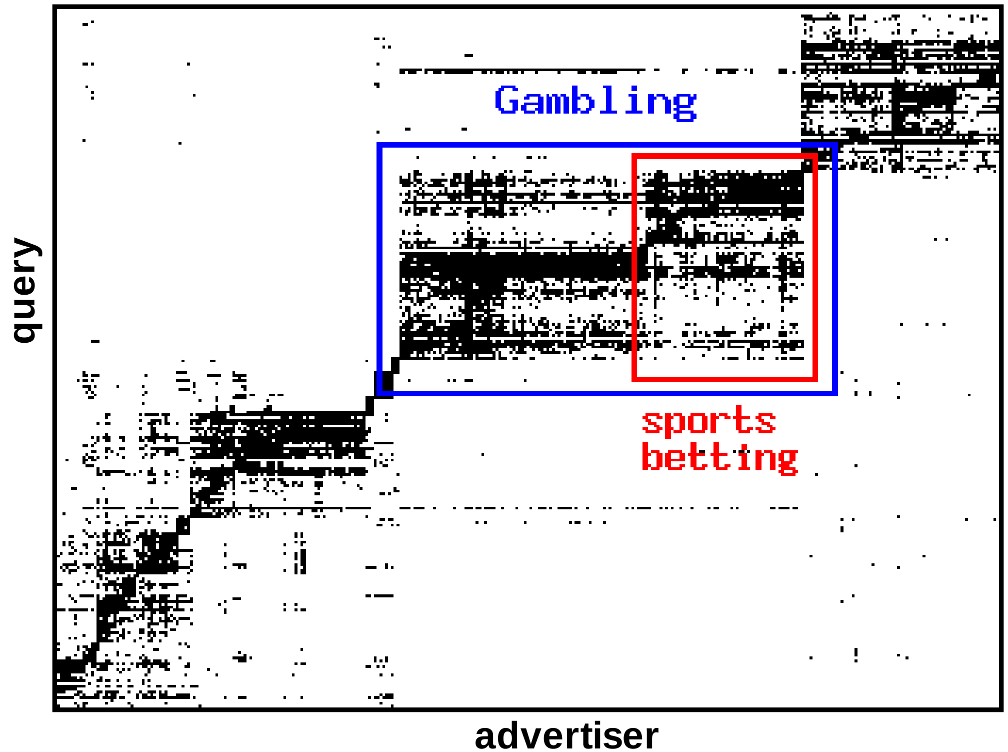
High dim. data	Graph data	Infinite data	Machine learning	Apps
Locality sensitive hashing	**PageRank,** SimRank	Filtering data streams	SVM	Recommen der systems
Clustering	Community Detection	Web advertising	Decision Trees	Association Rules
Dimensional ity reduction	Spam Detection	Queries on streams	Parallel SGD	Duplicate document detection

Online Algorithms

- **Classic model of algorithms**
  - You get to see the entire input, then compute some function of it
  - In this context, "**offline** algorithm"
- **Online Algorithms**
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way
  - **Similar to the data stream model**

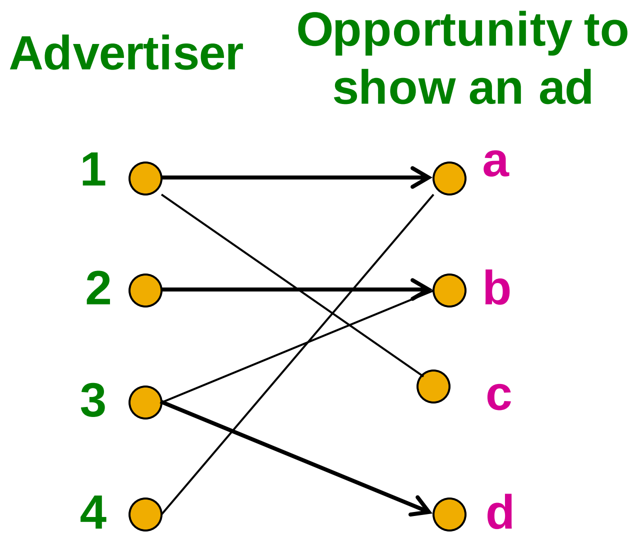
Sponsored Search: Ads

- **Query-to-advertiser graph:**



[Andersen, Lang: Communities from seed sets, 2006]

Graph Matching for Advertising



Advertiser X wants to show an ad for topic/query Y

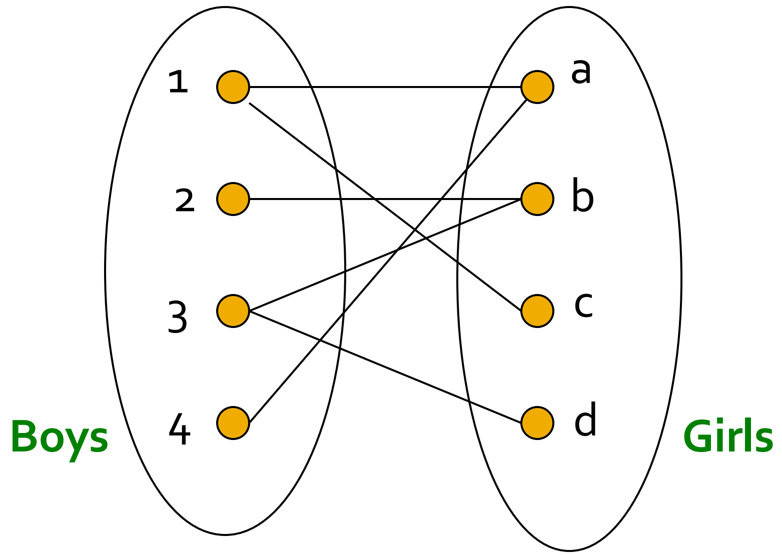
Which advertiser gets picked

(1,a)
(2,b)
(3,d)

This is an online problem: We have to make decisions as queries/topics show up. We do not know what topics will show up in the future.

Online Bipartite Matching

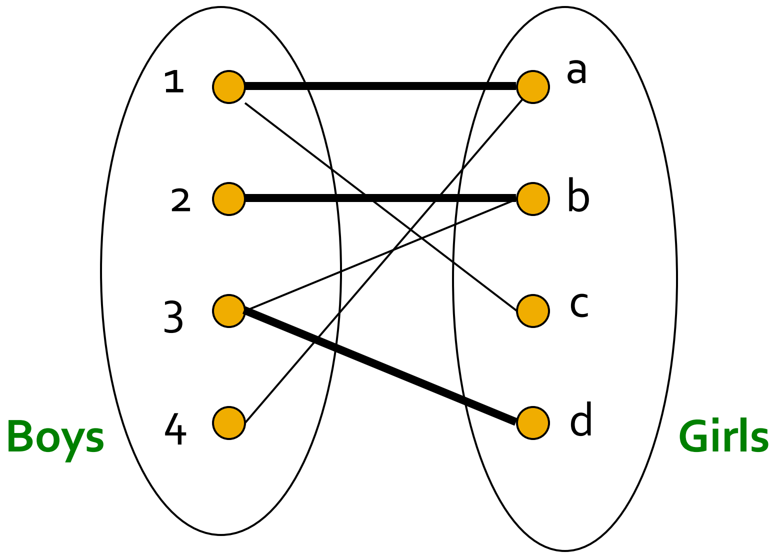
Example: Bipartite Matching



Nodes: Boys and Girls; Links: Preferences

Goal: Match boys to girls so that the most preferences are satisfied

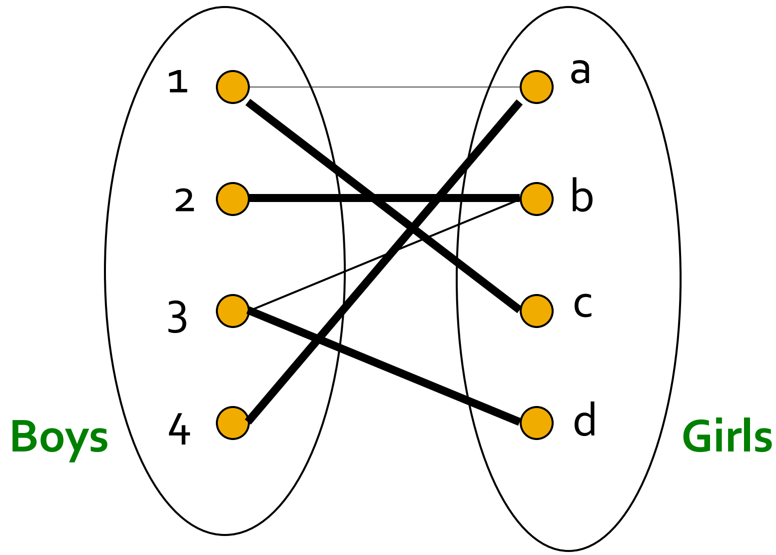
Example: Bipartite Matching



M = {(1,a),(2,b),(3,d)} is a **matching**

Cardinality of matching = |M| = 3

Example: Bipartite Matching



M = {(1,c),(2,b),(3,d),(4,a)} is a
perfect matching

Perfect matching … all vertices of the graph are matched
Maximum matching … a matching that contains the largest possible number of matches

Matching Algorithm

- **Problem: Find a maximum matching for a given bipartite graph**
  - A perfect one if it exists

- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm)

- **But what if we do not know the entire graph upfront?**

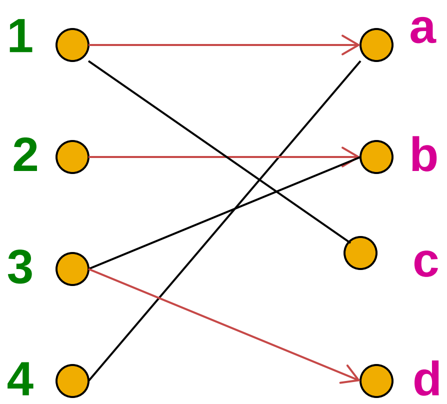
Online Graph Matching Problem

- Initially, we are given the set **boys**
- In each **round**, **one girl's choices are revealed**
  - That is, the girl's **edges** are revealed
- **At that time, we have to decide to either:**
  - Pair the **girl** with a **boy**
  - Do not pair the **girl** with any **boy**

- **Example of application:**

  Assigning tasks to servers

Online Graph Matching: Example



(1,a)
(2,b)
(3,d)

Greedy Algorithm

- **Greedy algorithm for the online graph matching problem:**
  - Pair the new girl with **any** eligible boy
    - If there is none, do not pair the girl
- **How good is the algorithm?**

Competitive Ratio

- For input I, suppose greedy produces matching M_{greedy} while an optimal matching is M_{opt}

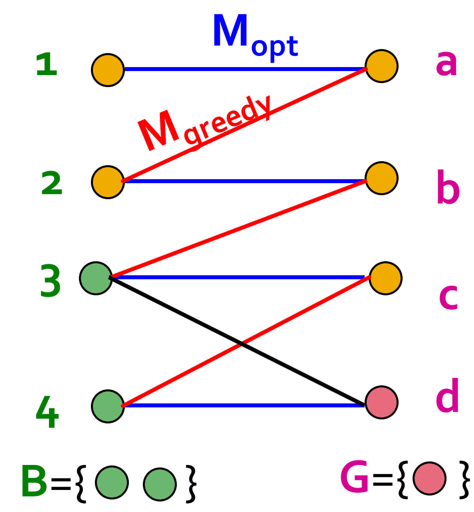
Competitive ratio =

$$\min_{\text{all possible inputs } I} (|M_{greedy}|/|M_{opt}|)$$

(what is greedy's worst performance over all possible inputs *I*)

Analyzing the Greedy Algorithm

- Consider a case: $M_{greedy} \neq M_{opt}$
- Consider the set G of girls matched in M_{opt} but not in M_{greedy}

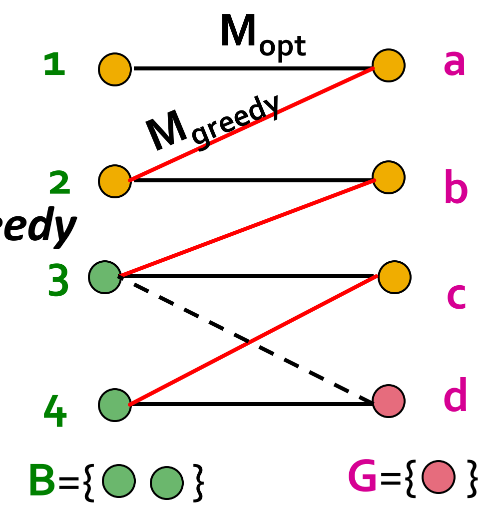


- **(1)** By definition of G:
  $|M_{opt}| \leq |M_{greedy}| + |G|$
- **(2)** Define set ***B of boys*** linked to girls in G
  - Notice boys in B are already matched in M_{greedy}. Why?
    - If there would exist such non-matched (by M_{greedy}) boy adjacent to a non-matched girl then greedy would have matched them

So: $|M_{greedy}| \geq |B|$

Analyzing the Greedy Algorithm

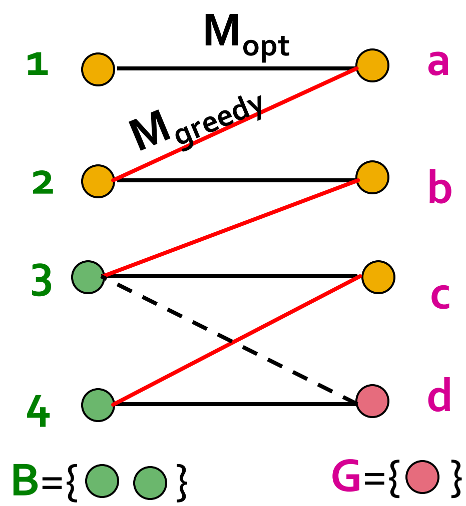
- **Summary so far:**
  - Girls G matched in M_{opt} but not in M_{greedy}
  - Boys B adjacent to girls in G
  - **(1)** $|M_{opt}| \leq |M_{greedy}| + |G|$
  - **(2)** $|M_{greedy}| \geq |B|$



- Optimal matches all girls in G to (some) boys in B
  - **(3)** $|G| \leq |B|$

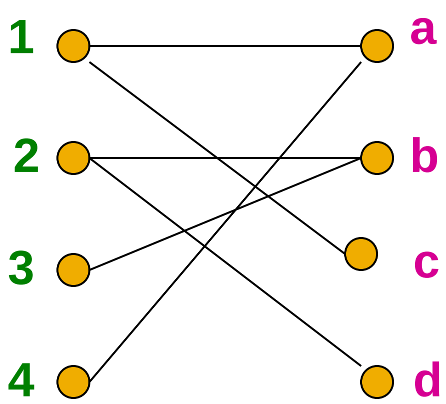
- Combining **(2)** and **(3)**:
  - $|G| \leq |B| \leq |M_{greedy}|$

Analyzing the Greedy Algorithm



- **So we have:**
  - **(1)** $|M_{opt}| \leq |M_{greedy}| + |G|$
  - **(4)** $|G| \leq |B| \leq |M_{greedy}|$
- Combining **(1)** and **(4)**:
  - Worst case is when $|G| = |B| = |M_{greedy}|$
  - $|M_{opt}| \leq |M_{greedy}| + |M_{greedy}|$
  - **Then** $|M_{greedy}|/|M_{opt}| \geq 1/2$

Worst-case Scenario



(1,a)
(2,b)

Web Advertising

History of Web Advertising

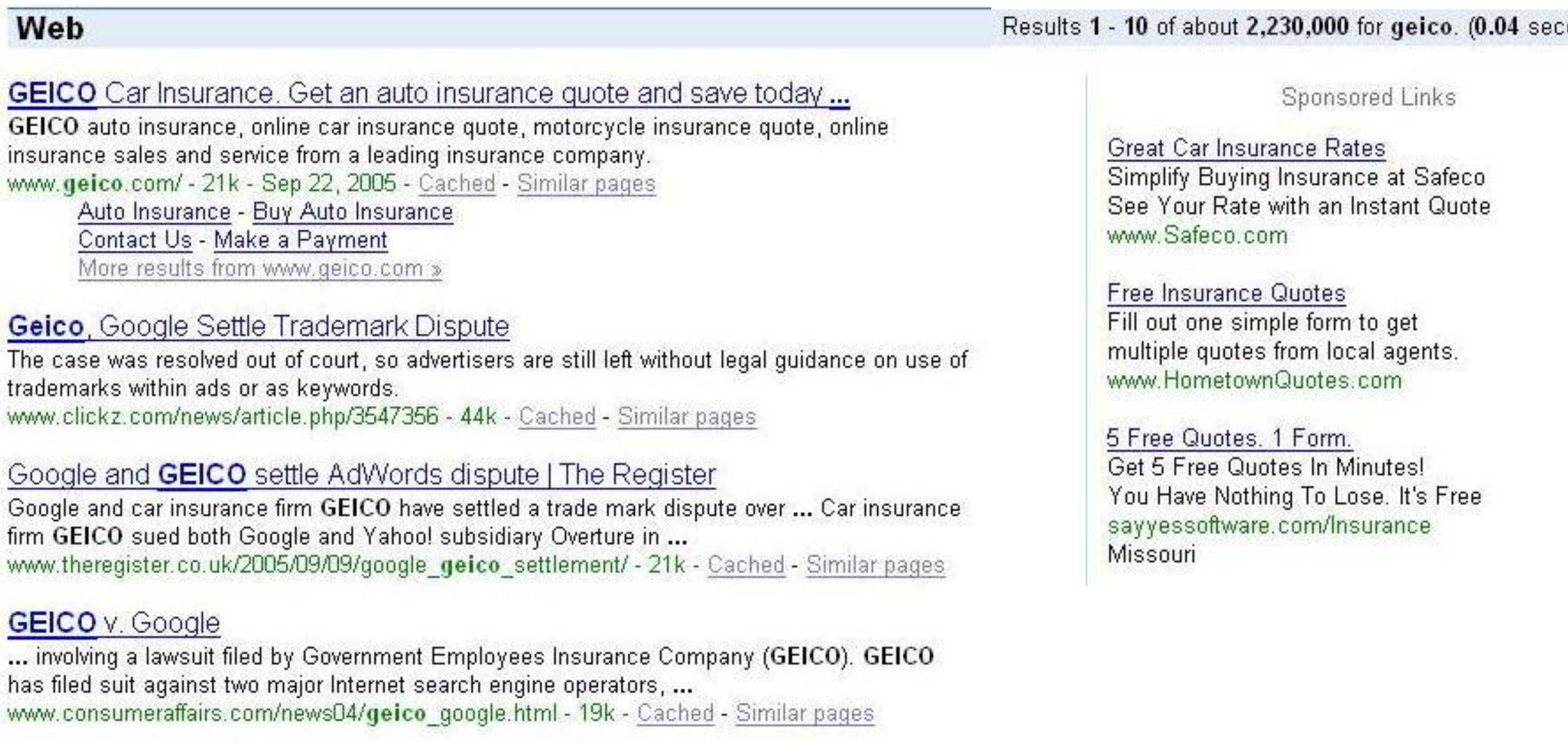
- **Banner ads (1995-2001)**
  - Initial form of web advertising
  - Popular websites charged $X for every 1,000 "impressions" of the ad
    - Called "**CPM**" rate (Cost per thousand impressions)
    - Modeled similar to TV, magazine ads
  - From **untargeted** to **demographically targeted**
  - **Low click-through rates**
    - Low ROI for advertisers

CPM…cost per *mille*
Mille…thousand in Latin

Performance-based Advertising

- **Introduced by Overture around 2000**
  - Advertisers **bid** on **search keywords**
  - When someone searches for that keyword, the **highest bidder's ad is shown**
  - Advertiser is charged only if the ad is clicked on
- Similar model adopted by Google with some changes around 2002
  - Called **Adwords**

Ads vs. Search Results



Web 2.0

- **Performance-based advertising works!**
  - Multi-billion-dollar industry
- **Interesting problem:**
  Which ads to show for a given query?
  - (Today's lecture)
- **If I am an advertiser, which search terms should I bid on and how much should I bid?**
  - (Not focus of today's lecture)

Adwords Problem

- A stream of queries arrives at the search engine: $q_1, q_2, \ldots$
- Several advertisers bid on each query
- When query q_i arrives, search engine must pick a subset of advertisers to show their ads

- Goal: **Maximize search engine's revenues**
  - Simple solution: Instead of raw bids, use the "**expected revenue per click**" (i.e., **Bid*CTR**)
- **Clearly we need an online algorithm!**

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.25 cents

Click through rate

Expected revenue

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.25 cents
A	$1.00	1%	1 cent

Instead of sorting advertisers by bid, sort by expected revenue

Limitations of Simple Algorithm

Instead of sorting advertisers by bid, sort by expected revenue

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.25 cents
A	$1.00	1%	1 cent

Challenges:

- CTR of an ad is unknown
- Advertisers have limited budgets and bid on multiple queries

Complications: Budget

- **Two complications:**
  - **Budget**
  - **CTR of an ad is unknown**

1) Budget: Each advertiser has a limited budget

- **Search engine guarantees that the advertiser will not be charged more than their daily budget**

Complications: CTR

- **2) CTR (Click-Through Rate): Each ad-query pair has a different likelihood of being clicked**
  - **Advertiser 1** bids \$2 on query A, click probability = 0.1
  - **Advertiser 2** bids \$1 on query B, click probability = 0.5
- **CTR** is predicted or measured historically
  - Averaged over a time period

- **Some complications we will not cover:**
  - **1) CTR is position dependent:**
    - Ad #1 is clicked more than Ad #2

Complications: CTR

- **Some complications we will cover (next lecture):**
  - **2) Exploration vs. exploitation**
    Exploit: Should we keep showing an ad for which we have good estimates of click-through rate?
    or
    Explore: Shall we show a brand new ad to get a better sense of its click-through rate?

Online Algorithms
The BALANCE Algorithm

Adwords Problem

- **Given:**
  - **1.** A set of bids by advertisers for search queries
  - **2.** A click-through rate for each advertiser-query pair
  - **3.** A budget for each advertiser (say for 1 month)
  - **4.** A limit on the number of ads to be displayed with each search query
- **Respond to each search query with a set of advertisers such that:**
  - **1.** The size of the set is no larger than the limit on the number of ads per query
  - **2.** Each advertiser has bid on the search query
  - **3.** Each advertiser has enough budget left to pay for the ad if it is clicked upon

Greedy Algorithm

- **Our setting: Simplified environment**
  - There is **1** ad shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Bid value of each ad is the same (=$**1**)
- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is 1/2**

Bad Scenario for Greedy

- **Two advertisers A and B**
  - ***A*** bids on query ***x***, ***B*** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream:** ***x x x x y y y y***
  - Worst case greedy choice: ***B B B B*** **_ _ _ _**
  - Optimal: **A A A A B B B B**
  - **Competitive ratio = ½**
- **This is the worst case!**
  - **Note:** Greedy algorithm is deterministic – it always resolves draws in the same way

BALANCE Algorithm [MSVV]

- **BALANCE** Algorithm by Mehta, Saberi, Vazirani, and Vazirani
  - **For each query, pick the advertiser with the <u>largest unspent budget</u>**
    - Break ties arbitrarily (**but in a deterministic way**)

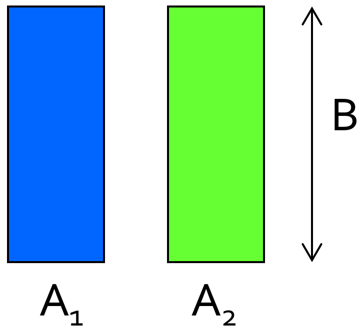
Example: BALANCE

- **Two advertisers A and B**
  - **A** bids on query ***x***, **B** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream:** ***x x x x y y y y***
- **BALANCE choice:** **A B A B B B _ _**
  - Optimal: **A A A A B B B B**
- **In general:** For **BALANCE** on **2** advertisers
  Competitive ratio = ¾

Analyzing BALANCE

- **Consider simple case (w.l.o.g.):**
  - **2** advertisers, $\mathbf{A_1}$ and $\mathbf{A_2}$, each with budget **B** ($\geq$1)
  - Optimal solution exhausts both advertisers' budgets
- **BALANCE must exhaust at least one budget:**
  - **If not, we can allocate more queries**
    - Whenever BALANCE makes a mistake (both advertisers bid on the query), advertiser's unspent budget only decreases
    - Since optimal exhausts both budgets, one will for sure get exhausted
  - Assume BALANCE exhausts $\boldsymbol{A_2}$'s budget, but allocates ***x*** queries fewer than the optimal
    - **So revenue of *BALANCE = 2B – x*** **(where OPT is 2B)**
  - **Let's work out what x is!**

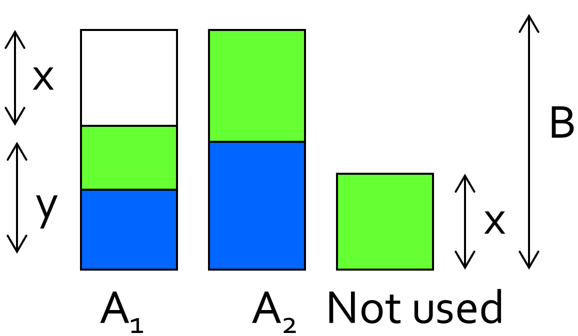
Analyzing Balance



Queries allocated to A_1 in optimal solution

Queries allocated to A_2 in optimal solution

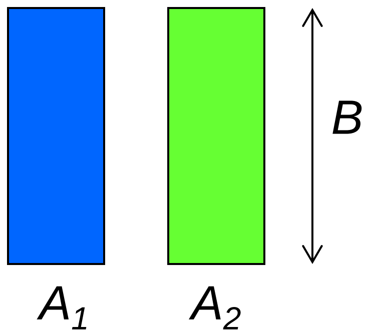
Opt revenue = 2B
Balance revenue = 2B-x = B+y



Balance allocation

We claim y ≥ x (next slide).
Balance revenue is minimum for x=y=B/2.
Minimum Balance revenue = 3B/2.
Competitive Ratio = 3/4.

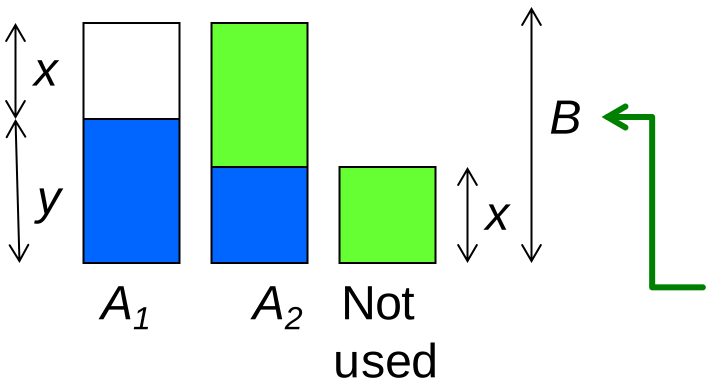
Analyzing BALANCE: What's x?



Queries allocated to A_1 in the optimal solution

Queries allocated to A_2 in the optimal solution

Optimal revenue = **2B**
Assume Balance gives revenue **= 2B-x = B+y**
Assume we exhausted A_2's budget

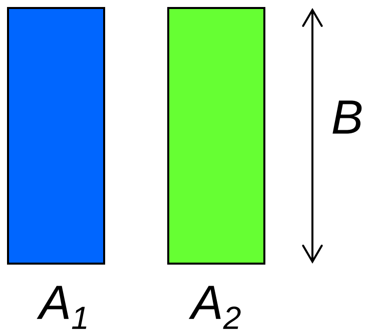


Notice: Unassigned queries should be assigned to A_2 (since if we could assign to $\mathbf{A_1}$ we would since we still have the budget)

Goal: Show we have $\mathbf{y \geq B/2}$

Case 1) BALANCE assigns at $\geq$B/2 blue queries to A_1.
Then trivially, $\boldsymbol{y} \geq \boldsymbol{B}/\mathbf{2}$

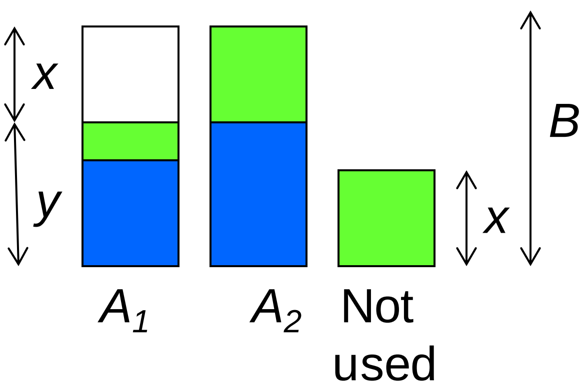
Analyzing BALANCE: What's x?



Queries allocated to $\boldsymbol{A_1}$ in the optimal solution

Queries allocated to $\boldsymbol{A_2}$ in the optimal solution

Optimal revenue = **2B**
Assume Balance gives revenue **= 2B-x = B+y**
Assume we exhausted A_2's budget



Unassigned queries should be assigned to A_2
(if we could assign to $\mathbf{A_1}$ we would since we still have the budget)

Goal: Show we have **y ≥ B/2**

Balance revenue is minimum for $x = y = B/2$
Minimum Balance revenue = $\mathbf{3B/2}$
Competitive Ratio: BAL/OPT = 3/4

Case 2) BALANCE assigns ≥B/2 blue queries to A_2.
Consider the last blue query assigned to A_2.
At that time, A_2's unspent budget must have been at least as big as A_1's.
That means at least as many queries have been assigned to A_1 as to A_2.
At this point, we have already assigned at least B/2 queries to A_2.
So, $x \le B/2$ **and** $x + y = B$ **then** $y > B/2$

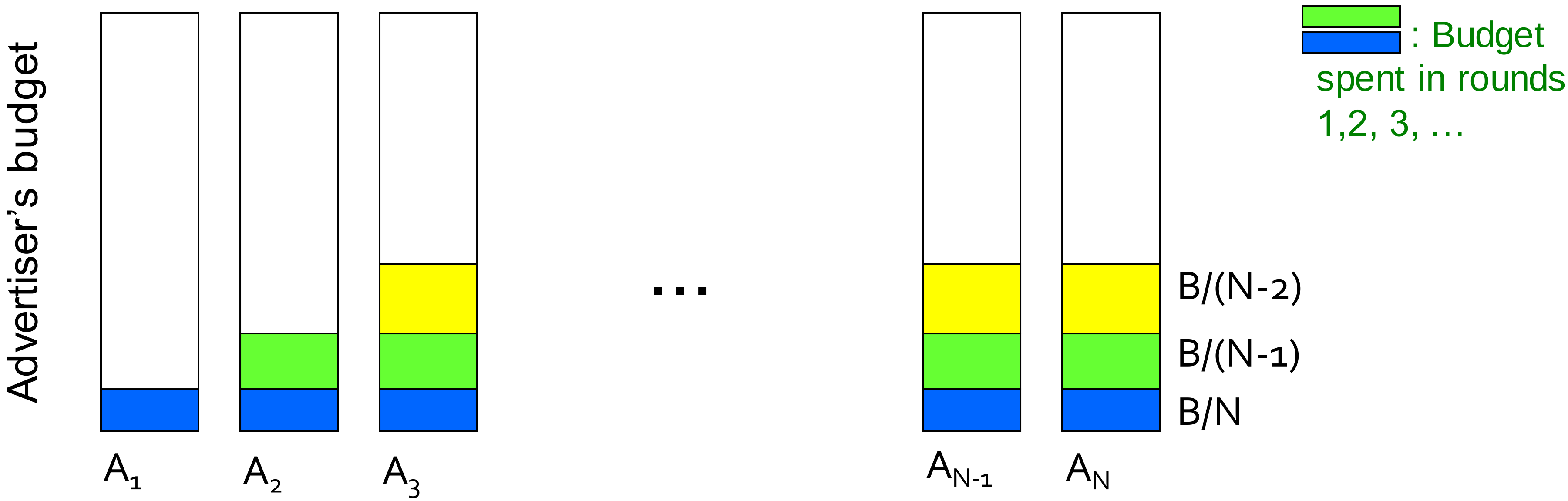
BALANCE: General Result

- **In the general case, worst competitive ratio of BALANCE is 1–1/e = approx. 0.63**
  - ***e* = 2.7182**
  - Interestingly, no online algorithm has a better competitive ratio!
- **Let's see the worst case example that gives this ratio**

Worst case for BALANCE

- ***N* advertisers:** $A_1, A_2, \ldots A_N$
  - Each with budget $B > N$
- **Queries:**
  - $N \cdot B$ queries appear in N rounds of B queries each
- **Bidding:**
  - Round **1** queries: bidders $A_1, A_2, \ldots, A_N$
  - Round **2** queries: bidders $A_2, A_3, \ldots, A_N$
  - Round ***i*** queries: bidders $A_i, \ldots, A_N$
- **Optimum allocation:**
  Allocate all round ***i*** queries to A_i
  - Optimum revenue $N \cdot B$

BALANCE Allocation



BALANCE assigns each of the queries in round 1 to **N** advertisers. After ***k*** rounds, sum of allocations to each of advertisers $\boldsymbol{A_k,\ldots,A_N}$ is

$$S_k = S_{k+1} = \cdots = S_N = \sum_{i=1}^{k} \frac{B}{N-(i-1)}$$

If we find the smallest *k* such that $S_k \geq B$, then after *k* rounds we cannot allocate any queries to any advertiser

BALANCE: Analysis

B/1 B/2 B/3 … B/(N-(k-1)) … B/(N-1) B/N

S_1

S_2

S_k **= B**

1/1 1/2 1/3 … 1/(N-(k-1)) … 1/(N-1) 1/N

S_1

S_2

S_k **= 1**

BALANCE: Analysis

- **Fact:** $\boldsymbol{H_n = \sum_{i=1}^{n} 1/i \approx \ln(n)}$ for large $\boldsymbol{n}$
  - Result due to Euler

1/1 1/2 1/3 … 1/(N-(k-1)) … 1/(N-1) 1/N

$$\xleftrightarrow{\hspace{6em}\ln(N)\hspace{6em}}$$

$$\underbrace{\hspace{4em}}_{\ln(N)-1}\ \underbrace{\hspace{4em}}_{S_k = 1}$$

- $\boldsymbol{S_k = 1}$ implies: $\boldsymbol{H_{N-k} = ln(N) - 1 = ln(\frac{N}{e})}$
- We also know: $\boldsymbol{H_{N-k} = ln(N-k)}$
- **So:** $\boldsymbol{N - k = \frac{N}{e}}$
- **Then:** $\boldsymbol{k = N(1 - \frac{1}{e})}$

N terms sum to ln(*N*).
Last *k* terms sum to 1.
First *N-k* terms sum
to ln(*N-k*) but also to ln(*N*)-1

BALANCE: Analysis

- So after the first **k=N(1-1/e)** rounds, we cannot allocate a query to any advertiser
- **Revenue = B·N (1-1/e)**
- **Competitive ratio = 1-1/e**
- **Note: So far we assumed:**
  - **All advertisers have the same budget B**
  - **All advertisers bid 1 for the ad**
  - **(but each advertiser can bid on any subset of ads)**

General Version of the Problem

- **Arbitrary bids and arbitrary budgets!**
- Consider we have 1 query q, advertiser i
  - Bid = x_i
  - Budget = b_i
- **In a general setting BALANCE can be terrible**
  - Consider two advertisers A_1 and A_2
  - A_1: $x_1 = 1$, $b_1 = 110$
  - A_2: $x_2 = 10$, $b_2 = 100$
  - Consider we see **10** instances of **q**
  - BALANCE always selects A_1 and earns **10**
  - Optimal earns **100**

Generalized BALANCE

- **Arbitrary bids:** consider query $\boldsymbol{q}$, bidder $\boldsymbol{i}$
  - Bid = $\boldsymbol{x_i}$
  - Budget = $\boldsymbol{b_i}$
  - Amount spent so far = $\boldsymbol{m_i}$
  - Fraction of budget left over $\boldsymbol{f_i = 1 - m_i/b_i}$
  - Define $\psi_i(q) = x_i(1-e^{-f_i})$
- Allocate query $\boldsymbol{q}$ to bidder $\boldsymbol{i}$ with largest value of $\psi_i(q)$
- **Same competitive ratio (1-1/*e*) = 0.63**