Continuous Fused Optimal Transport

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Abstract

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1 Introduction

2 Fused Optimal Transport

Notations. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and (S, d_S) be a compact metric space. A measurable map $X : \Omega \to S$ is called a random element, and its distribution is defined as $\mathbb{P}_X := \mathbb{P} \circ X^{-1}$. In addition, we introduce a feature space (M, d_M), which is also a compact metric space. Any measurable map $f : S \to M$ is called a feature function. For two random elements X and Y, we say a measurable map $T : S \to S$ pushes forward $\mathbb{P}_X$ to $\mathbb{P}_Y$, or simply X to Y, if $\mathbb{P}_X(T^{-1}(A)) = \mathbb{P}_Y(A)$ for all $A \in \mathcal{B}(S)$, where $\mathcal{B}(S)$ is the Borel σ-algebra of S. We denote $T_\# \mathbb{P}_X = \mathbb{P}_Y$ or $T(X) \overset{d}{=} Y$ if T pushes forward X to Y.

Fused optimal transport. For some known feature function f, we consider the following problem: for some $0 \leq \alpha \leq 1$,

$$\begin{aligned}\min_{T : T_\# \mathbb{P}_X = \mathbb{P}_Y} & (1-\alpha)\mathbb{E}_{X \sim \mathbb{P}_X}\Big[d_M(f(X), f(T(X)))\Big] \\ & + \alpha \mathbb{E}_{(X,X') \sim \mathbb{P}_X \otimes \mathbb{P}_X}\Big[K_h(X, X')\, |d_S(X, X') - d_S(T(X), T(X'))|^2\Big], \end{aligned} \tag{1}$$

where $K_h(\cdot, \cdot) : S \times S \to [0, \infty)$ is a bounded and symmetric ($K_h(x, x') = K_h(x', x)$) kernel with a bandwidth $h > 0$. A solution for (1), denoted as T^*, is called a fused optimal transport (OT) plan.

[*]https://junhyoung-chung.github.io/

39th Conference on Neural Information Processing Systems (NeurIPS 2025).