Fused Optimal Transport

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Abstract

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1 Introduction

2 Fused Optimal Transport

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $(\mathbb{F}, \mathcal{B}(\mathbb{F}))$ be a measurable space, where $\mathbb{F} \subset L^2(\mathcal{X}; dx)$ and $L^2(\mathcal{X}; dx)$ is a space of square integrable real-valued functions over a compact region $\mathcal{X} \subset \mathbb{R}^d$. The *norm* on $\mathbb{F}$ is defined as $\|f\| \coloneqq (\int_{\mathcal{X}} |f(x)|^2 dx)^{1/2}$. A measurable mapping $X : \Omega \to \mathbb{F}$ is called a *random function* and $\mathbb{P}_X \coloneqq \mathbb{P} \circ X^{-1}$ a *distribution* of X.

For two random functions X and Y, we say a measurable map $T : \mathbb{F} \to \mathbb{F}$ *pushes forward* $\mathbb{P}_X$ to $\mathbb{P}_Y$, or simply X to Y, if $\mathbb{P}_X(T^{-1}(A)) = \mathbb{P}_Y(A)$ for all $A \in \mathcal{B}(\mathbb{F})$. We denote $T_{\#}\mathbb{P}_X = \mathbb{P}_Y$ or $T(X) \overset{d}{=} Y$ if T pushes forward X to Y.

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\begin{aligned}
&\mathrm{FGW}(\mathbb{P}_X, \mathbb{P}_Y) \\
&\coloneqq \inf_{\pi \in \Pi(\mathbb{P}_X, \mathbb{P}_Y)} \left(\int_{\mathcal{X} \times \mathcal{X}} \left[(1-\alpha)(X(x) - Y(y))^2 + \alpha \int_{\mathcal{X} \times \mathcal{X}} \left| d(x, x') - d(y, y') \right|^2 \right] d\pi(x, y) d\pi(x', y') \right)^{1/2}.
\end{aligned} \tag{1}
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*https://junhyoung-chung.github.io/

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