1)

a) $\|\langle -3, 4\rangle\| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25} = 5$

b)

$\|\langle -1, 0, 1\rangle\| = \sqrt{(-1)^2 + 0^2 + 1^2}$

$= \sqrt{2}.$

c) $\|\langle 3, 0, -4\rangle\| = \sqrt{3^2 + 0^2 + (-4)^2}$

$= \sqrt{9 + 16}$

$= 5$

d) $\|\langle 5, -2, -2\rangle\|$

$= \sqrt{5^2 + (-2)^2 + (-2)^2}$

$= \sqrt{25 + 4 + 4}$

$= \sqrt{33}$

2)

a) $\langle -3, 4 \rangle$

$$\vec{u} = \frac{\langle -3, 4 \rangle}{\| \langle -3, 4 \rangle \|} = \boxed{\frac{-3i + 4j}{5.}}$$

b) $\langle -1, 0, 1 \rangle$

$$\vec{u} = \frac{\langle -1, 0, 1 \rangle}{\| \langle -1, 0, 1 \rangle \|} = \frac{-i + k}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}\, i + \sqrt{2}\, k}{2.}}$$

c) $\langle 1, 1, -1 \rangle$

$$\vec{u} = \frac{\langle 1, 1, -1 \rangle}{\| \langle -1, 0, 1 \rangle \|} = \frac{i + j + k}{\sqrt{1^2 + 0^2 + 1^2}} = \boxed{\frac{i + j - k}{3}}$$

d) $\langle 0, -3, 3 \rangle$

$$\vec{u} = \frac{\langle 0, -3, 3 \rangle}{\| \langle 0, -3, 3 \rangle \|} = \frac{\langle 0, -3, 3 \rangle}{\sqrt{0^2 + (-3)^2 + (3)^2}}$$

$$= \boxed{\frac{-3j + 3k}{\sqrt{18}}}$$

$$u \cdot v = \|u\| \|v\| \cos\theta$$

3) a) Angle between $\langle 4,3 \rangle$ & $\langle 2,-1 \rangle$

$\Rightarrow (4i+3j)\cdot(2i-j) = \|\langle 4,3\rangle\| \|\langle 2,-1\rangle\| \cos\theta$

$\Rightarrow 8-3 = \left(\sqrt{4^2+3^2}\right)\left(\sqrt{2^2+(-1)^2}\right)\cos\theta$

$\Rightarrow 5 = \sqrt{25}\sqrt{1} \cos\theta$

$\Rightarrow \cos\theta = 1$

$\Rightarrow \theta = \cos^{-1}(1)$

$\Rightarrow \theta = 2\pi K, \quad K \in \mathbb{Z}$

b) Angle between $\langle 1,-4,1 \rangle$ & $\langle 2,0,-2 \rangle$

$\langle 1,-4,1\rangle \cdot \langle 2,0,-2\rangle = \|\langle 1,-4,1\rangle\| \|\langle 2,0,-2\rangle\| \cos\theta$

$2+0-2 = \sqrt{1+16+1}\sqrt{4+0+4} \cos\theta$

$0 = \sqrt{18}\sqrt{8} \cos\theta$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \cancel{\theta = 2\pi K, \quad K \in \mathbb{Z}}$

$$\Rightarrow \theta = \begin{cases} \frac{\pi}{2} + 2\pi K \\ -\frac{\pi}{2} + 2\pi K \end{cases}, \quad K \in \mathbb{Z}$$

$\pi - \frac{\pi}{3}$ $\frac{2\pi}{3}$

c) Angle between

$\langle 1, 0, \sqrt{3} \rangle$ & $\langle -1, 0, 0 \rangle$

$\langle 1, 0, \sqrt{3} \rangle \cdot \langle -1, 0, 0 \rangle = \| \langle 1, 0, \sqrt{3} \rangle \| \| \langle -1, 0, 0 \rangle \| \cos\theta$.

$-1 + 0 + 0 = \left(\sqrt{1+0+3}\right)\left(\sqrt{1+0+0}\right) \cos\theta$

$-1 = 2\cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$$\theta = \begin{cases} \frac{2\pi}{3} + 2\pi K \\ -\frac{2\pi}{3} + 2\pi K \end{cases} \quad K \in \mathbb{Z}$$

4) a) $\langle 2,3,0\rangle \times \langle 1,0,5\rangle = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 1 & 0 & 5 \end{vmatrix}$

$\Rightarrow \; i(15) - j(10) + k(-3)$

$\boxed{15i - 10j - 3k}$

b) $\langle 1,0,5\rangle \times \langle 2,3,0\rangle = \begin{vmatrix} i & j & k \\ 1 & 0 & 5 \\ 2 & 3 & 0 \end{vmatrix}$

$= i(-15) - j(-10) + k(3)$

$= \boxed{-15i + 10j + 3k}$

c) $\langle 1,0,1\rangle \times \langle 0,1,0\rangle$

$= \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$

$i(-1) - j(0) + k(1)$

$= \boxed{-i + k}$

d) $\langle 2,-1,1\rangle \times \langle 4,-2,2\rangle = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 4 & -2 & 2 \end{vmatrix}$

$= i(-2+2) - j(4-4) + k(-4+4)$

$= \boxed{0}$

5) Area of triangle with vertices

$A(1,0,0)$, $B(-2,1,3)$, $C(4,2,5)$

$$\text{Area} = \frac{1}{2}\|AB \times AC\|$$

$$AB = B - A = (-2,1,3) - (1,0,0)$$
$$= (-3,1,3)$$

$$AC = C - A = (4,2,5) - (1,0,0)$$
$$= (3,2,5)$$

$$AB \times AC = \begin{vmatrix} i & j & k \\ -3 & 1 & 3 \\ 3 & 2 & 5 \end{vmatrix} = i(5-6) - j(-15-9) + k(-6-3)$$
$$= -i + 24j - 9k.$$

$$\|AB \times AC\| = \sqrt{1^2 + 24^2 + 9^2} = \sqrt{658}$$

$$\boxed{\text{Area} = \frac{1}{2}\sqrt{658}}$$

$1+3(3)+1$