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Katia Berti *

Abstract

Stochastic projections are a powerful tool to feature uncertainty in macroeconomic conditions into the analysis of public debt dynamics. They allow simulating a very large number of debt paths, corresponding to as many shock constellations to the non-fiscal determinants of debt evolution (short- and long-term interest rates, growth rate and exchange rate). Furthermore, random shocks are simulated in a way to reflect the size and the correlation of historical shocks. The specific approach for stochastic projections used here, based on the variance-covariance matrix of historical shocks, further allows defining a "central scenario" (for which we use European Commission DG ECFIN's Autumn 2012 forecasts), around which shocks apply. The paper applies this methodology to 24 EU countries over 2013-17. Cross-country differences in the variance of the debt-to-GDP ratio distributions (reflecting differences in historical volatility of macroeconomic conditions) emerge clearly from the simulations. This shows the importance of allowing for a more comprehensive and country-tailored assessment of downward and upward risks to debt dynamics. This stochastic framework also has the distinctive advantage of allowing for an explicit probabilistic assessment of debt projection results. A closer scrutiny of three EU countries in the case of temporary shocks reveals, for instance, that the most likely outcome for IT over 2013-17 is a decreasing path for the debt ratio (though this is projected to be still higher than 116% with a 50% probability in 2017). For ES, simulations show an increasing path over the projection horizon for all shock constellations, with an 80% probability of a debt ratio greater than 100% in 2017. Finally, for HU, we obtain a 60% probability that the debt ratio stabilises or reaches higher values from 2013 onwards, with a 40% probability of a debt ratio greater than 80% in 2017.

JEL classification: E62, H63, C15.

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1. Introduction

Debt sustainability analysis (DSA) is a key tool for the assessment of countries' fiscal stance. Typically, the analysis relies on public debt projections run under a set of macroeconomic assumptions (on actual and potential growth, interest rates and the government's primary balance) over a certain projection horizon. Governments' implicit liabilities from projected age-related costs can eventually be taken into account in a DSA.[1] The analysis allows drawing conclusions as to whether the debt-to-GDP ratio of a certain country is expected to be on a decreasing, increasing or broadly stable path over the projection horizon, under the assumptions made on the future macroeconomic environment. In this type of context, sustainability concerns generally arise when the debt ratio is projected to be on a steadily increasing path or to stabilise at a high level.

The macroeconomic assumptions are of course crucially important to such an analysis, as they can shape conclusions on debt sustainability in one sense or the other. Forecasts on GDP growth rate, interest rate and the government's structural primary balance (SPB) are generally used as underlying macroeconomic assumptions in DSAs (DG ECFIN's forecasts are used, for instance, in the European Commission's DSA[2]). Some convergence hypothesis to a long-run value is eventually made for some macroeconomic variables (growth and the interest rate, for instance) beyond the forecast horizon (this is indeed the case for the debt projection results presented in the European Commission's Fiscal Sustainability Report, FSR, 2012). As far as the SPB is concerned, the so called "no-policy change assumption" is commonly used in DSAs covering a time span longer than the forecast horizon. Given the key role played by these macroeconomic assumptions, all DSAs include a "sensitivity analysis" (or "bound tests"), aimed at illustrating how debt projections would change with changes in underlying assumptions.

One of the limitations of the traditional deterministic framework for public debt projections described above lies nonetheless in the very limited scope to account for uncertainty in underlying macroeconomic conditions. Although sensitivity tests are generally presented alongside baseline debt projections, these tests can only be based on a limited number of additional scenarios, in which macroeconomic assumptions are changed relative to the baseline, eventually in a way to reflect historical volatility (for instance, through the standard deviation of the variables in question). But of course uncertainty implies that an infinite number of such alternative scenarios could be designed to reflect possible macroeconomic shocks. At the same time, the sensitivity tests described above generally consist in simulating shocks to *each* of the determinants of debt dynamics *one at the time*, implying that correlations between shocks are fully neglected. And this is also the case when *ad-hoc* combinations of shocks are considered. Missing this dimension of shock correlation clearly represents a second limitation of the traditional deterministic DSA.

As will be better explained in the rest of the paper, stochastic projections can help overcoming both the aforementioned limitations. In a stochastic framework, uncertainty in macroeconomic conditions is accounted for in a much more satisfactory way, thanks to the explicit recognition of the probabilistic nature of the DSA. Practically, conducting stochastic

[1] This is done, for instance, in the European Commission's Fiscal Sustainability Report 2012.
[2] See European Commission's Fiscal Sustainability Report 2012.

debt projections (based on the methodology used in this paper) amounts to running a *very large number* of (randomly generated) sensitivity tests covering a whole range of *shock combinations* to the determinants of debt dynamics (reflecting historical volatility and shock correlations), so that a frequency distribution of the debt-to-GDP ratio is obtained for each year in the projection horizon. For each projection year, we will therefore have a whole set of possible values for the debt ratio, rather than a single value as in a deterministic framework. A fiscal policy assessment would therefore be based on the analysis of the whole "cone" of possible debt trajectories, to which probabilities can be assigned, rather than on individual debt trajectories.

The rest of the paper is structured as follows. Section 2 briefly discusses the approach for stochastic projections based on the variance-covariance matrix of historical shocks, adopted in this paper, versus the approach relying on VAR modelling used in other contributions. Section 3 provides a short description of the main features of deterministic public debt projections run by the European Commission's Directorate General for Economic and Financial Affairs (DG ECFIN), with particular attention to the underlying macroeconomic assumptions, as these are also used to define the central scenario in our stochastic projections. Section 4 describes in detail the methodology for stochastic debt projections based on the variance-covariance matrix of historical shocks (this methodology is now embedded in the debt sustainability analysis run by the European Commission DG ECFIN in the context of the Debt Sustainability Monitor and has been used in the FSR 2012), after shortly reviewing the advantages that this methodology provides relative to sensitivity analysis in deterministic contexts. The data used are discussed in Section 5, where stochastic projection results are reported for all EU countries but those under adjustment programmes (EL, IE and PT).[3] In the same section, results are presented and commented in more detail with specific focus on three countries (IT, ES and HU). Conclusions follow in Section 6.

2. Stochastic debt projections: the historical variance-covariance matrix approach versus the VAR model approach

Different methodological approaches have been proposed in the literature to run stochastic debt projections. A rather common approach makes use of a vector auto-regression (VAR) model to obtain an estimated variance-covariance matrix of shocks to the non-fiscal determinants of debt dynamics (growth rate, interest rate, and eventually the exchange rate). A number of contributions following this approach rely on the assumption of normality of the residuals and on the Choleski factorization of the variance-covariance matrix of the residuals (Garcia and Rigobon, 2004; Celasun, Debrun and Ostry, 2007; Celasun and Keim, 2010). Alternatively, or in addition to this, other contributions apply bootstrapping techniques on the residuals to avoid making the assumption of normality, which could be restrictive in some instances (Burger, Stuart, Jooste and Cuevas, 2011; Medeiros, 2012). Using one of these two techniques, random vectors of shocks to the non-fiscal determinants of debt dynamics are generated for each projection year and these, together with the estimated coefficients of the

[3] For these three programme countries, forecasts follow a different schedule, linked to the frequent country reviews, relative to standard DG ECFIN's forecasts. They would therefore require ad-hoc updates depending on availability of more recent forecasts from country review reports. For this reason, they have been excluded from the analysis.

VAR model, are used to project the (non-fiscal) macroeconomic variables into the future. The path of the debt-to-GDP ratio corresponding to each sequence of simulated shock vectors (one vector per projection year, each element in the vector referring to one variable) is then obtained through the standard debt evolution equation (which relates the debt ratio in a certain year with the primary balance and the stock-flow adjustment in the same year and the debt ratio in the previous year). As far as the primary balance is concerned, contributions based on VAR modelling generally make use of an estimated fiscal reaction function to be able to incorporate (discretionary) fiscal policy responses in debt projections (Celasun, Debrun and Ostry, 2007; Burger, Stuart, Jooste and Cuevas, 2011; Medeiros, 2012). The algorithm just described will of course produce as many debt paths as the number of simulated shock constellations.

Another (simpler) approach for stochastic debt projections proposed in the literature, and used in this paper, differs from the previous one in that the shocks to the non-fiscal determinants of debt dynamics are extracted from the variance-covariance matrix of historical shocks (rather than from the estimated variance-covariance matrix in the context of VAR modelling), assuming a joint normal distribution of the shocks (di Giovanni and Gardner, 2008; Beynet and Paviot, 2012). Random shocks obtained this way are then applied to "baseline" projected values of the corresponding variables, taken from some external (model-independent) projections. This gives as many different projected paths of the (non-fiscal) macroeconomic variables used in debt projections as the number of simulated shocks, and in turn generates as many corresponding paths for the debt-to-GDP ratio through the debt evolution equation. In this type of approach, external projections (on growth rate, interest rate and primary balance) are therefore used to define the central scenario, around which shocks apply. The methodology is therefore particularly suited to capture the impact of uncertainty in macroeconomic conditions on public debt projections *around a specific forecasted path*. This is the reason why we focus here on this approach.

3. The definition of the "central scenario" for stochastic projections

As anticipated, the methodology for stochastic debt projections used in this paper relies on the definition of a "central scenario" based on some external (in the sense of model-independent) projections. Specifically, the central scenario used in our analysis is based on DG ECFIN's forecasts and the macroeconomic assumptions agreed with the Economic Policy Committee Working Group on Ageing Populations and Sustainability (AWG). These forecasts and assumptions are used in deterministic debt projections run by DG ECFIN using its Debt Sustainability Monitor (DSM) model, from which the baseline projected debt path is taken as central scenario for our stochastic projections.

The DSM is a standard model for deterministic public debt projections based on the conventional stock-flow identity (linking the debt ratio in a certain year to the debt ratio in the previous year, the interest rate, the growth rate, the primary balance and the stock-flow adjustment in the same year). The model incorporates the distinction between short- and long-term interest rates on government bonds,[4] as well as public debt maturity structure. It therefore allows accounting for differences between interest paid on short- and long-term debt, as well as accounting for the shares of maturing and non-maturing debt in the way projected changes in interest rates affect debt dynamics (projected changes in a certain year will only impact debt evolution through the share of debt that is issued or rolled over in that year). The impact of governments' implicit liabilities from projected age-related spending (as from the AWG reference scenario in the 2012 Ageing Report) is also taken into account in DSM debt projections.

DG ECFIN's forecasts, currently till 2014, on implicit interest rate, actual and potential growth and the structural primary balance (SPB) are used in DSM debt projections. Beyond the forecast horizon, the no-policy change assumption is made (translating into a SPB constant at last forecast year), while the cyclical component of the balance is calculated as the output gap multiplied by the (country-specific) budget balance sensitivity to the cycle (based on EC-OECD estimates).[5] As a general rule, in the model the output gap is assumed to close within five years, after which the potential growth rate converges (within 10 years) to the baseline scenario agreed with the AWG. AWG long-run assumptions are also used beyond the forecast horizon for the implicit interest rate (assumed to converge to 3%, in real terms, by 2020) and the inflation rate (assumed to converge to 2% by the output gap closure year). The shares of short- and long-term debt over total are kept constant at latest available data over the whole projection horizon, while new and rolled-over long-term debt is assumed to have maturity identical to the (country-specific) weighted average maturity of public debt. Zero stock-flow adjustment is assumed in the model beyond forecasts.

4. Stochastic debt projections based on the historical variance-covariance matrix approach

4.1 Value added relative to standard sensitivity analysis in deterministic projections

The methodology for stochastic projections used in this paper provides a significantly stronger tool to feature uncertainty in macroeconomic conditions (interest rates, growth rate and exchange rate) into public debt projections, compared to traditional sensitivity analysis. First, the methodology allows simulating a very large number of possible debt paths (2000 in the simulation results presented in this paper) corresponding to different underlying macroeconomic conditions. The spectrum of simulated macroeconomic conditions is therefore enormously widened, to limits that are beyond what is conceivable for sensitivity analysis in the context of deterministic projections. Second important advantage of the methodology is that such spectrum of macroeconomic conditions is obtained by simulating shocks to short-term and long-term interest rates, growth rate and exchange rate, taking into account *the size of the historical shocks* to each of the variables in question, as well as *the correlation of the shocks across variables*. Results obtained therefore reflect the fact that

[4] Short-term and long-term debt is defined in the model as debt with maturity below the year and above the year respectively.

[5] Estimates for OECD countries are taken from Girouard and Andre' (2005), while estimates for non-OECD countries were calculated by DG ECFIN (see European Commission, 2005).

shocks are not independent one from the other but rather inter-related (with the correlation pinned down based on variables' historical behaviour).

The very large number of simulated debt paths makes it possible to construct "fan charts" of possible debt outcomes around the central (DSM) scenario. These charts can then be used to assess *the probability* of the debt ratio remaining on a stable or declining path over the projection horizon, despite possible shocks to interest rates, growth and exchange rates that are *outside government's control*. Interesting under this respect is the possibility to design two different types of scenarios depending on the assumed persistency of shocks to macroeconomic variables (temporary versus permanent shocks), as done in di Giovanni and Gardner (2008).

Making policy-makers able to feature country-specific volatility of macroeconomic conditions into the analysis of debt projections is therefore a distinctive advantage of this methodology. This makes it possible to assess debt dynamics also with regard to the volatility around the central outcome. Countries with very similar debt dynamics in the baseline scenario could still look very different due to the different size of upward and downward risks obtained through the application of the methodology. Policy recommendations could in case be "modulated" to take account of this additional dimension.

4.2 Description of the methodology

The methodology for stochastic debt projections used here follows di Giovanni and Gardner (2008) and Beynet and Paviot (2012). The difference relative to their contributions (apart for the fact that we apply the methodology to 24 EU countries) lies in the set of non-fiscal determinants of debt dynamics, to which random shocks apply. Consistently with the DSM model (from which figures are taken for our central scenario), we make the distinction here between short- and long-term interest rates on government bonds (rather than between interest rates on debt denominated in different currencies) and shocks are applied separately to the two. Other non-fiscal determinants of debt evolution, to which shocks apply in our model, are the nominal GDP growth rate and the nominal exchange rate for non-EA countries.

Shocks to the exchange rate have been introduced in the analysis following Beynet and Paviot (2012). As will be showed in more detail below, this is done by distinguishing (for non-EA countries) between debt denominated in national and foreign currency and allowing changes in the nominal exchange rate to impact on the debt ratio through the latter. This allows capturing the important impact that exchange rate shocks can have on debt evolution in some non-EA Member States having large shares of public debt denominated in euros (to the purpose of the analysis, the assumptions are made that all public debt denominated in a foreign currency is denominated in euros[6] and that the shares of public debt denominated in national and foreign currency remain constant over the projection period).

[6] ESTAT data for eastern non-EA Member States show that this is indeed rather close to reality for CZ and LV. The assumption is not too restrictive for BG, HU, LT and RO, where the largest share of foreign currency debt is indeed issued in euros.

In line with the relevant literature, our stochastic debt projection model covers a time span of 5 years (2013-17), the projection horizon over which this kind of methodology can provide meaningful results (Beynet and Paviot, 2012).

4.2.1 The method to obtain (annual) shocks to the non-fiscal determinants of debt dynamics

In our model shocks apply to the main macroeconomic variables entering the debt evolution equation (short-term interest rate, long-term interest rate, nominal growth rate and exchange rate). First, the methodology requires transforming the time series of quarterly data for each macroeconomic variable x into series of historical quarterly shocks δ_q^x as follows:

$$\delta_q^x = x_q - x_{q-1}$$

A Monte Carlo simulation is then run by extracting random vectors of quarterly shocks over the projection period (2013-17) from a joint normal distribution with zero mean and *variance-covariance matrix identical to that of the historical shocks*.

The quarterly shocks (ε_q^x) obtained this way are then aggregated into annual shocks to short-term interest rate, long-term interest rate, growth, and exchange rate for non-EA countries, as follows (so as to be able to apply these shocks to the series with annual frequency used in debt projections):

- the shock to nominal growth (g) in year t is given by the sum of the quarterly shocks to growth:

$$\varepsilon_t^g = \sum_{q=1}^{4} \varepsilon_q^g$$

- the shock in year t to the nominal exchange rate (e) is given by the sum of the quarterly shocks to the exchange rate:

$$\varepsilon_t^e = \sum_{q=1}^{4} \varepsilon_q^e$$

- the shock in year t to the nominal short-term interest rate (si) is given by the sum of the quarterly shocks to the short-term interest rate:

$$\varepsilon_t^{si} = \sum_{q=1}^{4} \varepsilon_q^{si}$$

The calculation of the shock in annual terms is based on the fact that the short-term interest rate is defined here as the interest rate on government bonds with maturity below the year. With the equation above, we rule out persistence of short-term interest rate shocks over time (consistently with assumptions made in DG ECFIN's DSM model).[7] In other words, unlike the case of the long-term interest rate (see below), a shock to the short-term interest rate occurring in any of the quarters of year t is not carried over beyond year t.

[7] Some persistence of short-term interest rate shocks could in fact be observed in the real world, linked to the fact that some short-term government bonds might be issued at the end of year t and mature in the course of year t+1. For simplicity reasons we did not take this possibility into account in the DSM model, and here we rule this out for consistency.

- the aggregation of the quarterly shocks to the nominal long-term interest rate (li) into annual shocks takes account of the persistence of these shocks over time. This is due to the fact that long-term debt issued/rolled over at the moment where the shock takes place will remain in the debt stock, for all years to maturity, at the interest rate conditions holding in the market at the time of issuance (implicit in this logic is the assumption that long-term government bonds are issued at fixed interest rates). A shock to the long-term interest rate in year t is therefore carried over to the following years in proportion to the share of maturing debt that is progressively rolled over (country-specific figures on weighted average maturity are used to implement this). For countries where average weighted maturity of debt T is equal or greater than the number of projection years (5 years, from 2013 to 2017), the annual shock to the long-term interest rate in year t is defined as:

$$\varepsilon_t^{li} = \frac{1}{T}\sum_{q=1}^{4}\varepsilon_q^{li} \quad \text{if } t = 2013$$

$$\varepsilon_t^{li} = \frac{2}{T}\sum_{q=-4}^{4}\varepsilon_q^{li} \quad \text{if } t = 2014$$

$$\varepsilon_t^{li} = \frac{3}{T}\sum_{q=-8}^{4}\varepsilon_q^{li} \quad \text{if } t = 2015$$

$$\varepsilon_t^{li} = \frac{4}{T}\sum_{q=-12}^{4}\varepsilon_q^{li} \quad \text{if } t = 2016$$

$$\varepsilon_t^{li} = \frac{5}{T}\sum_{q=-16}^{4}\varepsilon_q^{li} \quad \text{if } t = 2017$$

where q = -4, -8, -12, -16 respectively indicate the first quarter of years t-1, t-2, t-3 and t-4.

The set of equations above clearly allows for shocks to the long-term interest rate in a certain year to carry over to the following years, till when, on average, debt issued at those interest rate conditions will remain part of the stock.

For countries where the average weighted maturity of debt is smaller than the number of projection years, the equations above are adjusted accordingly to reflect a shorter carry over of past shocks. For instance, countries with average weighted maturity T = 3 years will have the annual shock to the long-term interest rate defined as follows:[8]

$$\varepsilon_t^{li} = \frac{1}{3}\sum_{q=1}^{4}\varepsilon_q^{li} \quad \text{if } t = 2013$$

$$\varepsilon_t^{li} = \frac{2}{3}\sum_{q=-4}^{4}\varepsilon_q^{li} \quad \text{if } t = 2014$$

$$\varepsilon_t^{li} = \sum_{q=-8}^{4}\varepsilon_q^{li} \quad \text{if } t \geq 2015$$

Finally, the weighted average of annual shocks to short- and long-term interest rates (with weights given by the country-specific shares of short-term debt, α^S, and long-term debt, α^L) gives the annual shock to the implicit interest rate (i):

[8] Annual shocks to the long-term interest rate for countries with weighted average maturities of 2 and 4 years are defined in a fully analogous way.

$$\varepsilon_t^i = \alpha^S \varepsilon_t^{si} + \alpha^L \varepsilon_t^{li}$$

4.2.2 Two different scenarios reflecting the temporary or permanent nature of the shocks

Following di Giovanni and Gardner (2008), two different scenarios are constructed based on different assumptions on the persistence of the shocks.

Temporary shocks

The first scenario assumes that all shocks are temporary. In this case, annual shocks (ε_t) are applied each year to the baseline values of the variables in question (implicit interest rate i_t, nominal growth rate g_t and exchange rate e_t) as follows (with t = 2013,…,2017):

$g_t = \overline{g}_t + \varepsilon_t^g$ with $\overline{g}_t$ = DSM baseline nominal GDP growth at year t

$i_t = \overline{i}_t + \varepsilon_t^i$ with $\overline{i}_t$ = DSM baseline implicit interest rate at year t

$e_t = \overline{e}_t + \varepsilon_t^e$ with $\overline{e}_t$ = nominal exchange rate as in DG ECFIN forecasts if t within forecast horizon (till 2014); nominal exchange rate identical to last forecasted value if t beyond forecast horizon

Permanent shocks

The second scenario, on the contrary, assumes that shocks to the implicit interest rate and the exchange rate are permanent. This is implemented by applying the annual shocks for these two variables to the values of the same variables in the year before, rather than to their (DSM) baseline values. Under this scenario, the series of projected implicit interest rate and exchange rate are therefore obtained as follows (with t > 2013, as first projection year):[9]

$$i_t = i_{t-1} + \varepsilon_t^i$$
$$e_t = e_{t-1} + \varepsilon_t^e$$

Shocks to growth are assumed to remain temporary also in this scenario.

As in di Giovanni and Gardner (2008) and Beynet and Paviot (2012), in both scenarios the temporary shock to GDP growth translates into a shock to the balance over GDP *through the budget cyclical component* (no fiscal policy reaction function is, on the contrary, assumed).[10] The impact on the balance is calculated using the EC-OECD (country-specific) estimated coefficients of budget balance sensitivity to the cycle (s), which are used also in DSM deterministic projections. Thus, the shock to the balance (as a ratio of GDP) b linked to the shock to GDP growth is given by the following:

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[9] A constraint of non-negative values for nominal interest rates is introduced in the model. This turns out to be relevant only for some countries in the case of permanent shocks.
[10] This is fully in line with the deterministic debt projection model (DSM) used to define the central scenario, around which shocks are applied.

$$\varepsilon_t^b = s \cdot \varepsilon_t^g$$

4.2.3 The debt evolution equation

Through the steps described above we obtain series, over the whole projection period, of simulated nominal growth rate, implicit interest rate, nominal exchange rate and changes in the budget cyclical component that can be used in the debt evolution equation to calculate debt ratios over a 5-year horizon, starting from the last historical value. The debt evolution equation takes the following form:

$$d_t = \alpha^n d_{t-1} \frac{1+i_t}{1+g_t} + \alpha^f d_{t-1} \frac{1+i_t}{1+g_t} \frac{e_t}{e_{t-1}} - b_t + c_t + f_t$$

where d_t = debt-to-GDP ratio in year t

α^n = share of total debt denominated in national currency

α^f = share of total debt denominated in foreign currency

b_t = primary balance over GDP in year t

c_t = change in age-related costs over GDP in year t relative to base year (2012)

f_t = stock-flow adjustment over GDP in year t

All the steps above (extraction of random vectors of quarterly shocks over the projection horizon; aggregation of quarterly shocks into annual shocks; calculation of the corresponding simulated series of implicit interest rate, nominal growth rate, exchange rate and change in the budget cyclical component; calculation of the corresponding path for the debt ratio) are repeated 2000 times. This allows us to obtain yearly distributions of the debt-to-GDP ratio over 2013-17, from which we extract the 10th, 20th, 40th, 50th, 60th, 80th and 90th percentiles to construct the fan charts presented in Section 5.

5. Data used and simulation results

5.1 The data used

For the calculation of the historical variance-covariance matrix used to obtain the simulation results that follow, quarterly data were taken from IFS and OECD for short- and long-term interest rates, from ESTAT and IFS for the nominal growth rate, and from ESTAT for the nominal exchange rate (for non-EA countries).

Results using the methodology described before were derived for 24 EU countries using both short- and long-term interest rates, whenever possible based on data availability (to keep in line with the central scenario from DSM projections). For the vast majority of EU countries it has indeed been possible to use both interest rates, the only exceptions being BG, CY, EE and RO (for EE we only used the short-term interest rate as quarterly data on the long-term rate were not available; for BG and CY we used the long-term interest rate only as data on the short-term rate were not available for most recent years; for RO we used the long-term interest rate only as a too short time series was available for the short-term rate).

In general, data starting from the late 70s-early 80s till the second quarter of 2012 were used to calculate the historical variance-covariance matrix for old Member States, whereas for the Member States that joined more recently the data used generally cover the period from the late 90s-early 2000 till the third quarter of 2010 or the second quarter of 2012 (a more precise overview country by country is provided in Annex 1, Table A.1).

Country-specific data on the weighted average maturity of government bonds were extracted from Bloomberg. The shares of short-term and long-term government debt over total (2009-11 average figures) were taken from ESTAT, as were the shares of public debt denominated in national and foreign currency (2011 ESTAT data were used for all countries but DK and SE, for which ESTAT data were not available, and 2010 OECD data were used instead). For the DSM central scenario, DG ECFIN's Autumn 2012 forecasts were used.

5.2 Simulation results

Stochastic debt projection results obtained by using the methodology presented above (that is now part of the European Commission DG ECFIN's debt sustainability analysis in the context of the Debt Sustainability Monitor) are presented in this section for 24 EU countries (all but the EU countries under adjustment programmes – EL, IE and PT) and the EA aggregate. Tables 1 and 2 report, country by country, summary results on the debt ratio distribution for 2017, last projection year, obtained from the simulations under the temporary and permanent shock scenarios[11] respectively (the 10th, 50th and 90th percentiles of the debt ratio distributions are provided in the tables, together with differences between selected percentiles). Graphical representations of simulation results for the debt ratio are also provided in the form of fan charts[12] for three countries (IT, ES and HU) in Figures 1a to 3b (fan charts for the corresponding simulated non-fiscal determinants of debt dynamics - growth rate, interest rate and exchange rate - are reported in Annex 2).

As explained in previous sections, the debt ratio distributions obtained through stochastic projections allow assigning probabilities to debt paths. In this framework it is possible, for instance, to attach a probability to the debt ratio of a certain country being higher than a specified value in a given projection year, or to the debt ratio being on a stable or declining path over the projection horizon.

Taking the case of IT in the temporary shock scenario, for instance, Table 1 and Figure1a show that the debt-to-GDP ratio in 2017 can be expected to lie between 106% and 128% with an 80% probability (as the two values correspond respectively to the 10th and the 90th distribution percentiles). Under the permanent shock scenario, the interval would widen to

[11] For simplicity, we label as "permanent shock scenario" the one, in which shocks to interest rate and exchange rate are permanent, though shocks to growth remain temporary.

[12] The fan charts report the 10th, 20th, 40th, 50th, 60th, 80th and 90th distribution percentiles. The debt path under the DSM deterministic projections is represented in the charts as a dashed line.

100-131%, as from Table 2 and Figure 1b. The Spanish debt ratio in 2017 would lie in the interval 96-118% with 80% probability under the temporary shock assumption, and in the interval 93-125% under the permanent shock assumption (Tables 1-2, Figures 2a-2b).[13]

Cross-country differences in the variance of the distribution of the debt ratio in 2017 (reflecting the country-specific volatility of macroeconomic conditions) are evident from Tables 1 and 2. Focussing the attention on the temporary shock scenario, for instance, Table 1 shows that, while 80% of the debt ratio distribution takes values between 86% and 97% for FR and between 25% and 36% for SE (with a difference of around 11 p.p. between the 10th and the 90th distribution percentiles for both countries), the same share of the distribution lies in the much wider interval of 62-94% for HU and 27-63% for LV (a difference respectively of 31.9 and 36.2 p.p. between the 10th and the 90th percentiles), with medians at around 77% and 41% respectively for the two countries.

Table 1 – Median and differences between percentiles of debt-to-GDP ratio distribution in 2017 (temporary shock scenario)

Country	Debt ratio in 2012	Proj. median debt ratio in 2017	10th percentile of debt ratio distribution in 2017	90th percentile of debt ratio distribution in 2017	Proj. diff. btw. percentiles 90th and 10th of debt ratio distribution in 2017	Proj. diff. btw. percentiles 60th and 40th of debt ratio distribution in 2017
BE	99.9	105.2	99.2	112.3	13.1	2.5
BG	19.5	17.0	13.5	21.7	8.1	1.6
CZ	45.1	51.1	45.9	56.5	10.7	2.0
DK	45.4	46.3	41.1	52.0	10.9	2.1
DE	81.7	71.3	62.6	82.0	19.4	3.9
EE	10.5	9.2	7.2	11.8	4.5	0.9
ES	86.1	106.5	96.4	118.3	21.9	4.4
FR	90	91.0	85.6	97.0	11.5	2.3
IT	126.5	116.1	106.1	128.0	21.8	4.1
CY	89.7	116.4	106.7	127.6	20.9	4.2
LV	41.9	40.9	27.1	63.3	36.2	7.3
LT	41.6	41.3	32.1	53.6	21.5	4.2
LU	21.3	30.3	25.9	36.5	10.6	1.9
HU	78.4	77.1	62.5	94.4	31.9	6.2
MT	72.3	73.0	63.3	86.5	23.1	4.4
NL	68.8	70.8	65.2	77.9	12.7	2.5
AT	74.6	74.9	71.4	79.2	7.8	1.6
PL	55.5	51.2	45.9	56.9	11.0	2.1
RO	34.6	33.3	23.8	47.6	23.8	4.5
SI	54	70.7	64.7	77.7	13.1	2.7
SK	51.7	58.7	51.4	68.0	16.5	3.4
FI	53.1	56.9	51.5	63.4	11.9	2.3
SE	37.4	29.9	25.0	35.8	10.8	2.1
UK	88.7	97.9	91.2	105.2	14.0	2.9
EA	92.8	91.1	86.4	96.2	9.8	2.0

[13] For all countries, the scenario with permanent shocks displays of course higher variance in the distribution of possible outcomes relative to the scenario with temporary shocks (see Tables 1 and 2).

Table 2 – Median and differences between percentiles of debt-to-GDP ratio distribution in 2017 (permanent shock scenario)

Country	Debt ratio in 2012	Proj. median debt ratio in 2017	10th percentile of debt ratio distribution in 2017	90th percentile of debt ratio distribution in 2017	Proj. diff. btw. percentiles 90th and 10th of debt ratio distribution in 2017	Proj. diff. btw. percentiles 60th and 40th of debt ratio distribution in 2017
BE	99.9	104.3	96.9	112.8	15.9	3.0
BG	19.5	16.5	12.6	22.9	10.3	2.0
CZ	45.1	50.7	44.4	58.1	13.7	2.4
DK	45.4	46.5	40.5	53.3	12.9	2.4
DE	81.7	70.9	61.5	82.7	21.2	4.0
EE	10.5	9.2	7.0	13.2	6.2	1.2
ES	86.1	106.8	92.9	124.6	31.7	6.1
FR	90.0	90.5	82.7	99.8	17.0	3.2
IT	126.5	114.0	99.7	131.4	31.8	6.0
CY	89.7	116.0	102.7	132.4	29.7	6.1
LV	41.9	40.1	25.5	69.2	43.7	8.2
LT	41.6	41.3	30.8	57.0	26.2	5.4
LU	21.3	30.1	25.4	36.4	11.0	2.2
HU	78.4	75.7	56.2	102.3	46.0	10.1
MT	72.3	73.3	63.2	86.9	23.7	4.5
NL	68.8	70.5	63.9	78.2	14.3	2.6
AT	74.6	74.7	70.0	79.8	9.8	1.9
PL	55.5	50.7	42.8	60.5	17.7	3.8
RO	34.6	33.9	21.3	55.2	33.9	6.4
SI	54.0	70.5	63.0	79.1	16.1	3.4
SK	51.7	58.7	50.3	69.3	19.0	3.9
FI	53.1	56.5	50.3	64.7	14.3	2.7
SE	37.4	29.7	24.1	39.5	15.4	2.7
UK	88.7	98.1	89.6	107.2	17.6	3.5
EA	92.8	90.6	84.4	97.0	12.6	2.6

For IT, based on Figure 1a, the most likely outcome appears to be a decreasing path for the debt ratio over the five-year projection horizon, despite possible adverse shocks to growth and interest rates. The Italian debt ratio in 2017 is anyway projected to be still higher than 120% with a 30% probability (as the 70^{th} distribution percentile is around 120%), and higher than 116% with a 50% probability (Figure 1a). In terms of upside risks to debt dynamics, with a probability of around 20%, the debt ratio is projected to broadly stabilise (above 130%) between 2013 and 2014, and start reaching lower values only afterwards (Figure 1a). But if shocks to interest rates were permanent rather than temporary, the debt ratio could also take higher values in the last projection year (2017) relative to the year before (Figure 1b).

For ES, there is an 80% probability of a debt ratio higher than 100% in 2017, when (temporary) macroeconomic shocks are accounted for (Figure 2a). The whole cone of possible projected debt ratios over the projection horizon 2013-17 lies above the value of the debt ratio in 2012, thus fostering debt sustainability concerns.

Finally, for HU, Figure 3a shows a 40% probability of a debt ratio higher than 80% in 2017 under the assumption of temporary shocks to growth, interest rates and the exchange rate. In terms of debt dynamics, from 2013 onwards, the debt ratio would broadly stabilise (at around 76-77%) or reach higher values with a rather high probability (60%).

Figure 1a – Fan chart based on stochastic debt projections (temporary shock scenario)

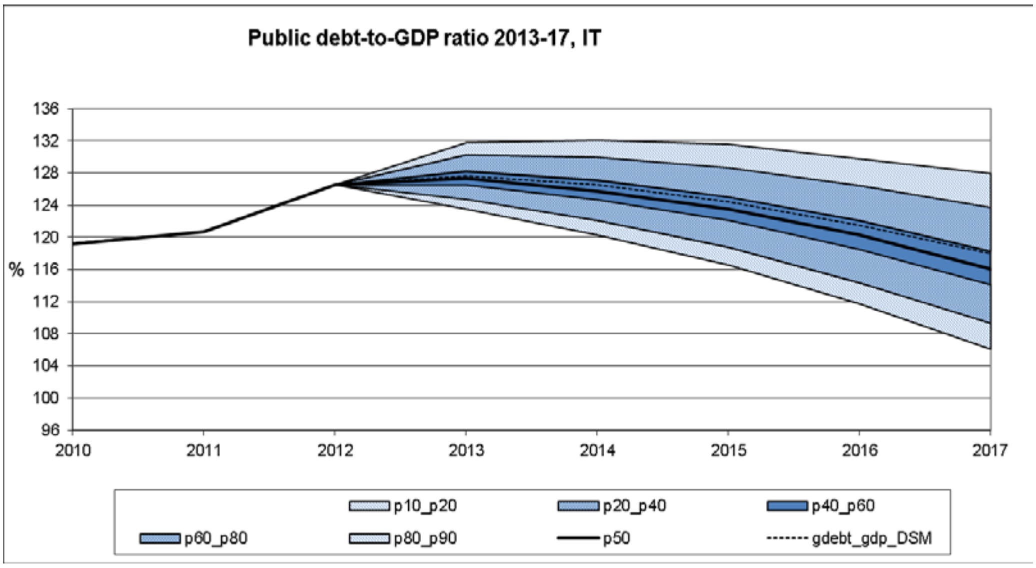


Figure 1b – Fan chart based on stochastic debt projections (permanent shock scenario)

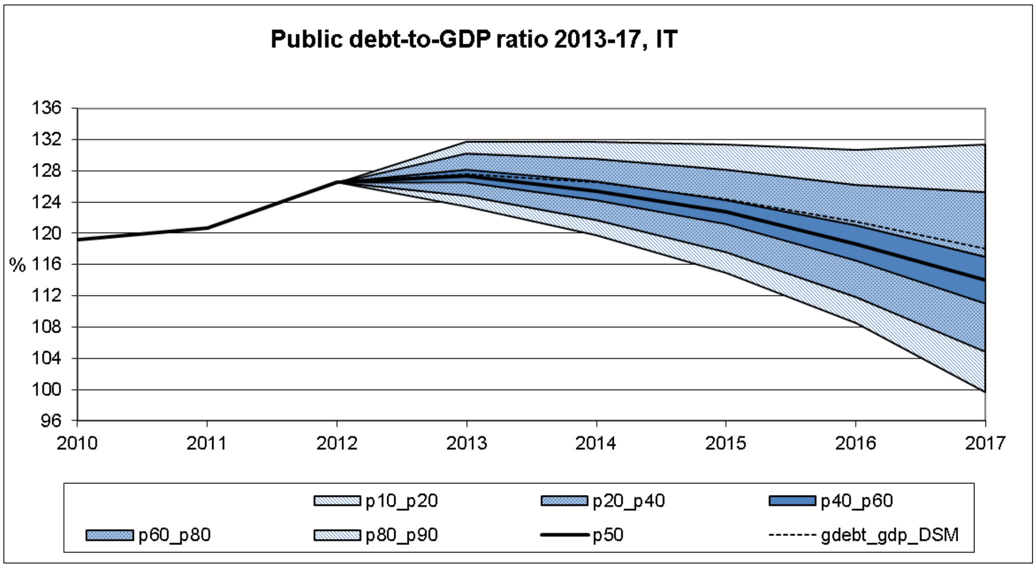


Figure 2a – Fan chart based on stochastic debt projections (temporary shock scenario)

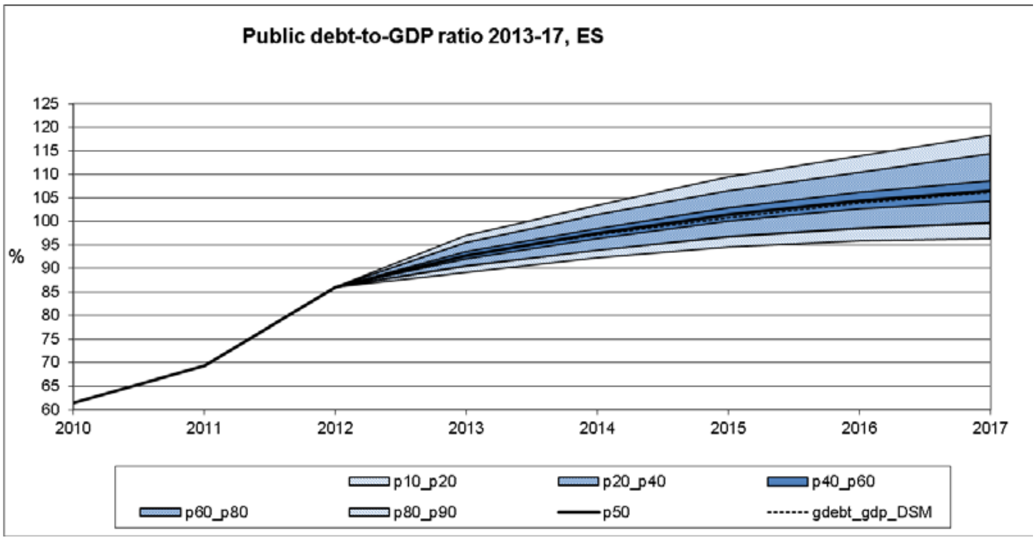


Figure 2b – Fan chart based on stochastic debt projections (permanent shock scenario)

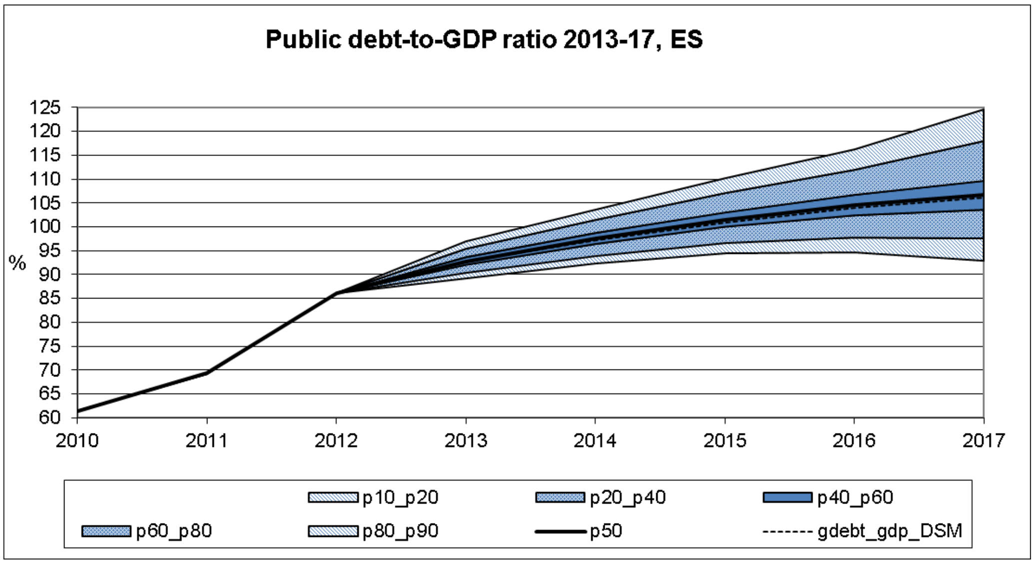


Figure 3a – Fan chart based on stochastic debt projections (temporary shock scenario)

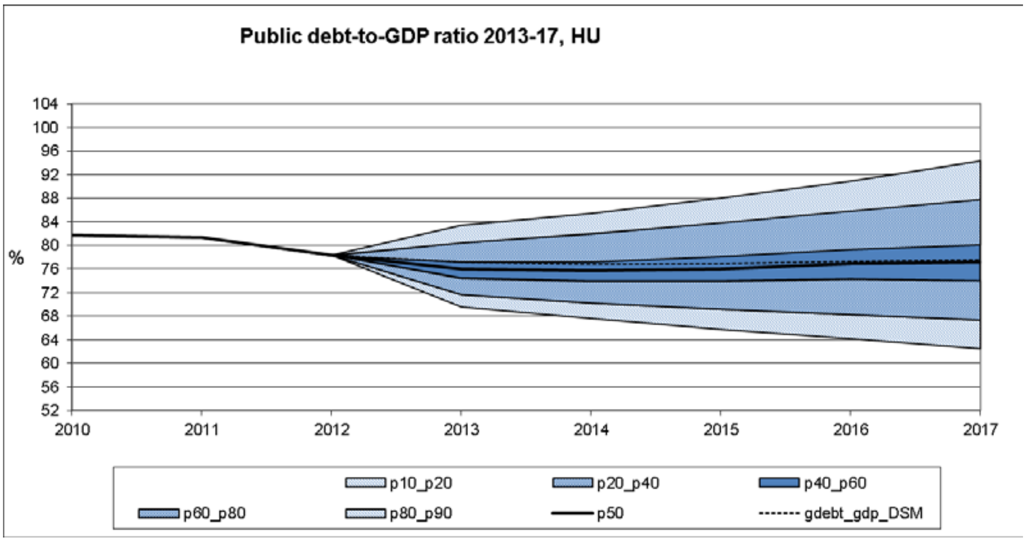
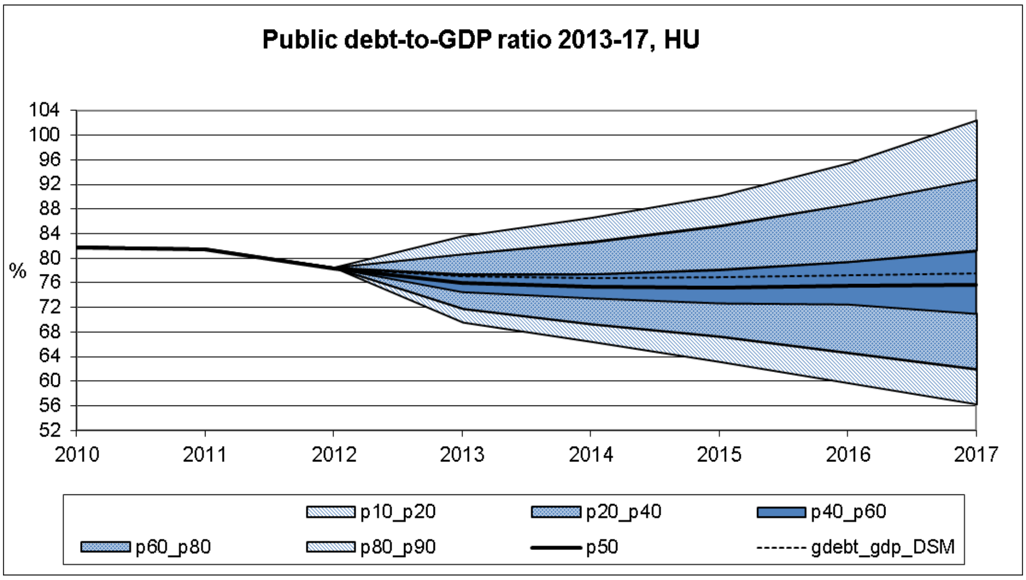


Figure 3b – Fan chart based on stochastic debt projections (permanent shock scenario)



6. Conclusions

The methodology for stochastic debt projections based on the variance-covariance matrix of historical shocks has been used in this paper to run projections for 24 EU countries (with the only exception of those under adjustment programmes). The methodology is particularly suited to the purpose of featuring macroeconomic uncertainty into the analysis of debt dynamics. Shocks to growth, interest rates and the exchange rate have an impact on the evolution of the debt-to-GDP ratio. It is indeed crucial to take account of these shocks, beyond governments' control, when running debt projections and producing debt sustainability assessments.

More traditional deterministic debt projections (that produce single debt trajectories corresponding to precise sets of macroeconomic assumptions) respond with sensitivity analysis to the need to account for the impact of possible macroeconomic shocks. While indeed providing some information on possible changes in debt evolution due to changes in underlying macroeconomic assumptions, sensitivity analysis does anyway only allow accounting for macroeconomic uncertainty in a very limited way, by considering alternative scenarios to the baseline, in which macroeconomic variables are shocked one at the time or ad-hoc combinations of shocks are assumed. The limitation of this kind of approach is twofold. First, only a limited number of alternative scenarios can be designed, while one could think of an infinite number of scenarios reflecting possible changes to growth, interest rates and the exchange rate. Second, the correlation of macroeconomic shocks is fully neglected. Both shortcomings are addressed in a stochastic debt projection framework, where a very large number of random shocks are simulated (2000 in this paper) and both the size of the shocks and their correlation (based on variables' historical behaviour) are taken into account through the variance-covariance matrix of shocks.

A distinctive advantage of the methodology based on the historical variance-covariance matrix approach used here, relative to the methodology relying on VAR modelling, lies in the possibility to use model-independent forecasts to define a central scenario, around which shocks are applied. To define our central scenario, we indeed used DG ECFIN's (Autumn 2012) forecasts, together with AWG agreed assumptions on the long-run convergence of the non-fiscal determinants of debt dynamics, consistently with the European Commission DG ECFIN's Debt Sustainability Monitor (DSM) model for deterministic debt projections. It is around this central scenario that we simulate shocks to growth, short-term and long-term interest rates, and the exchange rate for non-EA countries. In the model, the cyclical component of the government's balance adjusts to the shocks, while no (discretionary) fiscal policy response is assumed (this will shortly be object of an extension to the current modelling).

The stochastic framework presented in this paper produces probabilistic outcomes, like the probability that the debt ratio for a certain country is higher than a certain value in a given projection year or the probability that the debt ratio stabilises or decreases over the projection horizon. This improves the transparency of simulation results relative to deterministic projections, which provide single debt trajectories, dependent on the specific macroeconomic assumptions made, with no corresponding probabilistic assessment. In the latter case, the reader can only judge about the relevance of the presented debt projection results based on his expectations about the plausibility of the underlying macro-assumptions, while stochastic projections make such a probabilistic assessment explicit.

Results are provided in the paper for all 24 EU countries for both scenarios with temporary and permanent macroeconomic shocks. Cross-country differences in the variance of the distribution of the debt ratio in 2017 (end of projection horizon), reflecting the country-specific volatility of macroeconomic conditions, are evident from the results. This shows the relevance of the applied methodology in providing a more comprehensive and country-tailored assessment of upward and downward risks to debt dynamics, based on which policy recommendations could be "modulated".

Three countries (IT, ES and HU) have been subject to closer scrutiny in the paper. For IT, our simulation results allow to conclude that the most likely outcome is a decreasing path for the debt ratio over the projection horizon (2013-17), despite possible adverse shocks to growth and interest rates. The debt ratio in 2017 is anyway projected to be still higher than 120% with a 30% probability, and higher than 116% with a 50% probability, when temporary shocks are accounted for. For ES, simulation results point to debt sustainability concerns due to the increasing debt path over the whole projection period, for all simulated shock combinations. Under the assumption of temporary macroeconomic shocks, the probability of a Spanish debt ratio greater than 100% is as high as 80% in 2017. Finally, for HU, there is a rather high probability (60%) that the debt ratio broadly stabilises (at around 76-77%) or reaches higher values from 2013 onwards. Under the assumption of temporary shocks, HU would have a 40% probability of a debt ratio higher than 80% in 2017.

Annex 1

Table A.1 – Time interval used for the calculation of the historical variance-covariance matrix

Country	***Time interval***
AT	1989q3-2012q2
BE	1981q1-2012q2
BG	1998q3-2010q3
CY	1999q4-2010q3
CZ	2000q2-2012q2
DE	1975q4-2012q2
DK	1981q3-2012q2
EE	1998q1-2012q2
ES	1978q2-2012q2
FI	1988q1-2012q2
FR	1975q4-2012q2
HU	1999q2-2012q2
IT	1981q1-2012q2
LT	2001q1-2010q2
LU	1999q1-2012q1
LV	2001q1-2010q3
MT	2001q1-2010q3
NL	1978q2-2012q2
PL	2001q1-2012q2
RO	2005q2-2010q3
SE	1981q1-2010q3
SI	2002q2-2012q2
SK	2000q3-2012q2
UK	1975q4-2012q2

Annex 2 - Fan charts with simulated data on growth, interest rate and exchange rate

Figure A.1 – Fan chart based on stochastic debt projections

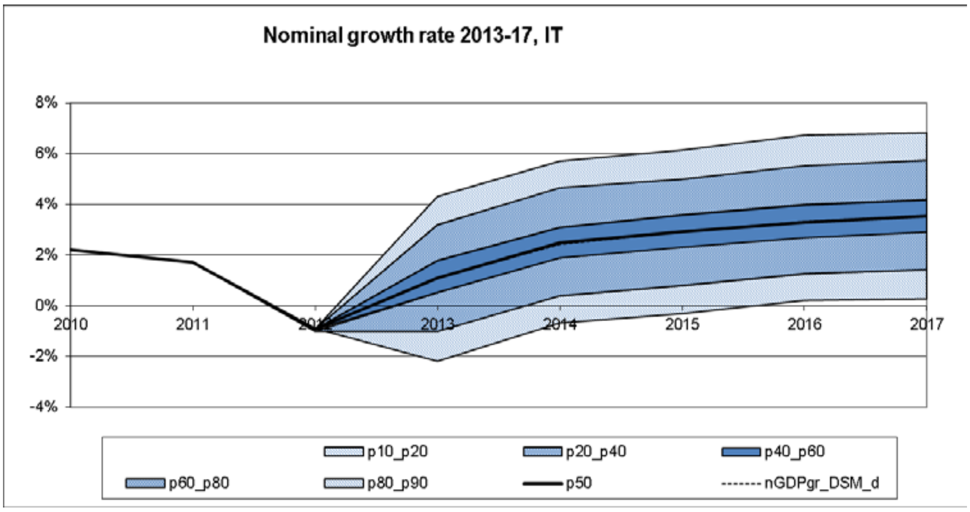


Figure A.2 – Fan chart based on stochastic debt projections

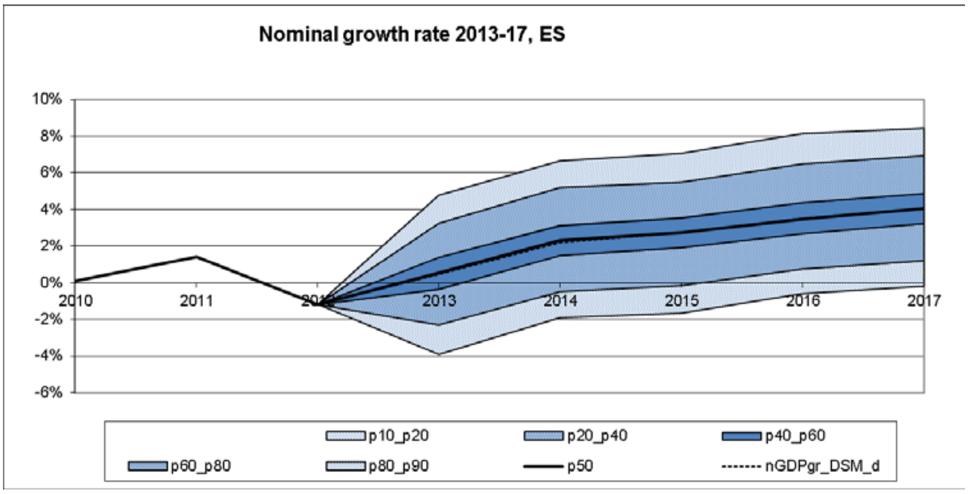


Figure A.3 – Fan chart based on stochastic debt projections

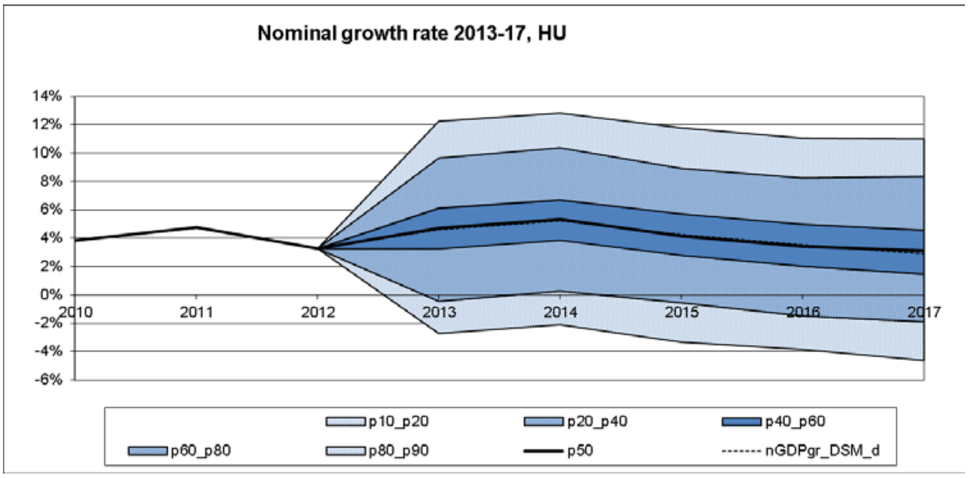


Figure A.4a – Fan chart based on stochastic debt projections (temporary shock scenario)

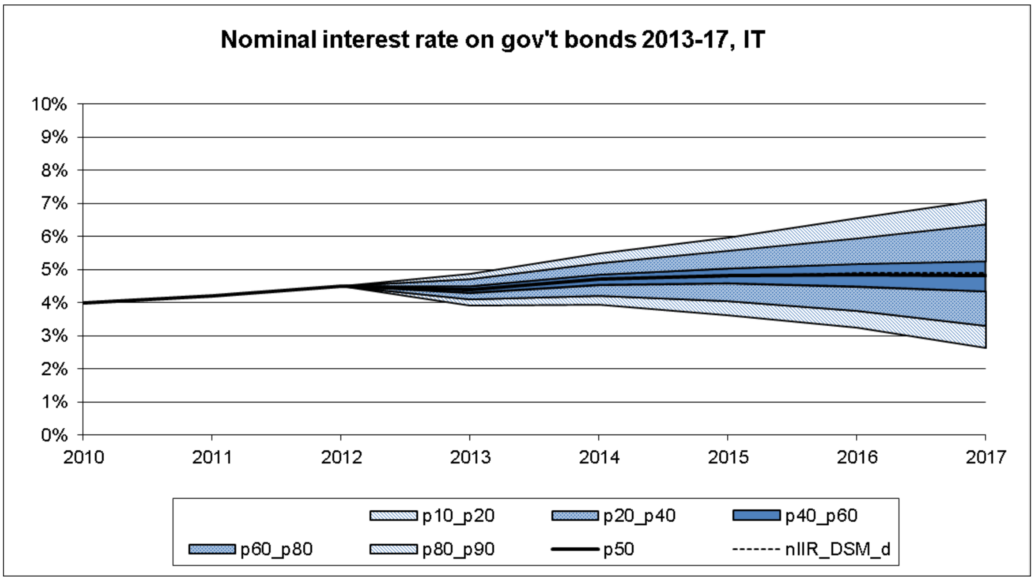


Figure A.4b – Fan chart based on stochastic debt projections (permanent shock scenario)

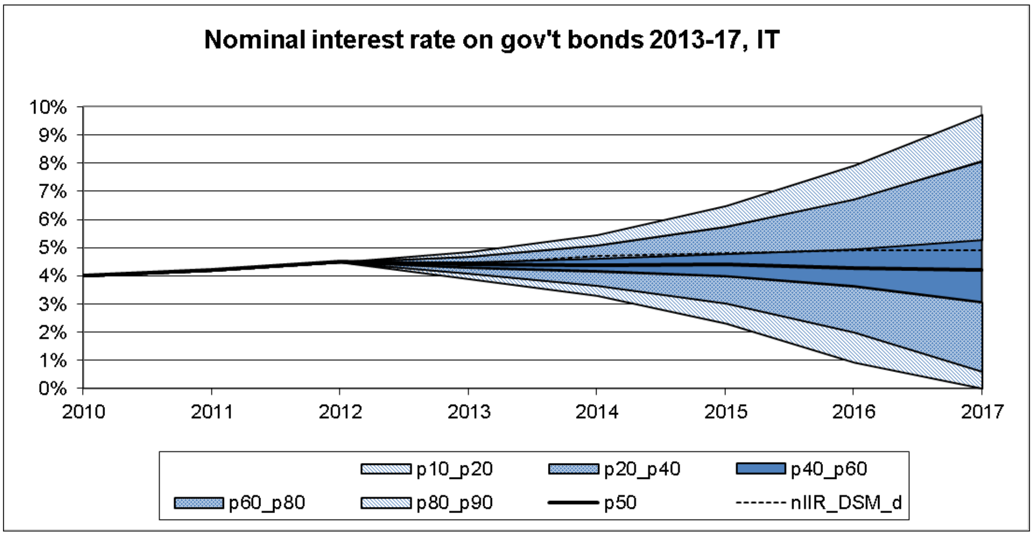


Figure A.5a – Fan chart based on stochastic debt projections (temporary shock scenario)

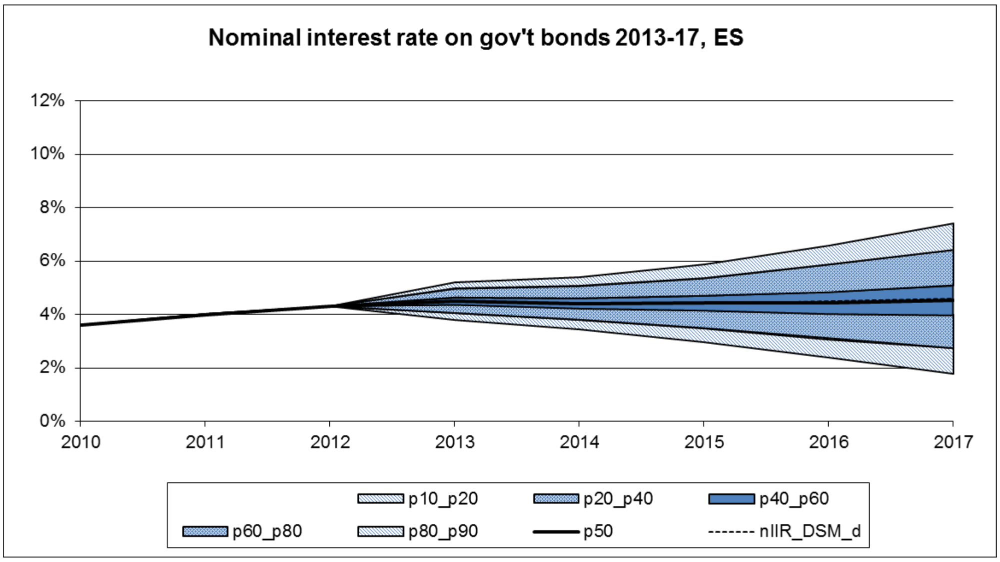


Figure A.5b – Fan chart based on stochastic debt projections (permanent shock scenario)

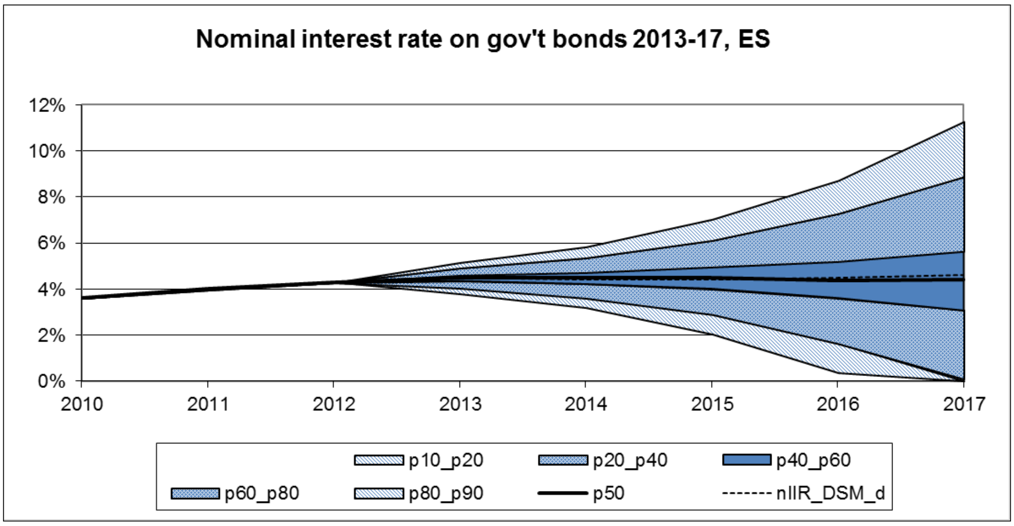


Figure A.6a – Fan chart based on stochastic debt projections (temporary shock scenario)

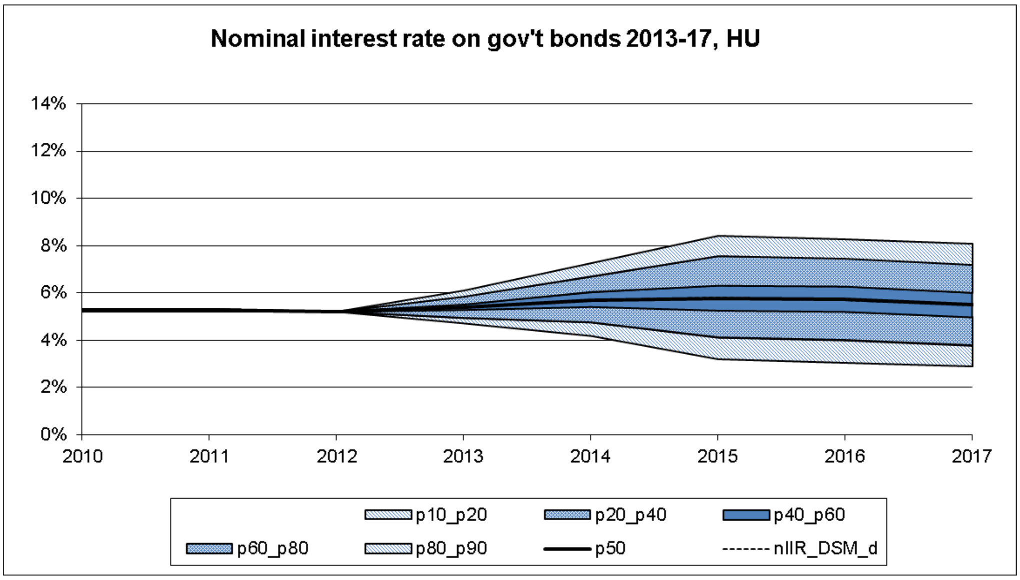


Figure A.6b – Fan chart based on stochastic debt projections (permanent shock scenario)

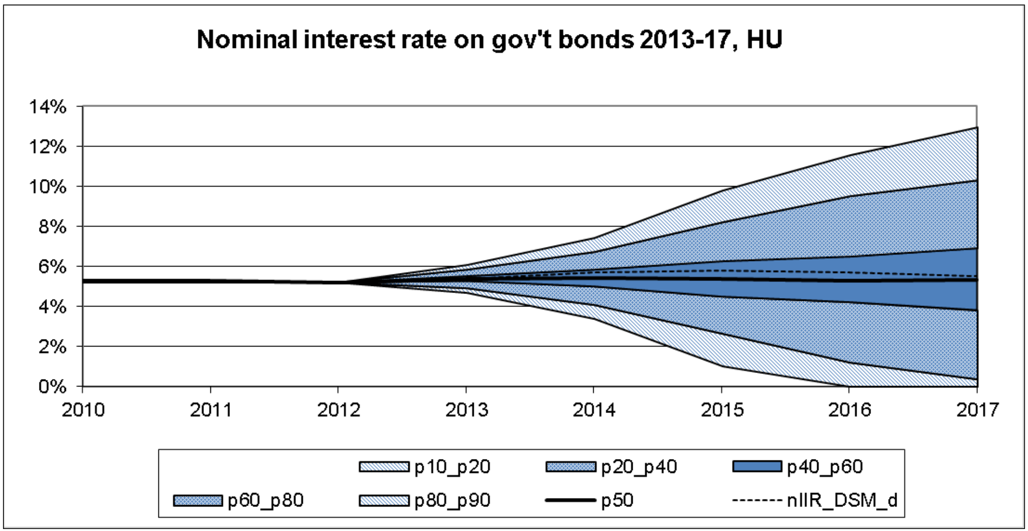


Figure A.7a – Fan chart based on stochastic debt projections (temporary shock scenario)

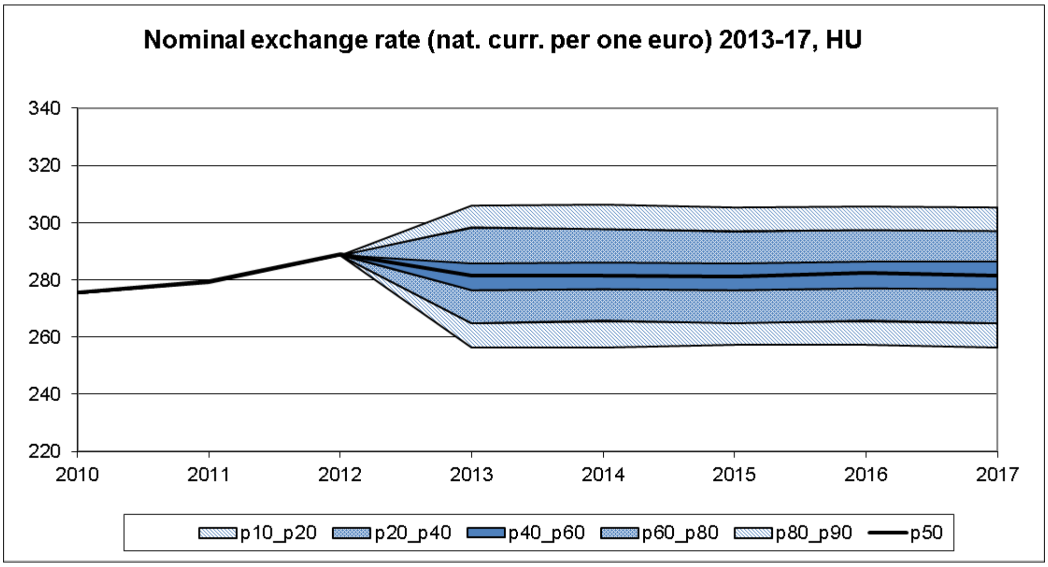
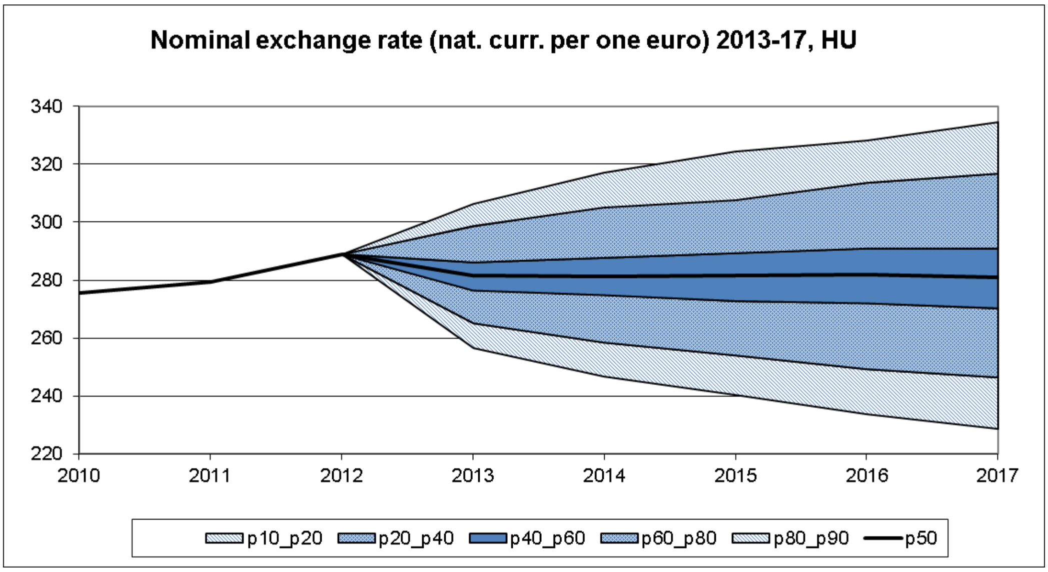


Figure A.7b – Fan chart based on stochastic debt projections (permanent shock scenario)



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