1- A certain sports car comes equipped with either an automatic or a manual transmission, and the car is available in one of four colors. Relevant probabilities for various combinations of transmission type and color are given in the accompanying table.

	Color			
Transmission Type	White	Blue	Black	Red
A	.13	.10	.11	.11
M	.15	.07	.15	.18

Let A = (automatic transmission), $B = \{black\}$, and $C = \{white\}$.

- a. Calculate P(A), P(B), and $P(A \cap B)$.
- b. Calculate both P(A|B) and P(B|A), and explain in context what each of these probabilities represents.
- c. Calculate and interpret P(A|C) and P(A/C').

Answers to Problem No. 1

a.
$$P(A) = .13 + .10 + .11 + .11 = .45,$$

 $P(B) = .11 + .15 = .26$
 $P(A \cap B) = .11$

b.
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.11}{.26} = .4231$$

Knowing that the car is black, the probability that it has an automatic transmission is .4231.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.11}{.45} = .2444$$

Knowing that the car has an automatic transmission, the probability that it is black is .2444.

c.
$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{.13}{.28} = .4643$$

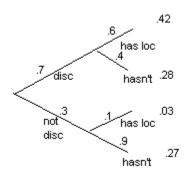
The probability that the car has automatic transmission, knowing that the car is white is .4643.

$$P(A \mid C') = \frac{P(A \cap C')}{P(C')} = \frac{.32}{.72} = .4444$$

Knowing that the car is not white, the probability that it has an automatic transmission is .4444.

- 2- Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared.
 - a. If it has an emergency locator, what is the probability that it will not be discovered?
 - b. If it does not have an emergency locator, what is the probability that that it will be discovered?

Answers to Problem No. 2



a. P(not disc | has loc) =
$$\frac{P(not.disc \cap has.loc)}{P(has.loc)} = \frac{.03}{.03 + .42} = .067$$

b.
$$P(\text{disc} \mid \text{no loc}) = \frac{P(\text{disc} \cap no.loc)}{P(\text{no.loc})} = \frac{.28}{.55} = .509$$

- 3- Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities.
 - a. P(all of the next three vehicles inspected pass)
 - b. P(at least one of the next three inspected fail)
 - c. P(exactly one of the next three inspected passes)
 - d. P(at most one of the next three vehicles inspected passes)
 - e. Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass?

Answers to Problem No. 3

P(pass) = .70

- a. P(three pass) = (.70)(.70)(.70) = .343
- b. P(at least one fails) = 1 P(all pass) = 1 .343 = .657
- c. P(exactly one passes) = (.70)(.30)(.30) + (.30)(.70)(.30) + (.30)(.30)(.70) = .189
- d. P (at most one passes) = $P(0 \text{ passes}) + P(\text{one passes}) = (.3)^3 + .189 = .216$
- e. $P(3 \text{ pass} | 1 \text{ or more pass}) = \frac{P(3 \text{ pass}) \ge 1 \text{ pass})}{P(\ge 1 \text{ pass})} = \frac{P(3 \text{ pass})}{P(\ge 1 \text{ pass})} = \frac{.343}{.973} = .353$