

1-A large company offers its employees two different health insurance plans and two different dental insurance plans. Plan 1 of each type is relatively inexpensive, but restricts the choice of providers, whereas plan 2 is more expensive but more flexible. The accompanying table gives the percentages of employees who have chosen the various plans:

Health Plan	Dental Plan	
	1	2
1	27%	14%
2	24%	35%

Suppose that an employee is randomly selected and both the health plan and dental plan chosen by the selected employee are determined.

- What are the four simple events?
- What is the probability that the selected employee has chosen the more restrictive plan of each type?
- What is the probability that the employee has chosen the more flexible dental plan?
- If the employee selects D2, what is the chance that the employee will go with H1?

ANSWER:

Let H1 and H2 represent the two health plans. Let D1 and D2 represent the two dental plans.

- The simple events are {H1,D1}, {H1, D2}, {H2,D1}, {H2,D2}.
- $P(\{H1,D1\}) = .27$
- $P(\{D2\}) = P(\{H1,D2\}, \{H2,D2\}) = .14 + .35 = .49$
- $P(H1 | D2) = .14 \div .49 = .286$

2-Let A denote the event that the next item checked out at a college library is a math book, and let B be the event that the next item checked out is a history book. Suppose that $P(A) = .40$ and $P(B) = .50$.

- Why is it not the case that $P(A) + P(B) = 1$?
- Calculate $P(A')$
- Calculate $P(A \cup B)$.
- Calculate $P(A' \cap B')$.

ANSWER:

- The probabilities do not add to 1 because there are other items besides math and history books to be checked out from the library.
- $P(A') = 1 - P(A) = 1 - .40 = .60$
- $P(A \cup B) = P(A) + P(B) = .40 + .50 = .90$ (since A and B are mutually exclusive events)
- $P(A' \cap B') = P[(A \cup B)']$ (De Morgan's law) $= 1 - P(A \cup B) = 1 - .90 = .10$

3-At Oxnard University, a student ID consists of two letters followed by four digits. How many unique student IDs can be created? Would one letter followed by three digits suffice for university with 40,000 student population?

ANSWER:

No. of arrangements = $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6,760,000$ number of IDs

$26 \times 10 \times 10 \times 10 = 26,000$ is not enough for a student population of 40,000.

4-Consider the following transportation engineering problem. You want to drive, in sequence, from a start point to each of 5 cities, and you want to compare the distances and average speeds of the different routings. How many different routings would have to be compared?

ANSWER:

This is a permutation problem of $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ different ways that 5 cities can be tested.

5- Shortly after being put into service, some buses manufactured by a certain company have developed cracks on the underside of the main frame. Suppose a particular city has 25 of these buses, and cracks have actually appeared in 8 of them.

- How many ways are there to select a sample of 5 buses from the 25 for a thorough inspection?
- In how many ways can a sample of 5 buses contain exactly 4 with visible cracks?
- If a sample of 5 buses is chosen at random, what is the probability that exactly 4 of the 5 buses will have visible cracks?

ANSWER:

a. $\binom{25}{5} = \frac{25!}{5!20!} = 53,130$

b. $\binom{8}{4} \binom{17}{1} = 1190$

c. $P(\text{exactly 4 have cracks}) = \frac{\binom{8}{4} \binom{17}{1}}{\binom{25}{5}} = \frac{1190}{53,130} = .022$