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1. This table shows a cross-tabulation of employee age versus frequency of absenteeism during January at the Zarthan Company.

Absences	Age	
	Under 25 (A)	25 or More (D)
Under 2 days (B)	50	38
2 or more days (C)	45	32
Total	95	70

88  
77  
165 total

- a. What is  $P(A \cup C)$ ? (10 pts)

$$\frac{50 + 45 + 32}{165} \rightarrow \frac{127}{165} \approx 0.7697 \text{ or } \frac{127}{165}$$

$$(50 + 45) + (45 + 32) - (45) = 127$$

- b. What is  $P(B | D)$ ? (10 pts)

$$\frac{38}{70} \approx 0.5429 \text{ or } \frac{19}{35}$$

$$50 + 38 + 45 + 32 = 165$$

$$\frac{38}{165}$$

$$P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{38}{38 + 32} \rightarrow \frac{38}{70}$$

2. 70% of the light aircraft that disappear while in flight in a certain country are subsequently discovered (D). Of the aircraft that are discovered, 60% have an emergency locator (L), whereas 90% of the aircraft not discovered (D') do not have such a locator (L'). Suppose a light aircraft has disappeared

- a. What is the probability that it will not be discovered, if it has an emergency locator? (10 pts)  
b. What is the probability that it will be discovered, if it does not have an emergency locator? (10 pts)

a)  $P(\text{discovered}) = 0.7$  and  $P(\text{not discovered}) = 0.3$

$P(\text{locator} | \text{discovered}) = 0.6$  and  $P(\text{no locator} | \text{discovered}) = 0.4$

$P(\text{locator}) = 0.6 \cdot 0.7 + 0.1 \cdot 0.3 = 0.42 + 0.03 = 0.45$

So,  $P(\text{no locator}) = 0.55$

$P(\text{not discovered} | \text{locator}) \cdot 0.45 = (0.1)(0.3)$

$P(\text{not discovered} | \text{locator}) = 0.03 / 0.45 = 0.0667 \text{ or } \frac{1}{15}$

b)  $P(\text{discovered} | \text{no locator}) = ?$

From Bayes theorem we get:

$P(\text{discovered} | \text{no locator}) P(\text{no locator}) = P(\text{no loc} | \text{dis}) P(\text{dis})$

Putting all the values above we get

$P(\text{discovered} | \text{no locator}) (0.55) = (0.4)(0.7)$

$P(\text{discovered} | \text{no locator}) = 0.28 / 0.55 = 0.5091$

3. An insurance company offers its policyholders a number of different payment options. For a randomly selected policyholder, let  $X$  = number of months between successive payments. The cdf is as follows:

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.25 & 1 \leq x < 3 \\ 0.45 & 3 \leq x < 4 \\ 0.55 & 4 \leq x < 6 \\ 0.6 & 6 \leq x < 12 \\ 1 & 12 \leq x \end{cases}$$

$$P(X=1) = 0.25$$

$$P(X=3) = 0.45 - 0.25 = 0.20$$

$$P(X=4) = 0.55 - 0.45 = 0.10$$

$$P(X=6) = 0.60 - 0.55 = 0.05$$

$$P(X=12) = 1 - 0.60 = 0.40$$

- A) Compute  $P(1 < X \leq 6)$ ? (10 pts)

- B) Compute the expected optional payment? (10 pts)

$$a) P(1 < X \leq 6) = P(3) + P(4) + P(6) = 0.20 + 0.10 + 0.05 = 0.35$$

- b)  $E(X)$  = expected payment

$$E(X) = \sum xP(x)$$

$$E(X) = 0 \times 0 + 1 \times 0.25 + 3 \times 0.20 + 4 \times 0.10 + 6 \times 0.05 + 12 \times 0.40$$

$$E(X) = 6.35$$

4. Suppose the number of tornadoes observed in Kansas is a Poisson process with rate  $\alpha = 10$  per year. What is the probability of observing more than mean minus one standard deviation of tornadoes over the next 18 months? (10 pts)

given data, for 18 months  $\lambda = 10 + 5 = 15$

Probability of observing more than 14 tornadoes in 18 months

$$P(X=15)$$

STD Dev.?

$$\frac{15^{15} e^{-15}}{15!} \rightarrow (334864.6277)(e^{-15}) \approx .1024$$

$$1 - e^{-15} \left[ \frac{15^0}{0!} + \frac{15^1}{1!} + \frac{15^2}{2!} + \frac{15^3}{3!} + \dots \right] = .5343$$

$$\approx 53.4 \text{ or } .5343$$

5. If 85 percent of automobiles in Orange County have both headlights working, what is the probability that in a sample of 10 automobiles, either 6 or 7 will have both headlights working? (10 pts)

$$P = 0.85 \quad n = 10 \quad X \sim \text{Bin}(10, 0.85)$$

$$P(X \text{ is at least } 7)$$

$$P(7 \leq X)$$

$$P(X=7) + P(X=10)$$

$$\binom{10}{7} (0.85)^7 (0.15)^3 + \binom{10}{10} (0.85)^{10} (0.15)^0 = \boxed{0.074}$$

*p. > 0.5 cannot be more than one!*

$$\frac{\binom{10}{6} (0.85)^6 (0.15)^4 + \binom{10}{7} (0.85)^7 (0.15)^3}{2} = 9.02204$$

6. A box in a certain supply room contains four 40-W lightbulbs (LB), five 60 W bulbs and six 75-W bulbs. Suppose that 4 bulbs are randomly selected.

- A) What is the probability of exactly two of the selected bulbs are rated 75-W? (10 pts)

$$P(\text{exactly two of selected bulbs are rated 75W})$$

given data,

4 → 40W lightbulbs

5 → 60W bulbs

6 → 75W bulbs

15 total and 4 are randomly selected

$$= \frac{6C_2 \times 9C_2}{15C_4} \approx \boxed{.0989 \text{ or } \frac{9}{91}}$$

- B) What is the probability that all four of the selected bulbs have the same rating? (10 pts)

$$P(\text{all 4 bulbs have the same rating})$$

$$\frac{4C_4 + 5C_4 + 6C_4}{15C_4} = \boxed{\frac{1}{65} \text{ or } 0.0153}$$

$$\frac{1 + 5 + \frac{6!}{4! \times (6-4)!}}{15! / (4! \times (15-4)!)}$$