

Sistemas Lineares - Regra de Cramer e Escalonamento:

Regra de Cramer:

1.

a)

Handwritten solution for a system of two linear equations in two variables:

$$\textcircled{1} a) \begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$
$$D = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} = 5 \quad D_x = \begin{bmatrix} 2 & -1 \\ -3 & 3 \end{bmatrix} = 3$$
$$D_y = \begin{bmatrix} 2 & 2 \\ -1 & -3 \end{bmatrix} = -4$$
$$D = 5 \quad D_x = 3 \quad D_y = -4$$
$$x = \frac{3}{5} \rightarrow x = 0,6$$
$$y = \frac{-4}{5} \rightarrow y = -0,8$$

R: $V = \{(3/5, -4/5)\}$ ou $V = \{(0,6; -0,8)\}$.

b)

Handwritten solution for a system of three linear equations in three variables:

$$\textcircled{2} \begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$$
$$D = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{bmatrix} = -23$$
$$D_x = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{bmatrix} = -23$$
$$D_y = \begin{bmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{bmatrix} = -23$$
$$D_z = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{bmatrix} = 23$$
$$D = -23 \quad D_x = -23 \quad D_y = -23 \quad D_z = 23$$
$$x = \frac{-23}{-23} \rightarrow x = 1 \quad y = \frac{-23}{-23} \rightarrow y = 1$$
$$z = \frac{23}{-23} \rightarrow z = -1$$
$$R: V = \{(1, 1, -1)\}$$

R: $V = \{(1, 1, -1)\}$.

2.

$$\textcircled{2} \begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases} \quad D = \begin{bmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{bmatrix} = 30$$

$$Dy = \begin{bmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{bmatrix} = 30 \quad y = \frac{30}{30} \rightarrow y = 1$$

$R: A) 1$

R: A) 1

3.

$$\textcircled{3} \begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases} \quad D = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{bmatrix} = 1$$

$$Dx = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 3 & -1 \end{bmatrix} = -1 \quad Dy = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{bmatrix} = 1 \quad Dz = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix} = 0$$

$$\begin{array}{lll} D = 1 & x = \frac{-1}{1} \rightarrow x = -1 & y = \frac{1}{1} \rightarrow y = 1 \\ Dx = -1 & & \\ Dy = 1 & z = \frac{0}{1} \rightarrow z = 0 & a + b + c = -1 + 1 + 0 = 0 \\ Dz = 0 & & \end{array}$$

$R: C) 0$

R: C) 0

4.

$$\textcircled{4} \begin{cases} x+2y-3z=29 \\ x+3y+2z=4 \\ x-y-2z=8 \end{cases} \quad D = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{bmatrix} = 16$$

$$D_x = \begin{bmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{bmatrix} = 16 \quad D_y = \begin{bmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{bmatrix} = 80 \quad D_z = \begin{bmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{bmatrix} = -96$$

$$\begin{aligned} D &= 16 & X &= \frac{16}{16} \rightarrow X=1 & Y &= \frac{80}{16} \rightarrow Y=5 \\ D_x &= 16 & Z &= \frac{-96}{16} \rightarrow Z=-6 \\ D_y &= 80 & & & & \\ D_z &= -96 & & & & \end{aligned}$$

$$X+Y+Z = 1+5-6 = 0$$

$R: a) 0$

R: A) 0

5.

$$\textcircled{5} \begin{cases} 2x+y=5 \\ 2y+z=3 \\ 3x+2y+z=7 \end{cases} \quad D = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix} = 3$$

$$D_x = \begin{bmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{bmatrix} = 4 \quad D_y = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{bmatrix} = 7 \quad D_z = \begin{bmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{bmatrix} = -5$$

$$\begin{aligned} D &= 3 & X &= \frac{4}{3} \rightarrow X=1,333... & Y &= \frac{7}{3} \rightarrow Y=2,333... \\ D_x &= 4 & Z &= \frac{-5}{3} \rightarrow Z=-1,666... \\ D_y &= 7 & & & & \\ D_z &= -5 & & & & \end{aligned}$$

$R: D) \frac{4}{3}; \frac{7}{3} \text{ e } -\frac{5}{3}$

R: D) $\frac{4}{3}$; $\frac{7}{3}$ e $-\frac{5}{3}$

6.

$$\textcircled{6} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix} \quad \begin{cases} X=3 \\ 2X+Y=7 \\ -X+2Y+2Z=-1 \end{cases} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} = 2$$

$$D_x = \begin{bmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} = 6 \quad D_y = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ -1 & -1 & 2 \end{bmatrix} = 2 \quad D_z = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 7 \\ -1 & 2 & -1 \end{bmatrix} = 0$$

$$\begin{array}{l} D=2 \\ D_x=6 \\ D_y=2 \\ D_z=0 \end{array} \quad \begin{array}{l} X = \frac{6}{2} \rightarrow X=3 \\ Y = \frac{2}{2} \rightarrow Y=1 \\ Z = \frac{0}{2} \rightarrow Z=0 \end{array} \quad \boxed{R: E) Z=0}$$

R: E) Z = 0

Escalonamento:

1.

$$\textcircled{1} \quad \begin{cases} 2X - Y - 3Z = -5 \\ X + 3Y - Z = 11 \\ X - 5Z = 3 \end{cases} \xrightarrow{-0,5} \begin{bmatrix} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{bmatrix}$$

$$\begin{cases} 0 & 3,5 & -0,5 & 13,5 \\ 0 & 0,5 & -3,5 & 5,5 \end{cases} \xrightarrow{\cdot 0,5} \begin{cases} 0 & 1,75 & 0,25 & 6,75 \\ 0 & -1,75 & 12,25 & -19,25 \end{cases}$$

$$\begin{cases} 0 & 0 & 12,5 & -12,5 \end{cases} \quad \begin{array}{l} 12,5Z = -12,5 \\ Z = \frac{-12,5}{12,5} \\ Z = -1 \end{array} \quad \begin{array}{l} X - 5Z = 3 \\ X - 5(-1) = 3 \\ X + 5 = 3 \\ X = 3 - 5 \\ X = -2 \end{array} \quad \begin{array}{l} X + 3Y - Z = 11 \\ -2 + 3Y - (-1) = 11 \\ -2 + 3Y + 1 = 11 \\ 3Y = 12 \\ Y = 4 \end{array}$$

$$\boxed{R: X=-2; Y=4; Z=-1}$$

R: X = -2; Y = 4; Z = -1.

2.

$$\begin{aligned} \textcircled{2} \begin{cases} X &= 2Y \\ 2Y &= 3Z \\ X+Y+Z &= 11 \end{cases} & \begin{aligned} X &= 2Y \\ 2Y &= 3Z \\ X &= 3Z \end{aligned} & X=2Y=3Z \\ & & X+Y+Z=11 \\ & & X+\frac{1}{2}X+\frac{1}{3}X=11 \\ & & \frac{3}{2}X+\frac{1}{3}X=11 \\ & & \frac{11}{6}X=11 \\ & & X=\frac{11}{\frac{11}{6}} \rightarrow X=6 \\ & & \frac{11}{6} \\ & Y=\frac{1}{2}X & Z=\frac{1}{3}X \\ & Y=\frac{1}{2} \cdot 6 & Z=\frac{1}{3} \cdot 6 \\ & Y=3 & Z=2 \\ & & X+2Y+3Z \\ & & 6+2 \cdot 3+3 \cdot 2 \\ & & 6+6+6 \\ & & 18 \\ & \text{R: B) } 18 \end{aligned}$$

R: B) 18

3.

$$\begin{aligned} \textcircled{3} \begin{cases} x+y+z=0 \\ 2x-y-2z=1 \\ 6y+3z=-12 \end{cases} & \begin{aligned} 6y+3z &= -12 \div 3 \\ 2y+z &= -4 \\ z &= -4-2y \end{aligned} \\ & \begin{aligned} 2x-y-2(-4-2y) &= 1 & x+y+(-4-2y) &= 0 \\ 2x-y+8+4y &= 1 & x-y-4 &= 0 \\ 2x+3y &= -7 & x &= y+4 \\ 2 \cdot (y+4)+3y &= -7 & x &= -3+y \\ 2y+8+3y & & x &= 1 \\ 5y &= -15 & x+y+z &= 0 \\ y &= \frac{-15}{5} \rightarrow y = -3 & 1-3+z &= 0 \\ & & z &= -1+3 \\ & & z &= 2 \end{aligned} \\ & \text{R: D) } 2 \end{aligned}$$

R: D) 2

4.

$$\begin{aligned}
 &\textcircled{4} \begin{cases} A+B+C=68 \\ B+0,2C=A \\ 0,2A+C=3B \end{cases} \quad \begin{cases} B+0,2C=A \cdot 5 \\ 5B+C=5A \end{cases} \quad \begin{cases} 0,2A+C=3B \cdot 5 \\ A+5C=15B \end{cases} \\
 &C=68-A-B \quad A+5 \cdot (5A-5B)=15B \\
 &C=5A-5B \quad A+25A-25B=15B \\
 &5A-5B=68-A-B \quad 26A=40B \\
 &6A-4B=68 \cdot (-10) \quad 26A-40B=0 \quad \begin{matrix} -34A=-680 \\ A=-680 \\ -34 \end{matrix} \\
 &-60A+40B=-680 \quad \text{---} \quad -60A+40B=-680 \quad A=-680 \\
 &26A-40B=0 \quad C=68-A-B \quad A=20 \\
 &26 \cdot 20=40B \quad C=68-20-13 \\
 &B=\frac{520}{40} \Rightarrow B=13 \quad C=35 \\
 &\quad \quad \quad C-A \\
 &\quad \quad \quad 35-20 \\
 &\quad \quad \quad 15 \\
 &\text{R: A) R\$15,00 a menos que Caco.}
 \end{aligned}$$

R: A) R\$15,00 a menos que Caco.

5.

$$\begin{aligned}
 & \textcircled{5} \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} 0+3y+4z & | & 134 \\ x+0+5z & | & 115 \\ 2x+y+0 & | & 48 \end{bmatrix} \quad X=115-5Z \\
 & y=48-2x \quad 3.(-182+10z)+4z=134 \\
 & y=48-2(115-5z) \quad -546+30z+4z=134 \\
 & y=48-230+10z \quad 34z=134+546 \\
 & y=-182+10z \quad z=\frac{680}{34} \rightarrow z=20 \\
 & y=-182+10 \cdot 20 \quad x=115-5 \cdot 20 \\
 & y=-182+200 \quad x=115-100 \\
 & y=18 \quad x=15 \\
 & x+y+z \\
 & 15+18+20 \\
 & 53 \quad \boxed{R:A)53}
 \end{aligned}$$

R: A) 53