

Matrizes Determinantes:

1.

a)

Handwritten calculation for the determinant of a 2x2 matrix:

$$\textcircled{1} \text{ a) } \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \quad \begin{array}{l} 2 \cdot 5 = 10 \\ 1 \cdot 3 = 3 \end{array} \quad \begin{array}{l} 10 - 3 = 7 \\ \end{array} \quad \text{R: } 7$$

R: 7

b)

Handwritten calculation for the determinant of a 2x2 matrix:

$$\textcircled{2} \text{ b) } \begin{bmatrix} -2 & -4 \\ 3 & 6 \end{bmatrix} \quad \begin{array}{l} -2 \cdot 6 = -12 \\ 3 \cdot -4 = -12 \end{array} \quad \begin{array}{l} -12 - (-12) \\ -12 + 12 = 0 \end{array} \quad \text{R: } 0$$

R: 0

c)

$$\begin{array}{l}
 c) \begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 4 & -2 \end{bmatrix} \begin{matrix} 3-1 \\ 21 \\ 14 \end{matrix} \quad \begin{matrix} 3 \cdot 1 \cdot -2 = -6 \\ -1 \cdot -1 \cdot 1 = 1 \\ 1 \cdot 2 \cdot 4 = 8 \end{matrix} \\
 -6 + 1 + 8 - 1 + 12 - 4 \quad -1 \cdot 1 \cdot 1 = -1 \\
 3 + 7 \quad -4 \cdot -1 \cdot 3 = 12 \\
 10 \quad + 2 \cdot 2 \cdot -1 = -4 \\
 \boxed{R: 10}
 \end{array}$$

R: 10

d)

$$\begin{array}{l}
 d) \begin{bmatrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix} \begin{matrix} 32 \\ 23 \\ 11 \end{matrix} \quad \begin{matrix} 3 \cdot 3 \cdot 4 = 36 \\ 2 \cdot 1 \cdot 1 = 2 \\ -1 \cdot 2 \cdot 1 = -2 \end{matrix} \\
 36 + 2 - 2 + 3 - 3 - 16 \quad -1 \cdot 3 \cdot -1 = 3 \\
 36 - 16 \quad -1 \cdot 1 \cdot 3 = -3 \\
 20 \quad -4 \cdot 2 \cdot 2 = -16 \\
 \boxed{R: 20}
 \end{array}$$

R: 20

2.

② $A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

$$\begin{aligned} -3 \cdot 0 &= 0 & -3 \cdot -3 \cdot -3 &= -27 \\ 0 \cdot -3 &= 0 & 0 \cdot 0 \cdot 0 &= 0 \\ 0 \cdot 0 &= 0 & 0 \cdot 0 \cdot 0 &= 0 \end{aligned}$$

$R: A) -27$

$$\begin{aligned} -0 \cdot -3 \cdot 0 &= 0 \\ -0 \cdot 0 \cdot -3 &= 0 \\ +3 \cdot 0 \cdot 0 &= 0 \end{aligned}$$

R: A) -27

3.

③ $\begin{bmatrix} X & 1 & X \\ 3 & X & 4 \\ 1 & 3 & 3 \end{bmatrix} = -3$

$$\begin{aligned} X \cdot X \cdot 3 &= 3X^2 \\ 1 \cdot 4 \cdot 1 &= 4 \\ X \cdot 3 \cdot 3 &= 9X \end{aligned}$$

$$\begin{aligned} 3X^2 + 4 + 9X - X^2 - 12X - 9 &= -3 \\ 3X^2 - X^2 - 12X + 9X - 9 + 4 + 3 &= 0 \\ 2X^2 - 3X - 2 &= 0 \end{aligned}$$

$$\begin{aligned} -1 \cdot X \cdot X &= -X^2 \\ -3 \cdot 4 \cdot X &= -12X \\ -3 \cdot 3 \cdot 1 &= -9 \end{aligned}$$

$$\begin{aligned} \Delta &= (-3)^2 - 4 \cdot 2 \cdot (-2) \\ \Delta &= 9 + 16 \\ \Delta &= 25 \end{aligned}$$

$$\frac{-b \pm \sqrt{\Delta}}{2 \cdot a}$$

$$\frac{3 \pm 5}{4} \begin{cases} x' = \frac{8}{4} = 2 \\ x'' = \frac{-2}{4} = -\frac{1}{2} \text{ ou } -0,5 \end{cases}$$

$R: E) \{-1/2; 2\}$

R: E) $\{-1/2; 2\}$

4.

④

$$\begin{bmatrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & -1 & x+1 \end{bmatrix}$$

$$(x-1) \cdot (x+1) \cdot (x+1) = x^3 + x^2 - x - 1$$

$$-1 \cdot -1 \cdot 2 = 2$$

$$0 \cdot 0 \cdot -1 = 0$$

$$-2 \cdot (x+1) \cdot 0 = 0$$

$$1 \cdot -1 \cdot (x-1) = -x+1$$

$$(x+1) \cdot 0 \cdot -1 = 0$$

$$x^3 + x^2 - x - 1 + 2 - x + 1 = 2$$

$$x^3 + x^2 - 2x + 2 = 2$$

$$x^3 + x^2 - 2x = 0$$

$$x(x^2 + 2x - x - 2) = 0$$

$$x(x(x+2) - (x+2)) = 0$$

$$x(x+2) \cdot (x-1) = 0$$

$$x=0 \rightarrow x=0 \quad 0+(-2)+1$$

$$x+2=0 \rightarrow x=-2 \quad -2+1$$

$$x-1=0 \rightarrow x=1 \quad -1$$

$R: \{1, -1\}$

R: C) -1

5.

$$\textcircled{5} \quad A = \begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{bmatrix} \quad \det A \cdot B = \begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{bmatrix} \begin{matrix} 4 & -1 \\ 2 & 1 \\ 0 & 3 \end{matrix}$$

$$(4 \cdot 1 \cdot 6) + (-1 \cdot 0 \cdot 0) + (-6 \cdot 2 \cdot 3) + (-0 \cdot 1 \cdot -6) + (-3 \cdot 0 \cdot 4) -$$

$$24 + 0 - 36 + 0 + 0 + (-6 \cdot 2 \cdot -1)$$

$$-12 + 12$$

$$0$$

$$R: C) 0$$

R: C) 0

6.

$$\textcircled{6} \quad A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 & -4 \\ -2 & 2 \end{bmatrix} \quad \det A \cdot B = (2 \cdot 2) - (-2 \cdot -4)$$

$$4 - 8$$

$$-4$$

$$R: D) -4$$

R: D) -4