

**Matriz Inversa:**

1.

①  $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$   $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$   $A \cdot B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 3x+y & -x+2 \\ 3y+15 & 1 \end{bmatrix}$   $-x+2=0$   $3x+y=1$

$x=2$   $y=1-3 \cdot 2$

$y=-5$

$x+y$   
 $2+(-5)$   
 $-3$

$R: C) -3$

R: C) -3

2.

②  $\begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix}$   $1 \cdot 1 \cdot 3 = 3$   $-1 \cdot 1 \cdot 1 = -1$

$0 \cdot 3 \cdot 1 = 0$   $-k \cdot 3 \cdot 1 = -3k$

$1 \cdot k \cdot k = k^2$   $-3 \cdot k \cdot 0 = 0$

$3+k^2-3k-1$   $\Delta = (3)^2 - 4 \cdot 1 \cdot 2$   $\frac{3 \pm 1}{2}$

$k^2-3k+2$   $\Delta = 9-8$

$\Delta = 1$   $x' = \frac{4}{2} \rightarrow x' = 2$

$x'' = \frac{2}{2} \rightarrow x'' = 1$

$R: C) 1 \text{ e } 2$

R: C) 1 e 2

3.

$$\textcircled{3} \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \quad 3 \cdot 4 - 2 \cdot 5 = 2 \quad B = \frac{1}{2} \cdot \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \quad \det A = 2$$

$$B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix} \quad R: C) \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$$

R: C)

4.

$$\textcircled{4} \begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix} \quad \begin{array}{l} x \cdot 1 \cdot x = x^2 \\ 1 \cdot 2 \cdot 10 = 20 \\ 2 \cdot 3 \cdot 1 = 6 \end{array} \quad \begin{array}{l} -10 \cdot 1 \cdot 2 = -20 \\ -1 \cdot 2 \cdot x = -2x \\ -x \cdot 3 \cdot 1 = -3x \end{array}$$

$$x^2 + 20 + 6 - 20 - 2x - 3x$$

$$x^2 - 5x + 6$$

$$\Delta = (5)^2 - 4 \cdot 1 \cdot 6$$

$$\Delta = 25 - 24$$

$$\Delta = 1$$

$$\frac{5 \pm 1}{2} \rightarrow x' = \frac{6}{2} \rightarrow x' = 3$$

$$\Delta x'' = \frac{4}{2} \rightarrow x'' = 2$$

$$R: A) \{x \neq 3 \text{ e } x \neq 2\}$$

R: A)  $\{x \neq 3 \text{ e } x \neq 2\}$

5.

$$\textcircled{5} \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \quad \begin{array}{l} -1 \cdot 1 \cdot -1 = 1 \quad -1 \cdot 1 \cdot 2 = -2 \quad 7-6 \\ -1 \cdot 2 \cdot 1 = 2 \quad -1 \cdot -2 \cdot -1 = -2 \quad 1 \\ 2 \cdot 2 \cdot 1 = 4 \quad 1 \cdot 2 \cdot -1 = -2 \quad \det A = 1 \end{array}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \cdot \frac{1}{1} \quad A + A^{-1} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix} \quad \text{R: B) } \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

R: B)

6.

$$\textcircled{6} \quad \begin{array}{l} (XA)^{\dagger} = B^{\dagger} \cdot A \\ ((XA)^{\dagger})^{\dagger} = B^{\dagger} \\ XA = B^{\dagger} \cdot A^{-1} \\ XA \cdot A^{-1} = B^{\dagger} A^{-1} \\ 1X = B^{\dagger} \cdot A^{-1} \\ X = B^{\dagger} \cdot A^{-1} \end{array} \quad \text{R: B) } X = B^{\dagger} \cdot A^{-1}$$

R: B)  $X = B^{\dagger} \cdot A^{-1}$

7.

$$\textcircled{7} \quad B = \begin{bmatrix} x \\ y \end{bmatrix} \quad C = \begin{bmatrix} 4x+5y \\ 5x+6y \end{bmatrix} \quad AB=C \rightarrow A = \frac{C}{B}$$

$$A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \quad 4 \cdot 6 - 5 \cdot 5 = -1 \quad \det A = -1 \quad A^{-1} = \frac{1}{-1} \cdot \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix} \quad \textcircled{R:D) \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}}$$

R: D)

8.

$$\textcircled{8} \quad A = \begin{bmatrix} 2 & K \\ -2 & 1 \end{bmatrix} \quad 2 \cdot 1 - (-2) \cdot K = 2K+2 \quad \det A = 2K+2 \quad A^{-1} = \begin{bmatrix} 1-K & 2 \cdot 1 - 2 \cdot -K \\ 2 & 2 \end{bmatrix} \quad \det A^{-1} = 2K+2$$

$$2K+2=0 \rightarrow 2K=-2 \rightarrow K = \frac{-2}{2} \rightarrow K=-1$$

$$K+K = -1+(-1) \rightarrow K+K=-2$$

$$\textcircled{R:B)-2}$$

R: B) -2

9. a)

$$\textcircled{9} \text{ a) } (A+B) \cdot (A-B) \quad \textcircled{R: A^2 - A \cdot B + B \cdot A - B^2}$$

ou  
 $A^2 - B^2$

R:  $A^2 - AB + BA - B^2$  ou  $A^2 - B^2$

b)

Handwritten work for part b. It shows the expansion of  $(A+B)^2 = A^2 + 2AB + B^2$ . Below this, the expression  $A^2 + AB + BA + B^2$  is written and crossed out with a large 'X', followed by  $A^2 + 2AB + B^2$ . To the left, a cloud-shaped box contains the response:  $R: BA = AB$  and  $AB = BA$ .

R:  $AB = BA$  ou  $BA = AB$

c)

Handwritten work for part c. It shows the calculation  $\frac{\det(A)}{\det(-A)} = (-1)^2 = 1$ . To the right, a cloud-shaped box contains the response:  $R: 1$ .

R: 1

d)

Handwritten work for part d. It shows the derivation  $\det AB = 1 \rightarrow \det A \cdot \det B = 1$ , followed by  $\det B = \frac{1}{\det A}$ . To the right, a cloud-shaped box contains the response:  $R: \det B = \frac{1}{\det A}$ .

R:  $\det B = 1/\det A$