Velocity Vector:	$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$	
Acceleration Vector:	$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z}$	
Shear Stress:	$\tau_{w} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \Big _{y=0} = \mu \frac{\partial u}{\partial y} \Big _{y=0}$	
Density:	$\rho = \frac{\text{mass}}{\text{volume}} = \left(\frac{slug}{ft^3}, \frac{kg}{m^3}\right)$	
Specific Weight:	$\gamma = \rho g = \frac{\text{weight}}{\text{volume}} \qquad \left(\frac{lbf}{ft^3}, \frac{N}{m^3}\right)$	
Specific Gravity:	$SG = \frac{ ho_{liq}}{ ho_{water}},   ho_{liq} = SG \;  ho_{water}$	
Mass flow rate	$\dot{m} = \rho V A$	
Volumetric flow rate:	$\dot{V} = VA$	
Conservation of Mass:	$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$	
Incompressible Continuity Equation in Cartesian Coordinates:	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$	

Navier Stokes Equations in Cartesian Coordinates:

$$\rho g_{x} - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^{2} u}{\partial x^{2}} + \frac{\partial^{2} u}{\partial y^{2}} + \frac{\partial^{2} u}{\partial z^{2}} \right) = \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)$$

$$\rho g_{y} - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^{2} v}{\partial x^{2}} + \frac{\partial^{2} v}{\partial y^{2}} + \frac{\partial^{2} v}{\partial z^{2}} \right) = \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right)$$

$$\rho g_{z} - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^{2} w}{\partial x^{2}} + \frac{\partial^{2} w}{\partial y^{2}} + \frac{\partial^{2} w}{\partial z^{2}} \right) = \rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right)$$

## Method of Repeating Variables ( $\Pi$ Theorem):

## Six steps:

- 1. List the parameters in the problem and count their total number n.
- 2. List the primary dimensions of each of the n parameters. The number of primary dimensions appearing is j.
- 3. Calculate k, the expected number of  $\Pi$  groups, k = n i.
- 4. Choose *j* repeating parameters.
- 5. Construct the  $k \Pi$  groups and manipulate as necessary.
- 6. Write the final functional relationship and check algebra.

	Water	Mercury	Air
$\rho$ , kg/m <sup>3</sup>	998	13,550	1.20
ρg, N/m <sup>3</sup>	9790	132,900	11.77
μ, kg/(m·s)	1.00 E-3	1.56 E-3	1.8 E-5

$$g = 9.81 \text{ m/s}^2$$
  $P_{atm} = 101 \text{ kPa}$   $0^{\circ}\text{C} = 273 \text{ K}$