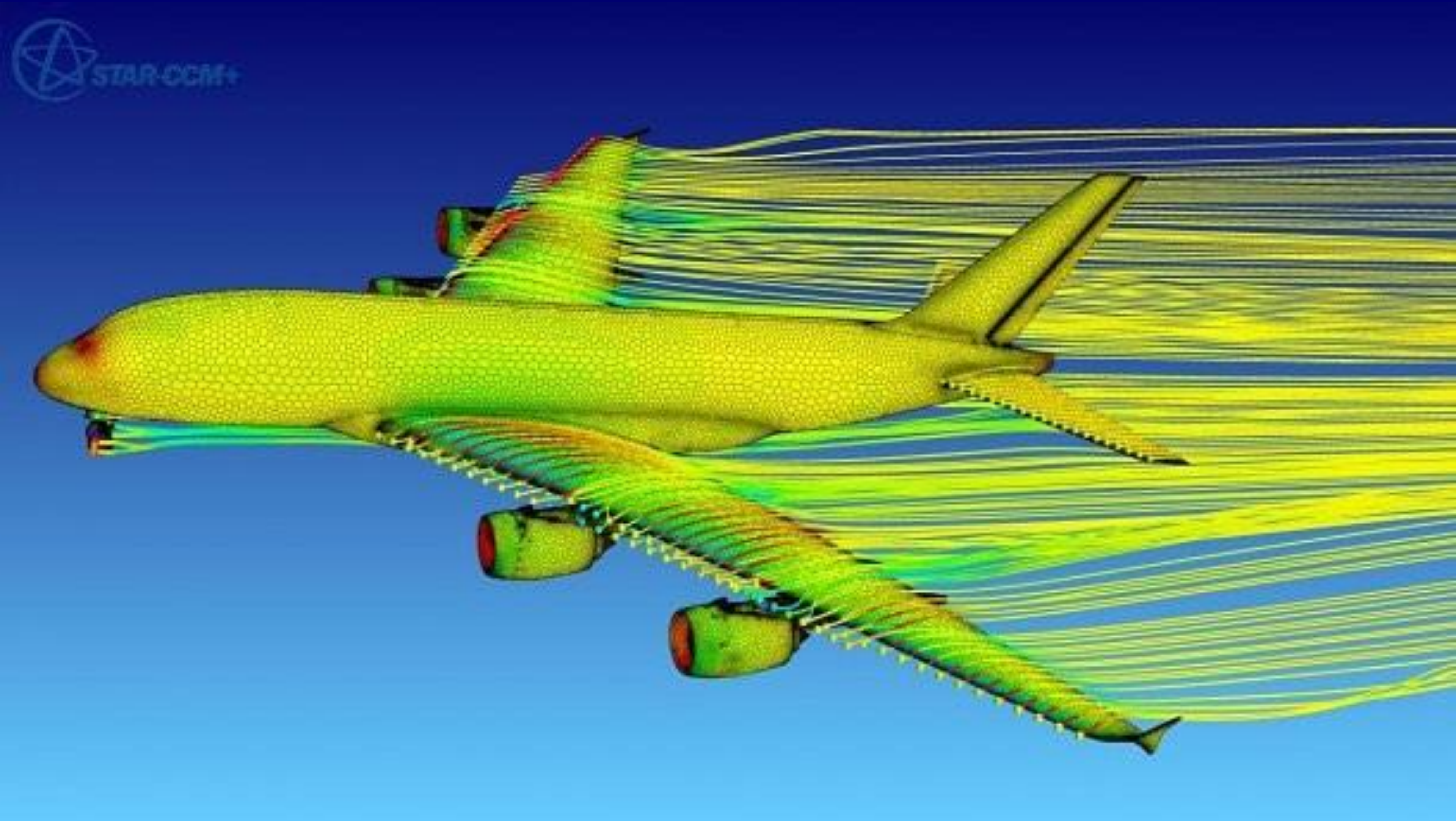
Lecture 21: Exam 2 Review



Exam 2

- **Exam Questions:**
  - **3-4 Problems:** Examples/Homework Problems
    - Only SI units ☺
  - **Short Conceptual Questions:** Lecture Slides
- **What to bring:**
  - Pen/Pencil
  - Eraser
  - Calculator
  - Formula Sheet is provided on Canvas and will be attached to the Exam
- **Date/Time/Location:**
  - Tuesday, November 12
  - 1:35-2:50 pm (75 Mins)
  - Gaige 121 (Our Regular Classroom)
- **Office Hours:**
  - Monday, 12:15-1:15pm, Gaige 245

Velocity and Acceleration

Velocity Vector:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

Substantial Derivative:

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

Shear Stress at wall (y-0):

$$\tau_w = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\Bigg|_{y=0} = \mu\frac{\partial u}{\partial y}\Bigg|_{y=0}$$

Conservation of Mass for a Differential CV

- Conservation of mass for a small CV:

$$\boxed{\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0}$$

- If the flow is incompressible:

(steady or unsteady)

$$\boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0}$$

In general, continuity equation cannot be used by itself to solve for flow field, however it can be used to:

1. Determine if velocity field is incompressible
2. Find missing velocity component

Navier Stokes Equations

Cartesian

X-Momentum Equation: $$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

Y-Momentum Equation: $$\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

Z-Momentum Equation: $$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

Steps to solve a problem:

1. Continuity & Navier Stokes Equations
2. Simplify the equations based on appropriate assumptions
3. Integrate the differential equation
4. Apply **Boundary Conditions** to find the final answer

Don’t forget to define the coordinate system first!

Appropriate Assumptions

- 1D, 2D, 3D Flow
- Steady State Flow
- Incompressible Flow
- Fully Developed Flow in One Direction
- Pressure Changes
- Gravity Effects

Boundary Conditions

Solid Surface: $\vec{V} = \vec{V}_{wall}$

→ No slip condition, No transpiration

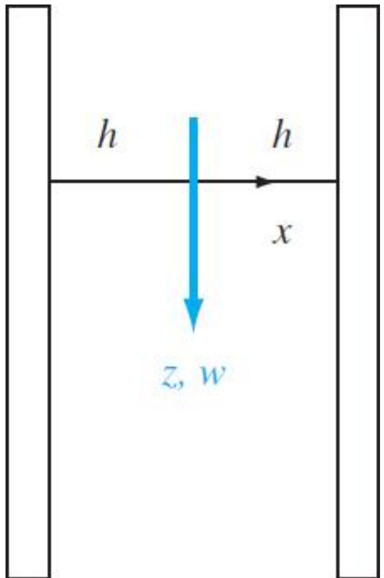
Inlet or Outlet: Known $\vec{V}$ and P

Free Surface: P = P_{atm} and $\tau_{surface} = 0$

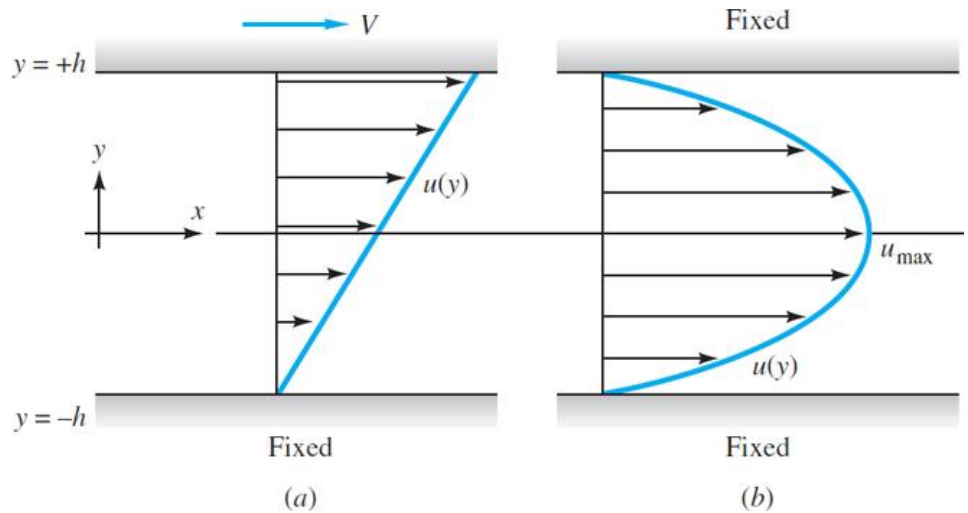
Fully developed flow in x direction: $\frac{\partial}{\partial x} = 0$

Fully developed flow in y direction: $\frac{\partial}{\partial y} = 0$

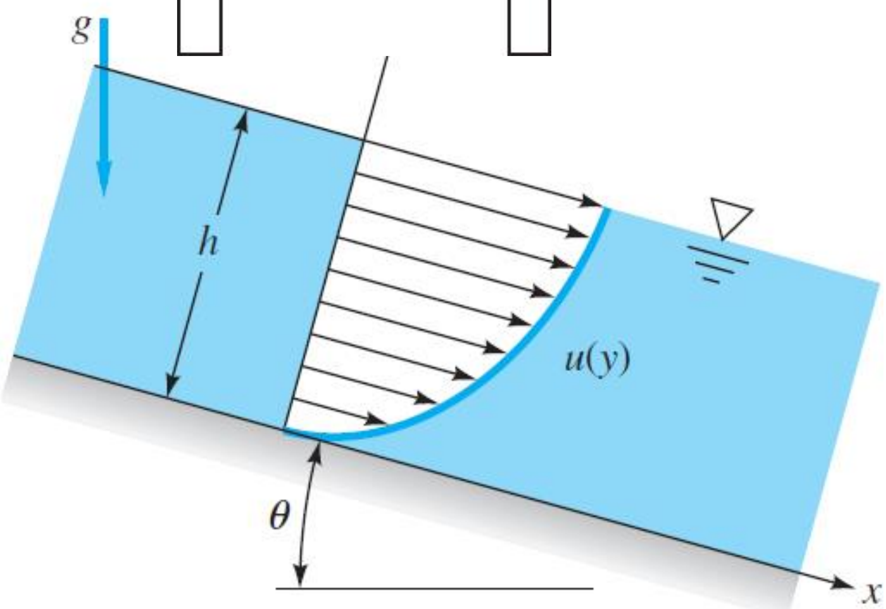
Examples



h
h
x
z, w



V
y = +h
y
x
u(y)
y = −h
Fixed
(a)
Fixed
u(y)
u_{max}
Fixed
(b)



g
h
u(y)
θ
x

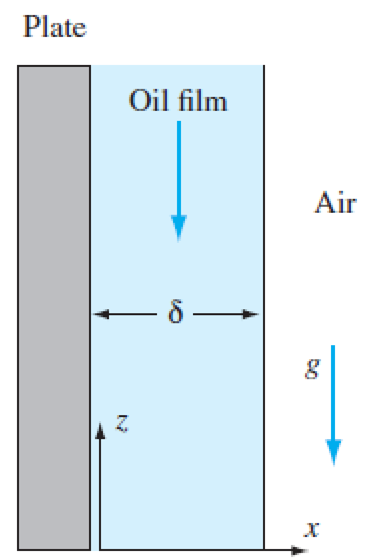


Plate
Oil film
Air
δ
g
z
x

Dimensional Analysis
Experimental Testing

Wanapum Dam on Columbia River

Physical Model at
Iowa Institute of Hydraulic Research (IIHR)

DDG-51 Destroyer

1/20th scale model

Dimensional Analysis

A method for reducing the number and complexity of experimental variables that affect a given physical phenomenon

n dimensional variables → k non-dimensional variables

Dimensions and Units

Dimension: Measure of a physical quantity,

e.g.: **length**, **time**, **mass**

Units: Assignment of a number to a dimension, e.g., (m), (sec), (kg)

7 Primary Dimensions:

1.	Mass	m	(kg)
2.	Length	L	(m)
3.	Time	t	(sec)
4.	Temperature	T or θ	(K)
5.	Current	I	(A)
6.	Amount of Light	C	(cd)
7.	Amount of matter	N	(mol)

All non-primary dimensions can be formed by a combination of the 7 primary dimensions

Examples

- {Velocity} = {Length/Time} = {L/t}
- {Force} = {Mass Length/Time^2} = {mL/t^2}

Dimensions of Fluid Mechanics Properties

Quantity	Symbol	Dimensions $MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\varepsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

Method of Repeating Variables

- For most problems, there will be more than one dimensionless group. Need a systematic method to find groups.
- We will use the **Method of Repeating Variables (**Π Theorem)

Six steps:

1. List the parameters in the problem and count their total number ***n***.
2. List the primary dimensions of each of the *n* parameters. The number of primary dimensions appearing is ***j***.
3. Calculate ***k***, the expected number of Π groups, $k = n - j$. (This is called the Buckingham Pi Theorem.)
4. Choose *j* ***repeating parameters.***
5. Construct the *k* Π groups and manipulate as necessary.
6. Write the final functional relationship and check algebra.

Dimensional Analysis and Similarity

- **Geometric Similarity** - the model must be the same shape as the prototype. Each dimension must be scaled by the same factor.
- **Kinematic Similarity** - velocity at any point in the model and prototype must be proportional by the same factor
- **Dynamic Similarity** - *all forces* in the model flow scale by a constant factor to corresponding forces in the prototype flow.
- **Complete Similarity** is achieved only if all 3 conditions are met. This is not always possible, e.g., river hydraulics models.

Modeling and Similarity

Flow conditions for a model test are completely similar if all relevant dimensionless parameters have the same corresponding values for the model and the prototype.

$$\text{If } \mathrm{Re}_m = \mathrm{Re}_p \quad \text{then} \quad C_{Fm} = C_{Fp}$$

Table 5.2 Dimensionless Groups in Fluid Mechanics

Parameter	Definition	Qualitative ratio of effects	Importance
Reynolds number	$\mathrm{Re} = \dfrac{\rho U L}{\mu}$	$\dfrac{\text{Inertia}}{\text{Viscosity}}$	Almost always
Mach number	$\mathrm{Ma} = \dfrac{U}{a}$	$\dfrac{\text{Flow speed}}{\text{Sound speed}}$	Compressible flow
Froude number	$\mathrm{Fr} = \dfrac{U^2}{gL}$	$\dfrac{\text{Inertia}}{\text{Gravity}}$	Free-surface flow
Weber number	$\mathrm{We} = \dfrac{\rho U^2 L}{\Upsilon}$	$\dfrac{\text{Inertia}}{\text{Surface tension}}$	Free-surface flow
Rossby number	$\mathrm{Ro} = \dfrac{U}{\Omega_{\text{earth}} L}$	$\dfrac{\text{Flow velocity}}{\text{Coriolis effect}}$	Geophysical flows
Cavitation number (Euler number)	$\mathrm{Ca} = \dfrac{p - p_v}{\frac{1}{2}\rho U^2}$	$\dfrac{\text{Pressure}}{\text{Inertia}}$	Cavitation
Prandtl number	$\mathrm{Pr} = \dfrac{\mu c_p}{k}$	$\dfrac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$\mathrm{Ec} = \dfrac{U^2}{c_p T_0}$	$\dfrac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific-heat ratio	$k = \dfrac{c_p}{c_v}$	$\dfrac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow
Strouhal number	$\mathrm{St} = \dfrac{\omega L}{U}$	$\dfrac{\text{Oscillation}}{\text{Mean speed}}$	Oscillating flow

Table 5.2 Dimensionless Groups in Fluid Mechanics

Parameter	Definition	Qualitative ratio of effects	Importance
Roughness ratio	$\frac{\varepsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent, rough walls
Grashof number	$\mathrm{Gr} = \frac{\beta \Delta T g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Rayleigh number	$\mathrm{Ra} = \frac{\beta \Delta T g L^3 \rho^2 c_p}{\mu k}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Temperature ratio	$\frac{T_w}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Friction factor	$f = \frac{h_f}{(V^2/2g)(L/d)}$	$\frac{\text{Friction head loss}}{\text{Velocity head}}$	Pipe flow
Skin friction coefficient	$c_f = \frac{\tau_{wall}}{\rho V^2/2}$	$\frac{\text{Wall shear stress}}{\text{Dynamic pressure}}$	Boundary layer flow

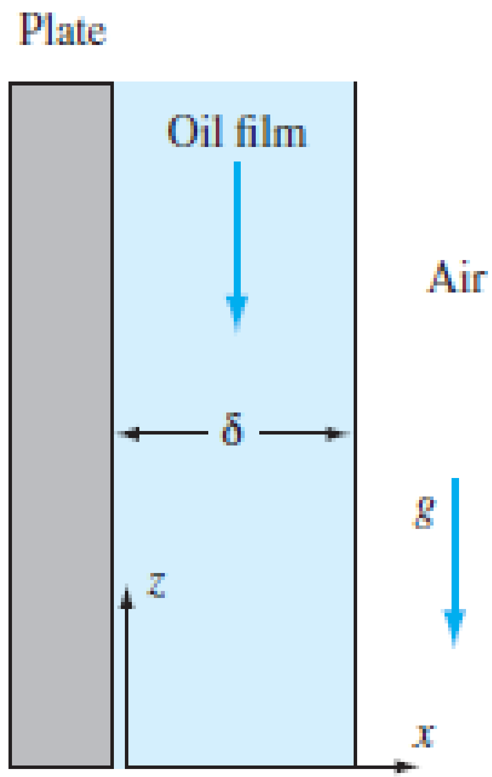
Review Problems

P4.29: Differential Analysis

Consider a steady, two-dimensional, incompressible flow of a newtonian fluid in which the velocity field is known: $u = -2xy$, $v = y^2 - x^2$, $w = 0$. (a) Does this flow satisfy conservation of mass? (b) Find the pressure field, $p(x, y)$ if the pressure at the point ($x = 0$, $y = 0$) is equal to p_a.

P4.80: Differential Analysis

Oil, of density ρ and viscosity μ, drains steadily down the side of a vertical plate, as in Fig. P4.80. After a development region near the top of the plate, the oil film speed will become independent of z and of constant thickness δ. Assume that w = w(x) only and that the atmosphere offers no shear resistance to the surface of the film. (a) Solve the Navier-Stokes equation for w(x), and sketch its approximate shape. (b) Suppose that film thickness δ and the slope of the velocity profile at the wall [∂w/∂x]wall are measured with a laser-Doppler anemometer (Chap. 6). Find an expression for oil viscosity μ as a function of (ρ, δ, g, [∂w/∂x]wall).



P 5-22

As will be discussed in Turbomachinery, the power *P* developed by a wind turbine is a function of diameter *D,* air density *ρ,* wind speed *V,* and rotation rate *ω.* Viscosity effects are negligible. Rewrite this relationship in dimensionless form.

Assume *ρ, V, and D are the repeating variable.*

P 5-62

For the previous example, assume that a small model wind turbine of diameter 90 cm, rotating at 1200 r/min, delivers 280 watts when subjected to a wind of 12 m/s. The data is to be used for a prototype of diameter 50 m and winds of 8 m/s. For dynamic similarity, estimate (*a*) the rotation rate, and (*b*) the power delivered by the prototype. Assume sea-level air density.