Homework 4

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Page 77-78 #1, 3, 6, 7, 9, 12

Page 88-89 #3, 6, 8, 10

Page 96 #1, 4, 5, 10

Page 77 Problem 1

> 1. A person is selected at random from the population of Verizon wireless subscribers. Let A be the event that the chosen subscriber has friends or family added to his/her plan, and B denote the event that the subscriber has unlimited text messaging. Extensive records suggest that P(A) = 0.37, P(B) = 0.23, and P(A ∪ B) = 0.47. Find P(A ∩ B).

Events:

- A: chosen subscriber has friends or family added to his/her plan
- B: subscriber has unlimited text messaging

Given:

$P(A) = 0.37$

$P(B) = 0.23$

$P(A \cup B) = 0.47$

Find:

$P(A \cap B)$

Assume: $P(A)$ is independent of $P(B)$

$$P(A \cap B) = P(A)P(B) = (0.37)(0.23) = 0.0851$$

Check Assumption?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.37 + 0.23 - 0.47$$

$$P(A \cap B) = 0.13$$

$0.13 \neq 0.0851 \Rightarrow P(A)$ is not independent of $P(B)$

$$\boxed{P(A \cap B) = 0.13}$$

Page 78 Problem 3

3. A new generation of hybrid vehicles achieves high- way gas mileage in the range of 50 to 53 MPG. The gas mileage of three such cars, during a pre-specified 100-mile drive, will be rounded to the nearest integer, resulting in the sample space S = {(x 1 , x 2 , x 3) : x i = 50, 51, 52, 53}, i = 1, 2, 3.

(a) Assume that the outcomes of the sample space S are equally likely, and use R commands similar to those used in Example 2.4-1 to find the probability mass function of the experiment that records the average mileage the three cars achieve.

```
x=c(50,51,52,53)
Sx <- expand.grid(X1=x, X2=x, X3=x)
attach(Sx)
pmf.s <- table((X1+X2+X3)/3)/dim(Sx)[1]
pmf.s
```

```
##
##               50 50.3333333333333 50.6666666666667               51
##         0.015625         0.046875         0.093750         0.156250
## 51.3333333333333 51.6666666666667               52 52.3333333333333
##         0.187500         0.187500         0.156250         0.093750
## 52.6666666666667               53
##         0.046875         0.015625
```

(b) Use the PMF obtained in part (a) to compute the probability that the average gas mileage is at least 52 MPG.

```
cat(paste0('The probability that the average gas milage is at least 52 MPG is: ',pmf.s["52"]))
```

```
## The probability that the average gas milage is at least 52 MPG is: 0.15625
```

Page 78 Problem 6

6. Each of the machines A and B in an electronics fabrication plant produces a single batch of 50 electrical components per hour. Let E 1 denote the event that, in any given hour, machine A produces a batch with no defective components, and E 2 denote the corresponding event for machine B. The probabilities of E 1 , E 2 , and E 1 ∩ E 2 are 0.95, 0.92, and 0.88, respectively. Express each of the following events as set operations on E 1 and E 2 , and find their probabilities.
(a) In any given hour, only machine A produces a batch with no defects.
(b) In any given hour, only machine B produces a batch with no defects.
(c) In any given hour, exactly one machine produces a batch with no defects.
(d) In any given hour, at least one machine produces a batch with no defects.

Events :

E_1 : in any given hour, machine A produces a batch with no defective parts.

E_2 : corresponding event for machine B

Given :

$P(E_1) = 0.95$

$P(E_2) = 0.92$

$P(E_1 \cap E_2) = 0.88$

Find:

a) In any given hour, only machine A produces a batch with no defects

$$E_3 = \{\bar{E}_2 \cup E_1\}$$

$$P(E_3) = P(E_1) - P(E_1 \cap E_2) = 0.95 - 0.88 = 0.07$$

b) In any given hour, only machine B produces a batch with no defects

$$E_4 = \{\bar{E}_1 \cup E_2\}$$

$$P(E_4) = P(E_2) - P(E_1 \cap E_2) = 0.92 - 0.88 = 0.04$$

c) In any given hour exactly one machine produces a batch with no defects

$$E_5 = \{(E_1 \cup \bar{E}_2) \cup (\bar{E}_1 \cup E_2)\} = \{E_3 \cup E_4\}$$

$$P(E_5) = P(E_3 \cup E_4) = P(E_3) + P(E_4) = P(E_1) + P(E_2) - 2P(E_1 \cap E_2)$$

$$= 0.07 + 0.04 = 0.95 + 0.92 - 2(0.88)$$

c) $\boxed{P(E_5) = 0.11}$

d) In any given hour at least one machine produces a batch with no defects

$$E_6 = \{E_1 \cup E_2\}$$

Page 78 Problem 7

> 7. The electronics fabrication plant in Exercise 6 has a third machine, machine C, which is used in periods of peak demand and is also capable of producing a batch of 50 electrical components per hour. Let E 1 , E 2 be as in Exercise 6, and E 3 be the corresponding event for machine C. The probabilities of E 3 , E 1 ∩ E 3 , E 2 ∩ E 3 , and E 1 ∩ E 2 ∩ E 3 are 0.9, 0.87, 0.85, and 0.82, respectively. Find the probability that at least one of the machines will produce a batch with no defectives.

Events:

E_1: in any given hour, machine A produces a batch with no defective parts.

E_2: corresponding event for machine B

E_3: corresponding event for machine C

Given:

$P(E_3) = 0.9$

$P(E_1 \cap E_3) = 0.87$

$P(E_2 \cap E_3) = 0.85$

$P(E_1 \cap E_2 \cap E_3) = 0.82$

From exercise 6

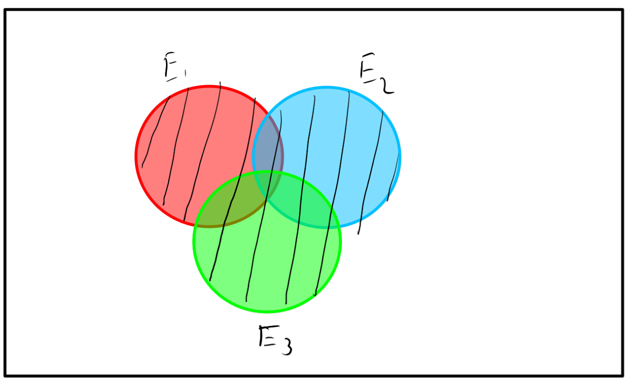
$P(E_1) = 0.95$

$P(E_2) = 0.92$

$P(E_1 \cap E_2) = 0.88$

Find:

Probability that at least one of the machines will produce a batch with no defectives.



E_4: represents the event of at least one machine produces a batch with no defectives

$E_4 = \{E_1 \cup E_2 \cup E_3\}$

$$P(E_4) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

$P(E_3) = 0.9$

$P(E_1 \cap E_3) = 0.87$

$P(E_2 \cap E_3) = 0.85$

$P(E_1 \cap E_2 \cap E_3) = 0.82$

$P(E_1) = 0.95$

$P(E_2) = 0.92$

$P(E_1 \cap E_2) = 0.88$

$P(E_4) = 0.95 + 0.92 + 0.9 - 0.88 - 0.85 - 0.87 + 0.82$

$$\boxed{P(E_4) = 0.99}$$

Pages 78-79 Problem 9

9. A type of communications system works if at least half of its components work. Suppose it is possible to add a fifth component to such a system having four components. Show that the resulting five-component system is not necessarily more reliable. (Hint. In the notation of Example 2.4-2, it suffices to show that E 4 ⊂ / E 5 . Expressing E 5 as the union of {at least three of the original four components work} and {two of the original four components work and the additional component works}, it can be seen that the event {two of the original four components work and the additional component does not work} is contained in E 4 but not in E 5 .)

Events:

E_4: {at least 2 of the 4 original components work}

E_5: {at least 3 of the original 4 components work} ∪ {2 of original 4 work & the 5th works}

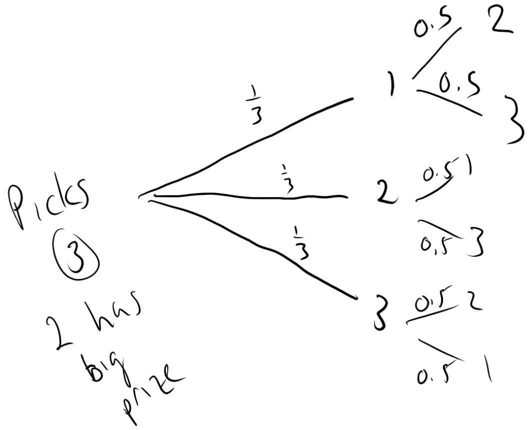
4 component system needs 2 of 4 to function if 3 fail it does not function

5 component system needs 3 of 5 to function if 3 fail it does not function

Clearly both systems will not function if 3 components fail which shows the the 5 component system is not necessarily more reliable.

Page 79 Problem 12

12. Let's make a deal. In the game Let's Make a Deal, the host asks a participant to choose one of three doors. Behind one of the doors is a big prize (e.g., a car), while behind the other two doors are minor prizes (e.g., a blender). After the participant selects a door, the host opens one of the other two doors (knowing it is not the one having the big prize). The host does not show the participant what is behind the door the participant chose. The host asks the participant to either
(a) stick with his/her original choice, or
(b) select the other of the remaining two closed doors.
Find the probability that the participant will win the big prize for each of the strategies (a) and (b).



$$P(a) = \left(\frac{1}{3}\right)(0.5) = \frac{1}{6}$$

$$P(b) = \left(\frac{1}{3}\right)(0.5) = \frac{1}{6}$$

$$P(a) = P(b) = \frac{1}{6}$$

Page 88 Problem 3

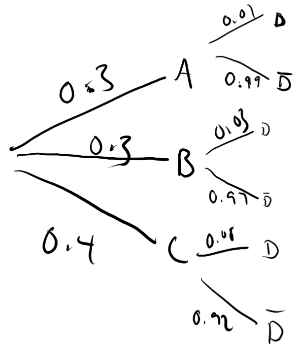
3. The moisture content of batches of a chemical sub- stance is measured on a scale from 1 to 3, while the impurity level is recorded as either low (1) or high (2). Let X and Y denote the moisture content and the impu- rity level, respectively, of a randomly selected batch. The probabilities for each of the six possible outcomes of the experiment that records X and Y for a randomly selected batch are given in the following table.

	Impurity Level	
Moisture	1	2
1	0.132	0.068
2	0.24	0.06
3	0.33	0.17

Let A and B be the events that X = 1 and Y = 1, respectively. (a) Find the probability of A. (b) Find the conditional probability of B given A. (c) Find the probability mass function of the random variable X.

Page 88 Problem 6

6. A particular consumer product is being assembled on production lines A, B, and C, and packaged in batches of 10. Each day, the quality control team selects a production line by probability sampling with probabilities P(A) = P(B) = 0.3 and P(C) = 0.4 and inspects a randomly drawn batch from the selected production line. The probability that no defects are found in a batch selected from production line A is 0.99, and the corresponding probabilities for production lines B and C are 0.97 and 0.92, respectively. A tree diagram may be used for answering the following questions.
(a) What is the probability that a batch from production line A is inspected and no defects are found?
(b) Answer the above question for production lines B and C.
(c) What is the probability that no defects are found in any given day?
(d) Given that no defects were found in a given day, what is the probability the inspected batch came from production line C?



Intersection Rule a) $P(\bar{D}\cap A) = P(A) \times P(\bar{D}|A) = (0.3)(0.99) = \boxed{0.33}$

b) $P(\bar{D}\cap B) = P(B) \times P(\bar{D}|B) = (0.3)(0.97) = \boxed{0.291}$

$P(\bar{D}\cap C) = P(C) \times P(\bar{D}|C) = (0.4)(0.92) = \boxed{0.368}$

Law of total probability c) $P(\bar{D}) = P(\bar{D}\cap A) + P(\bar{D}\cap B) + P(\bar{D}|C) = \boxed{0.989}$

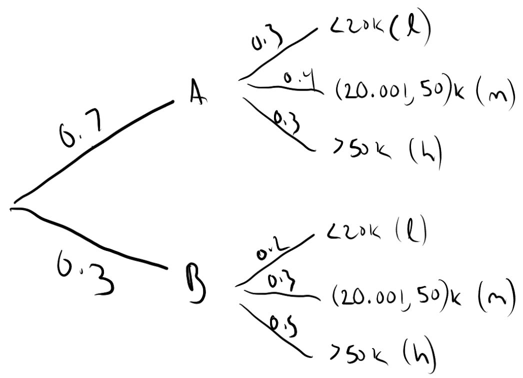
Bayes Theorem d) $P(C|\bar{D}) = \dfrac{P(C\cap\bar{D})}{P(\bar{D})} = \dfrac{0.368}{0.989} = \boxed{0.372}$

Page 89 Problem 8

8. Thirty percent of credit card holders carry no monthly balance, while 70% do. Of those card holders carrying a balance, 30% have annual income \$20,000 or less, 40% between \$20,001 and \$50,000, and 30% over \$50,000. Of those card holders carrying no balance, 20%, 30%, and 50% have annual incomes in these three respective categories.
(a) What is the probability that a randomly chosen card holder has annual income \$20,000 or less?
(b) If this card holder has an annual income that is \$20,000 or less, what is the probability that (s)he carries a balance?

$A = \{\text{have monthly balance}\}$; $B = \{\text{No monthly balance}\}$

$P(A) = 0.7$ $\quad$ $P(B) = 0.3$



a) $l = \{\text{Annual Income} < 20k\}$

$$P(l) = P(l \cap A) + P(l \cap B) = P(A) \times P(l|A) + P(B) \times P(l|B)$$
$$= (0.7)(0.3) + (0.3)(0.2) = \boxed{0.27}$$

b) $$P(A|l) = \frac{P(A \cap l)}{P(\ )} = \frac{(0.3)(0.7)}{0.27} = \boxed{0.78}$$

Page 89 Problem 10

10. A batch of 10 fuses contains three defective ones. A sample of size two is taken at random and without replacement.

(a) Find the probability that the sample contains no defective fuses.

```
x = c(rep(1,3),rep(0,7)) # 1 is for defective, 0 for good fuses in a batch
Sx = expand.grid(x1=x,x2=x)
attach(Sx)
pmf.s <- table(x1+x2)/dim(Sx)[1]
pmf.s
```

```
##
##    0    1    2
## 0.49 0.42 0.09
```

```
cat(paste0('The probability that the sample contains no defective fuses is: ',pmf.s['0']))
```

```
## The probability that the sample contains no defective fuses is: 0.49
```

(b) Let X be the random variable denoting the number of defective fuses in the sample. Find the probability mass function of X.

```
X = pmf.s
X
```

```
##
##    0    1    2
## 0.49 0.42 0.09
```

(c) Given that X = 1, what is the probability that the defective fuse was the first one selected?

```
pmf.Xc = table(x1==1,x2==0)/(dim(Sx)[1])
pmf.Xc
```

```
##
##          FALSE TRUE
##   FALSE  0.21 0.49
##   TRUE   0.09 0.21
```

```
cat('Given that X = 1, the probability that the defective fuse was the first one selected is 21%')
```

```
## Given that X = 1, the probability that the defective fuse was the first one selected is 21%
```

Page 96 Problem 1

1. In a batch of 10 laser diodes, two have efficiency below 0.28, six have efficiency between 0.28 and 0.35, and two have efficiency above 0.35. Two diodes are selected at random and without replacement. Are the events E 1 = {the first diode selected has efficiency below 0.28} and E 2 = {the second diode selected has efficiency above 0.35} independent? Justify your answer.

```
x = c(rep('l',2),rep('m',6),rep('h',2))
Sx = expand.grid(x1=x,x2=x)
attach(Sx)
```

```
## The following objects are masked from Sx (pos = 3):
##
##     x1, x2
```

```r
pmf.x = table(x1=='l',x2=='h')/dim(Sx)[1]
pmf.x
```

```
##
##         FALSE TRUE
##   FALSE  0.64 0.16
##   TRUE   0.16 0.04
```

The events E1 and E2 are independent because E1 can happen or not happen for the first diode selected and the second diode can be any of the three levels. There are enough of each level diode in any given batch to make them independent for a sample of size 2.

Page 96 Problem 4

4. An experiment consists of inspecting fuses as they come off a production line until the first defective fuse is found. Assume that each fuse is defective with a probability of 0.01, independently of other fuses. Find the probability that a total of eight fuses are inspected.

$d = \{$defective fuse is inspected$\}$

$\bar{d} = \{$fuse inspective is not defective$\}$

$P(d) = 0.01$

$P(\bar{d}) = 1 - P(d) = 0.99$

$E_1 = \{$7 good fuses inspected 8th fuse is defective$\}$

$P(E_1) = (0.99)^7(0.01) = \boxed{0.0093}$

Page 96 Problem 5

5. Quality control engineers monitor the number of nonconformances per car in an automobile production facility. Each day, a simple random sample of four cars from the first assembly line and a simple random sample of three cars from the second assembly line are inspected. The probability that an automobile produced in the first shift has zero nonconformances is 0.8. The corresponding probability for the second shift is 0.9. Find the probability of the events (a) zero nonconformances are found in the cars from the first assembly line in any given day, (b) the corresponding event for the second assembly line, and (c) zero nonconformances are found in any given day. State any assumptions you use.

$d = \{\text{defect}\}$

$\bar{d} = \{\text{not defect}\}$

Given:

$A = \{\text{auto produced in } 1^{st} \text{ shift has zero defect}\}$

$B = \{\text{auto produced in } 2^{nd} \text{ shift has zero defect}\}$

$P(A) = 0.8 \quad ; \quad P(\text{line1}) = \frac{4}{7}$

$P(B) = 0.9 \quad ; \quad P(\text{line 2}) = \frac{3}{7}$

Find:

a) $P(A \cap B \cap \text{line1}) = (0.8)(0.9)(\frac{4}{7}) = \boxed{0.41}$

b) $P(A \cap B \cap \text{line2}) = (0.8)(0.9)(\frac{3}{7}) = \boxed{0.31}$

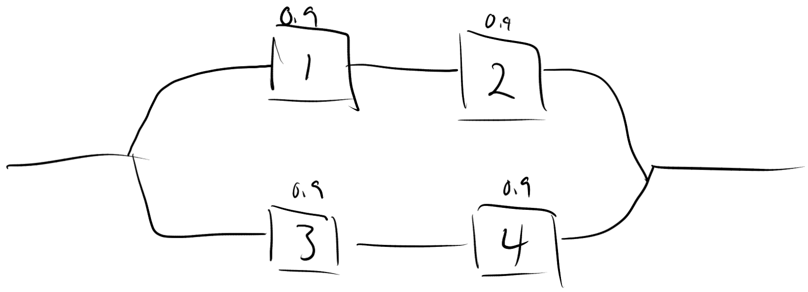
c) Assume any line & shift

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A)P(B)$$
$$= 0.8 + 0.9 - (0.8)(0.9)$$
$$= 0.98 \quad \text{Seems High}$$

$$P(A \cap B) = P(A)P(B) = \boxed{0.72}$$

Page 97 Problem 10

10. The system of components shown in Figure 2-15 below functions as long as components 1 and 2 both function or components 3 and 4 both function. Each of the four components functions with probability 0.9 independently of the others. Find the probability that the system functions.



$$P(\text{system functions}) = P(1)P(2) = (0.9)^2 = 0.81$$
$$= P(3)(P4) = \boxed{0.81}$$