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function Question4(~)

    Alpha = sqrt(2)/2;
    A = zeros(13, 13);

    A(1,2) = 1;
    A(1,6) = -1;
    A(2,3) = 1;
    A(3,1) = Alpha;
    A(3,4) = -1;
    A(3,5) = -Alpha;
    A(4,1) = Alpha;
    A(4,3) = 1;
    A(4,5) = Alpha;
    A(5,4) = 1;
    A(5,8) = -1;
    A(6,7) = 1;
    A(7,5) = Alpha;
    A(7,6) = 1;
    A(7,9) = -Alpha;
    A(7,10) = -1;
    A(8,5) = Alpha;
    A(8,7) = 1;
    A(8,9) = Alpha;
    A(9,10) = 1;
    A(9,13) = -1;
    A(10,11) = 1;
    A(11,8) = 1;
    A(11,9) = Alpha;
    A(11,12) = -Alpha;
    A(12,9) = Alpha;
    A(12,11) = 1;
    A(12,12) = Alpha;
    A(13,13) = 1;
    A(13,12) = Alpha;

    b = zeros(13,1);
    b(2) = 10;
    b(8) = 15;
    b(10) = 20;

    f = A\b %#ok<NOPRT>

    condA = cond(A, 2);

    r = b - A*f;
    r_residue = norm(r,2) / norm(b,2);
    r_error_lower = (1 / condA) *r_residue;
    r_error_upper = condA * r_residue;

    fprintf('Lower bound of relative error is %e.\n', r_error_lower);
    fprintf('Upper bound of relative error is %e.\n', r_error_upper);

end

>> Question4

f =

  -28.2843
   20.0000
   10.0000
  -30.0000
   14.1421
   20.0000
         0
  -30.0000
    7.0711
   25.0000
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   20.0000
  -35.3553
   25.0000

Lower bound of relative error is 1.925969e-17.
Upper bound of relative error is 2.088024e-15.



function Question5(~)

    n = 1;
    relative_error = 0;
    fprintf('Part1: \n');
    fprintf('\tn\trelative_error\n');
    warning('off');

    while (relative_error < 1)
            n = n+1;
        [x_hat, relative_error] = hat_error(n); %relative error and absolute error are the same.
            fprintf('\t%d\t%e\n', n, relative_error);
    end

    fprintf('\nPart2: \n');
    fprintf('\tn\tcond(H)\t\tlog(cond(H))\n');

    n_2 = (2:13)';
    cond_H = zeros(12,1);
    log_cond_H = zeros(12,1);
    correct = zeros(12,1);

    for i = 2:13
        cond_H(i-1) = cond(hilb(i), Inf);
        log_cond_H(i-1) = log(cond_H(i-1));
        fprintf('\t%d\t%e\t%f\n', i, cond_H(i-1), log_cond_H(i-1));
    end

    plot(n_2, log_cond_H);

    slope_intersect = [ones(12,1), n_2] \ log_cond_H;

    fprintf('By Linear Regression we can write log(cond(H)) = %fn%f\n', slope_intersect(2),
slope_intersect(1));

    fprintf('\nPart3: \n');
    fprintf('\tn\tnorm(x_hat)\tcorrect digits\tcond(H)\n');

    for i = 2:13
        [x_hat, relative_error] = hat_error(i);
        x_hat_norm = norm(x_hat, Inf);
        correct(i-1) = 0-floor(log10(relative_error));
        fprintf('\t%d\t%e\t%d\t\t%e\n', i, x_hat_norm, correct(i-1), cond_H(i-1));
    end
end

function [x_hat,err] = hat_error(n)
    H = hilb(n);
    x = ones(n, 1);
    b = H*x;
    x_hat = H\b;
            err = norm(x_hat-x, Inf);
end


(Below are the results of the script running on CDF)
>> Question5
Part1:
            n           relative_error
            2           7.771561e-16
            3           7.438494e-15
            4           4.130030e-13
            5           1.130429e-12
            6           5.025884e-10
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            7           1.367104e-08
            8           6.551379e-07
            9           6.465747e-06
            10          6.634040e-04
            11          7.171209e-03
            12          2.687440e-01
            13          3.079042e+00

Part2:
            n           cond(H)                 log(cond(H))
            2           2.700000e+01            3.295837
            3           7.480000e+02            6.617403
            4           2.837500e+04            10.253264
            5           9.436560e+05            13.757517
            6           2.907028e+07            17.185227
            7           9.851949e+08            20.708350
            8           3.387279e+10            24.245878
            9           1.099649e+12            27.726013
            10          3.535233e+13            31.196386
            11          1.229477e+15            34.745365
            12          3.841961e+16            38.187344
            13          7.490658e+17            41.157603
By Linear Regression we can write log(cond(H)) = 3.476950n-3.654113

Part3:
            n           norm(x_hat)             correct digits          cond(H)
            2           1.000000e+00            16                      2.700000e+01
            3           1.000000e+00            15                      7.480000e+02
            4           1.000000e+00            13                      2.837500e+04
            5           1.000000e+00            12                      9.436560e+05
            6           1.000000e+00            10                      2.907028e+07
            7           1.000000e+00            8                       9.851949e+08
            8           1.000000e+00            7                       3.387279e+10
            9           1.000006e+00            6                       1.099649e+12
            10          1.000663e+00            4                       3.535233e+13
            11          1.007171e+00            3                       1.229477e+15
            12          1.227167e+00            1                       3.841961e+16
            13          4.079042e+00            0                       7.490658e+17

(Below are the results of the script running on MatLab online, for larger n, the results are
quite different!)
>> Question5
Part1:
            n           relative_error
            2           7.771561e-16
            3           7.438494e-15
            4           4.678480e-13
            5           3.441691e-13
            6           3.988309e-10
            7           1.539426e-08
            8           5.777080e-07
            9           2.109568e-05
            10          2.773788e-04
            11          2.129638e-02
            12          1.143963e-01
            13          9.452850e+00

Part2:
            n           cond(H)                 log(cond(H))
            2           2.700000e+01            3.295837
            3           7.480000e+02            6.617403
            4           2.837500e+04            10.253264
            5           9.436560e+05            13.757517
            6           2.907028e+07            17.185227
            7           9.851949e+08            20.708350
            8           3.387279e+10            24.245878
            9           1.099649e+12            27.726013
            10          3.535233e+13            31.196386
            11          1.229477e+15            34.745365
            12          3.841961e+16            38.187344
            13          7.490658e+17            41.157603
By Linear Regression we can write log(cond(H)) = 3.476950n-3.654113
```

Part3:

n	norm(x_hat)	correct digits	cond(H)
2	1.000000e+00	16	2.700000e+01
3	1.000000e+00	15	7.480000e+02
4	1.000000e+00	13	2.837500e+04
5	1.000000e+00	13	9.436560e+05
6	1.000000e+00	10	2.907028e+07
7	1.000000e+00	8	9.851949e+08
8	1.000001e+00	7	3.387279e+10
9	1.000018e+00	5	1.099649e+12
10	1.000277e+00	4	3.535233e+13
11	1.021296e+00	2	1.229477e+15
12	1.114396e+00	1	3.841961e+16
13	1.045285e+01	0	7.490658e+17