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Output and answers for Q1, Q2c, Q4:

```
>> Question1a
gamma           error(1)        error(2)
1.000000e-02    8.881784e-16    0.000000e+00
1.000000e-04    -1.101341e-13   0.000000e+00
1.000000e-06    2.875566e-11    0.000000e+00
1.000000e-08    5.024759e-09    0.000000e+00
1.000000e-10    8.274037e-08    0.000000e+00
1.000000e-12    -2.212172e-05   0.000000e+00
1.000000e-14    -7.992778e-04   0.000000e+00
1.000000e-16    1.102230e-01    0.000000e+00
1.000000e-18    -1.000000e+00   0.000000e+00
1.000000e-20    -1.000000e+00   0.000000e+00
```

The error of the first component is increasing as gamma approaches zero, until the error gets to -1.
The error of the second component is always 0, regardless of the value of gamma.

The computed solution becomes more inaccurate as the absolute value of gamma decreases, until the error of the first component gets to -1. At this time, the computed solution is very inaccurate, as the absolute value of the relative error is 1 if we choose infinity-norm. After this stage, the accuracy will not drop further when gamma decreases.

```
>> Question1b
gamma           error(1)        error(2)
1.000000e-02    0.000000e+00    0.000000e+00
1.000000e-04    0.000000e+00    0.000000e+00
1.000000e-06    0.000000e+00    0.000000e+00
1.000000e-08    0.000000e+00    0.000000e+00
1.000000e-10    0.000000e+00    0.000000e+00
1.000000e-12    0.000000e+00    0.000000e+00
1.000000e-14    0.000000e+00    0.000000e+00
1.000000e-16    0.000000e+00    0.000000e+00
1.000000e-18    0.000000e+00    0.000000e+00
1.000000e-20    0.000000e+00    0.000000e+00
```

The computed solution is very accurate. The relative error of x is always 0 for gamma as small as 10^(-20). The accuracy is not changed when the absolute value of gamma decreases.

Pivoting avoids the rounding errors caused by extremely small leading digonal entry, making the computed solution much more accurate.

```
>> Question1c
gamma           error(1)        error(2)
1.000000e-02    0.000000e+00    0.000000e+00
1.000000e-04    0.000000e+00    0.000000e+00
1.000000e-06    0.000000e+00    0.000000e+00
1.000000e-08    0.000000e+00    0.000000e+00
1.000000e-10    0.000000e+00    0.000000e+00
```

```
1.000000e-12      0.000000e+00      0.000000e+00
1.000000e-14      0.000000e+00      0.000000e+00
1.000000e-16      0.000000e+00      0.000000e+00
1.000000e-18      0.000000e+00      0.000000e+00
1.000000e-20      0.000000e+00      0.000000e+00
```

The "better" approximate solution is also accurate. The relative error of x is always 0 for gamma as small as 10^(-20). The accuracy is not changed when the absolute value of gamma decreases.

The iterative refinement is very effective in this context. It can recover the full accuracy for badly scaled systems by repeatedly reducing the residue.

```
>> Question2c
The element with largest absolute value in U2 is 2.
norm(x-x1, inf) = 1
norm(x-x2, inf) = 0
```

Using infinity-norm, the relative error of the poor approximation x1 is 1, which means some entry of x1 is 100% deviated from the exact solution.

The relative error of the good approximation x2 is 0, which means all entry are exact (i.e. same as the exace solution).
x2 corrects all inaccurate entries in x1.

```
>> Question4

y1 =

     3
     5
     9
     4
    10
     8
     7
     1
     2
     6

q =

     3     5     9     4    10     8     7     1     2     6

y2 =

     3
     5
     9
     4
    10
     8
     7
     1
     2
     6
```