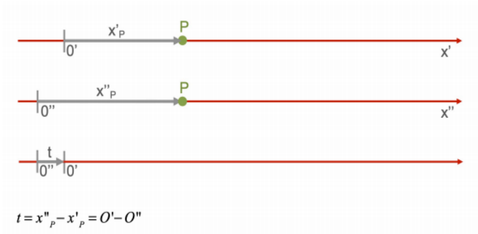
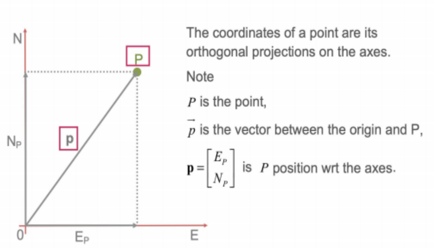
Reference Frames

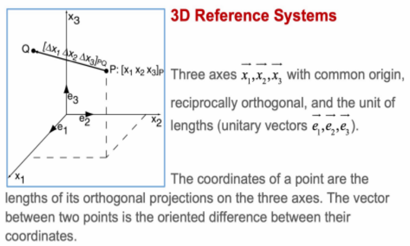
RF in 1, 2 and 3 dimensions



Case 1D

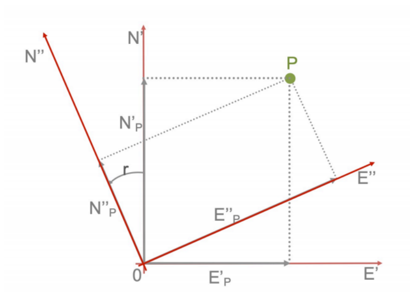


Case 2D



Case 3D

Rototranslation in 2D



Effect of a rotation of the axes

$$P'' = \begin{bmatrix} E'' \\ N'' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos r & \sin r \\ -\sin r & \cos r \end{bmatrix}}_{R} P'$$

$$P'' = \underset{\text{origin change}}{t} + \underset{\text{scale factor}}{\lambda} R P'$$

Helmert transformation in 3D

degrees of freedom (DOF): 3 DOF for origin translation and 3 DOF for axes rotation

Rotate x_2-x_3 plane around x_1

$$R_1(r_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos r_1 & \sin r_1 \\ 0 & -\sin r_1 & \cos r_1 \end{bmatrix}$$

Rotate x_1-x_3 plane around x_2

$$R_2(r_2) = \begin{bmatrix} \cos r_2 & 0 & -\sin r_2 \\ 0 & 1 & 0 \\ \sin r_2 & 0 & \cos r_2 \end{bmatrix}$$

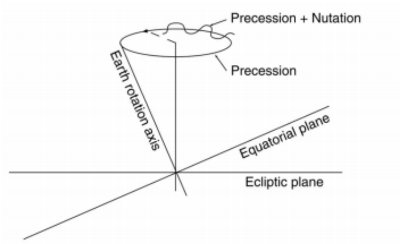
Rotate x_1-x_2 plane around x_3

$$R_3(r_3) = \begin{bmatrix} \cos r_3 & \sin r_3 & 0 \\ -\sin r_3 & \cos r_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x''_P = \underset{\text{origin change}}{t} + \boxed{\underset{\text{Rotation}}{\lambda R}\, x'_P}$$

$$R = R(r_1, r_2, r_3) = R_3(r_3) R_2(r_2) R_1(r_1)$$

Precession, nutation and Chandler wobble.



the rotation axis is moving in the space with two seperate conic motions (圆锥运动)

precession (岁差): a motion with an angle of 23°27′ and period of 25800 years

nutation (章动): a fluctuation around the precession motion, with angle of about 9.2" and period of 18.6 years

Chandle wobble: The planet fluctuates around its rotation axis. This motion is approximately conic.(自转轴在地球表面移动)

The definition of ITRS

ITRS: International Terrestrial Reference System
Origin: mass center of Earth 地球质心
Z: Toward the conventional Earth rotation axis 地球自转轴
X: Orthogonal to z, toward fundamental meridian(Greenwich) 与Z正交，朝向本初子午线
Unit of length: meter

Terrestrial reference frames: ITRF and ETRF.

System: geometric definition 几何定义
Frame: the realization

The IGS and EPN GNSS permanent networks

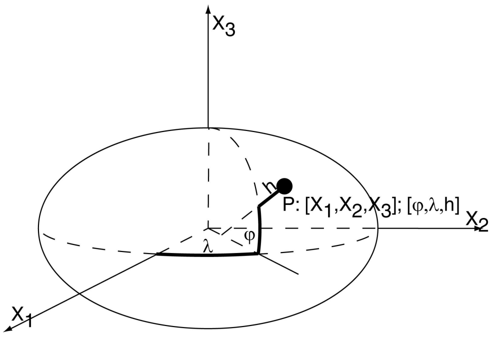
Permanent Station (PS): a continuously operating instrument which publishes raw data, continuously be monitored and provides its coordinates estimates.

Permanent Networks (PNs): consists of
a set of permanent stations
one or more control centers

- ***Global GNSS Network: IGS (International GNSS Service)***
  - It is a service of the International Association of Geodesy (IAG), established in 1993. Its goals are
    - contribute to the realization and distribution of ITRS
    - distribute GNSS products (ephemerides, EOP, ...)
    - define the standard for GNSS PNs and support GNSS research
  - consists of about 500 PSs, serveral Analysis centers, Working groups, Pilot projects, Services, and a Central Bureau
- ***EPN (European Permanent Network)***
  - about 330 GNSS PSs

Definition of global cartesian and geodetic coordinates

Global Cartesian [x, y ,z]
- Origin: mass center of earth
- Z: toward the conventional earth rotation axis
- X: orthogonal to Z, toward fundamental meridian(Greenwich)



Geodetic Coordinates [φ(fai), λ, h] 大地坐标
- geodetic latitude (φ): angle between the norml to ellipsoid and the equatorial plane 法线到椭球体和赤道面之间的夹角
- geodetic longitude (λ): angle between the meridian plane passing per P and the origin meridian plane 通过P的子午线面与原点子午线面之间的夹角
- geodetic height (h): distance between P and the ellipsoid surface

Transformation between them

1.大地转笛卡尔

Geodetic coordinates $[\varphi, \lambda, h]$ → Cartesian Coordinates $[x, y, z]$

$$X_P = (R_N + h_P)\cos\varphi\cos\lambda$$
$$Y_P = (R_N + h_P)\cos\varphi\sin\lambda$$
$$Z_P = [R_N(1-e^2)]\sin\varphi$$

$$R_N = \frac{a}{\sqrt{(1-e^2\sin^2\varphi)}}$$

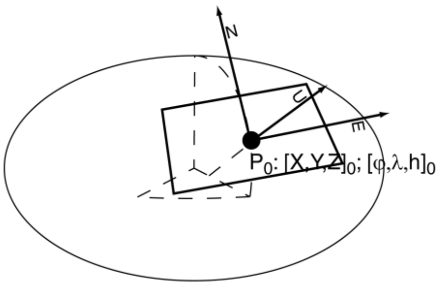
2.笛卡尔转大地

Cartesian Coordinates $[x, y, z]$ → Geodetic coordinates $[\varphi, \lambda, h]$

$$e_b^2 = \frac{a^2-b^2}{b^2}$$
$$r = \sqrt{X^2+Y^2}$$
$$\psi = \arctan\left(\frac{Z}{r\sqrt{1-e^2}}\right)$$

$$\lambda = \arctan\frac{Y}{X}$$
$$\varphi = \arctan\frac{(Z + e_b^2 b\sin^3\psi)}{(r - e^2 a\cos^3\psi)}$$
$$h = \frac{r}{\cos\varphi} - R_N$$

Definition of local cartesian



Local Cartesian coordinates (East, North, Up): origin in a point P0; X and Y orthogonal to the normal to the ellipsoid in P0; X points to East, Y points to North, Z points to the normal to the ellipsoid.

Transformations between Geocentric Cartesian and LC

LC → GC

$$X_{GC(P)} = X_{GC(P_0)} + R_0^T X_{LC(P)}$$

$$\begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + R_0^T \begin{bmatrix} E \\ N \\ U \end{bmatrix}_P$$

$$R_0 = \begin{bmatrix} -\sin\lambda_0 & \cos\lambda_0 & 0 \\ -\sin\varphi_0\cos\lambda_0 & -\sin\varphi_0\sin\lambda_0 & \cos\varphi_0 \\ \cos\varphi_0\cos\lambda_0 & \cos\lambda_0\sin\lambda_0 & \sin\varphi_0 \end{bmatrix}$$

Definition of Local Level: differences between LC and LL.

LL: Origin in a point P0, Z axis oriented along the gravity vertical in P0, X axis aligned to a specific direction (another point P1) that has [X, 0, Z] coordinates

LC TO LL

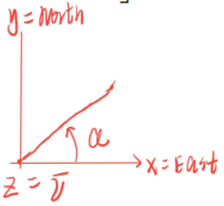
$$\mathbf{x}_{LL}(P_i) = \mathbf{R}\mathbf{x}_{LC} = \mathbf{R}_z(\alpha)\mathbf{R}_y(\eta)\mathbf{R}_x(-\xi)\mathbf{x}_{LC}(P_i)$$

Vertical deflection components 垂直偏转分量

$$\mathbf{R}_x(-\xi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\xi) & -\sin(\xi) \\ 0 & \sin(\xi) & \cos(\xi) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\xi \\ 0 & \xi & 1 \end{bmatrix}$$

$$\mathbf{R}_y(\eta) = \begin{bmatrix} \cos(\eta) & 0 & -\sin(\eta) \\ 0 & 1 & 0 \\ \sin(\eta) & 0 & \cos(\eta) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\eta \\ 0 & 1 & 0 \\ \eta & 0 & 1 \end{bmatrix}$$

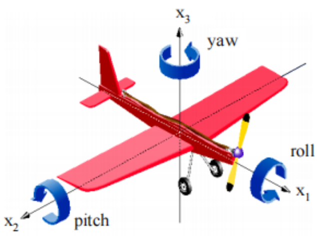
$$\mathbf{R}_z(\alpha) = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



The body frame for inertial navigation

The body frame for navigation

Origin in a generic point P inside the body (e.g. barycenter);



- x1 axis: oriented in the motion direction; 垂直的
- x3 axis: perpendicular to the vehicle plane and in the up direction;
- x2 axis: to complete the right-handed triad.

Diagram flux of the transformation between local systems (LC, LL, body frame) and geocentric systems

The covariance propagation in RF transformations

origin in P_0 LL $P_j = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{LL}$, $C_{LLj} = \begin{bmatrix} \sigma_{x_j}^2 & 0 & 0 \\ 0 & \sigma_{y_j}^2 & 0 \\ 0 & 0 & \sigma_{z_j}^2 \end{bmatrix}$

$\downarrow R_{(LL\to LC)}$

Same origin in P_0 LC $\begin{bmatrix} E \\ N \\ U \end{bmatrix}$ $C_{LCj} = R_{(LL\to LC)} C_{LLj} R^T_{(LL\to LC)}$

$\downarrow R_{(LC\to GC)}$

GC, ITRF $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GC}$ $C_{GCj} = R_{(LC\to GC)} C_{LCj} R^T_{(LC\to GC)}$

$\downarrow$

GC, Geodetic $\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix} \leftrightarrow \begin{bmatrix} E \\ N \\ U \end{bmatrix}$ $C_{\left|\begin{smallmatrix} E \\ N \\ U \end{smallmatrix}\right|_j} = R_{(GC\to LC)} C_{GCj} R^T_{(GC\to LC)}$

$\rightarrow \begin{cases} \sigma_{\lambda_j} = \sqrt{C_{\left|\begin{smallmatrix} E \\ N \\ U \end{smallmatrix}\right|_j (1,1)}} \\ \sigma_{\varphi_j} = \sqrt{C_{\left|\begin{smallmatrix} E \\ N \\ U \end{smallmatrix}\right|_j (2,2)}} \\ \sigma_{h_j} = \sqrt{C_{\left|\begin{smallmatrix} E \\ N \\ U \end{smallmatrix}\right|_j (3,3)}} \end{cases}$

Starting point:

$$\boldsymbol{C_{LL}}(P_i) = \begin{bmatrix} \sigma_E^2 & 0 & 0 \\ 0 & \sigma_N^2 & 0 \\ 0 & 0 & \sigma_U^2 \end{bmatrix}$$

Covariances in LC:

$$\boldsymbol{C_{LC}}(P_i) = R_{LL\to LC}\, \boldsymbol{C_{LL}}(P_i)\, R_{LL\to LC}{}^T$$

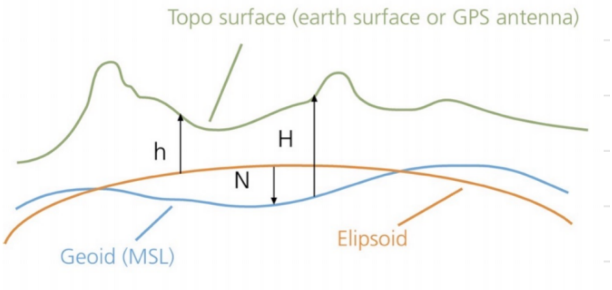
Covariances in GC:

$$\boldsymbol{C_{GC}}(\Delta\boldsymbol{x}) = R_0{}^T\, \boldsymbol{C_{LC}}\, R_0$$
$$\boldsymbol{C_{GC}}(P_i) = \boldsymbol{C_{GC}}(P_O) + \boldsymbol{C_{GC}}(\Delta\boldsymbol{x})$$

Final covariance matrices in $[\varphi, \lambda, h]$

$$C_{[\varphi,\lambda,h]} = R_0(\varphi,\lambda,h)\boldsymbol{C_{GC}}R_0(\varphi,\lambda,h)^{\boldsymbol{T}}$$

Geoid undulation, orthometric height and heights transformations

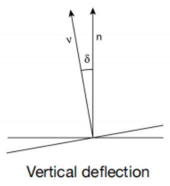


h=elipsoid height 椭球高（地面到椭球体）
H=orthometric height 正高（地面到大地水准面）
N=geoid height 大地水准面高（椭球体到大地水准面的法线）
height transformations

H = h + N

正高=椭球高+大地水准面高

Vertical deflection



Angle between the normal vectors to the ellipsoid and the geoid

GNSS Point positioning

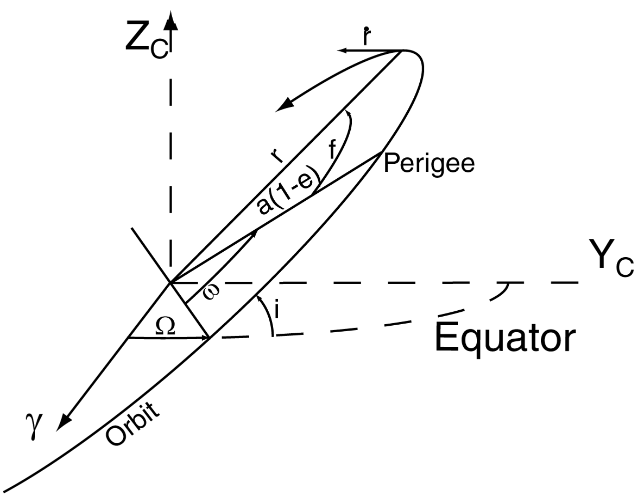
GPS, GLONASS, Compass / Beidou and Galileo: general description

GNSS (Global Navigation Satellite System): the set of all the satellites system aimed at positioning of users.
Inucludes:
·Several constellations of artificial satellites around the Earth
·A receiver observes signal travel times from all the in-view satellites
GPS (Global Positioning Service): United States
·Development started in 1970s, global fully operating from 1995 and completely renewed after 2000
·MEO orbits: Medium Earth Orbits
GLONASS: Russia (MEO)
·First project and development in 1970s, global fully operating from 2011, but still few operational problems
Compass/Beidou: China
·Beidou 1: 4 geostationary(对地静止) satellites, limited coverage to China and part of Assia
·Beidou 2: 5 geostationary and 30 MEO satellites
·Beidou 3: 3 geostaionary satellites, 3 geosynchronous and 24 MEO satellites

GPS: constellation, ephemerides and relevant errors

Consteallation: A satellite constellation is a group of artificial satellites working together as a system.
Ephemerides: The set of 16 parameters necessary and sufficient to compute the satellite position in time
· Broadcast ephemerides: have an accuracy of about 1m.
·Precise ephemerides: Final ephemerides have an accuracy better than 2.5 cm.

Diagram flux of orbit computation



i orbit inclination:angle between the orbital plane and the reference equatorial plane.
轨道倾角:轨道平面和赤道平面之间的夹角

Ω: right ascension of the ascending node: angle, on the reference equatorial plane, between the equinox node and the ascending node. orientation
升交点赤经:春分点和升交点之间的夹角

W: perigee argument: angle between the intersection of the orbital plane with the equatorial plane and the orbit perigee direction.
近地点：轨道与赤道平面交点、与近地点之间夹角 Position

M_0 mean anomaly angle between the satellite mean motion and the perigee
平近点角:

a and e major semi-axis and orbiteccentricity. size and shape
长半轴和轨道偏心度

1. $a, e, M_0 \longrightarrow x(t), y(t), r(t)$ of satellite in ORS

mean velocity : $n = \sqrt{\frac{GM_E}{a^3}}$

↓

eccentric anomaly : $M(t) = M_0 + n(t - t_0)$
偏近点角

↓

$\eta(t) = M(t) + e\sin(\eta(t))$

↓

true anomaly : $\psi(t) = \tan^{-1}\left(\frac{\sin(\eta(t))\sqrt{1-e^2}}{\cos(\eta(t)) - e}\right)$
真近点角

↓

$r(t) = \frac{a(1-e^2)}{1+e\cos(\psi(t))}$ 卫星到地球的距离

$x(t) = r(t)\cos(\psi(t))$

$y(t) = r(t)\cos(\psi(t))$

2. $i, \Omega, W \longrightarrow$ ITRF

ORS $\longrightarrow$ ICRS (celestial Reference System 天体参考系统)

$$\begin{bmatrix} x_s(t) \\ Y_s(t) \\ Z_s(t) \end{bmatrix}_{ICRS} = R_3(-\Omega) R_1(-i) R_3(-W) \begin{bmatrix} x_s(t) \\ y_s(t) \\ 0 \end{bmatrix}_{ORS}$$

ICRS $\longrightarrow$ ITRF

$$\begin{bmatrix} x_s(t) \\ Y_s(t) \\ Z_s(t) \end{bmatrix}_{ITRF} = R_1(y_p) R_2(-x_p) R_3(GAST) \begin{bmatrix} x_s(t) \\ Y_s(t) \\ Z_s(t) \end{bmatrix}_{ICRS}$$

angles in x, y direction

The oscillator: frequency, phase and time.

Ideal oscillator

$$A(t) = A_0 \sin(\omega t + \varphi_0) = A_0 \sin(\varphi(t)) \qquad T = 2\pi / \omega \qquad f = 1/T = \omega / 2\pi$$

A_0: signal amplitude, ω: angular velocity (rad/s), φ_0: initial phase (rad), $\omega t + \varphi_0$ instantaneous phase.

$\varphi(t)$ is the state of the phenomenon at epoch t, can be expressed in radians, or, taking

$$\phi(t) = \varphi(t)/2\pi$$

in part of a cycle.

Real oscillato

·Frequency $f(t) = \dfrac{d\phi(t)}{dt}$

·Phase $\phi(t) = \int_{t_0}^{t} f(\tau)d\tau + \phi_0$

·Time $t_i(t) = \dfrac{\phi_i(t) - \phi_i(t_0)}{f_0} = \dfrac{\phi_i(t)}{f_0} - \dfrac{\phi_i(t_0)}{f_0}$

GPS binary codes and phase carriers

Sinusoidal phase carries

Name	f (MHz)	& λ (cm)
L1	154 f_0 = 1575.42	≅ 19
L2	120 f_0 = 1227.60	≅ 24

Binary codes

·Used for point positioning
 Pseudo Random C/A: f0=1.023MHz, λ = 293m. Only on L1
 P(Precise)(Y) code: f0=10.26MHz, λ = 29.3m. On both L1 and L2
·Used for Navigation
 Navigational message D: contains ephemerides and other data. f=50Hz,λ = 6*10^3m

Propagation in the atmosphere and atmospheric effects

$$c \cdot \tau_R^S = \rho_R^S = \sqrt{(X_R - X^S)^2 + (Y_R - Y^S)^2 + (Z_R - Z^S)^2}$$

X_R, Y_R, Z_R: receiver cartesian coordinates
X^S, Y^S, Z^S: satellite cartesian coordinates

Atmospheric effects:In the atmosphere, the signal propagation velocity varies according to the physical state of the medium

$$\tau_R^S = \int_{\rho_R^S} \frac{dx}{v(\mathbf{x})}$$

ρ is the geometric trajectory of the signal,
$\mathbf{x}$ is the integration point of the path,
$v(\mathbf{x})$ is the propagation speed.

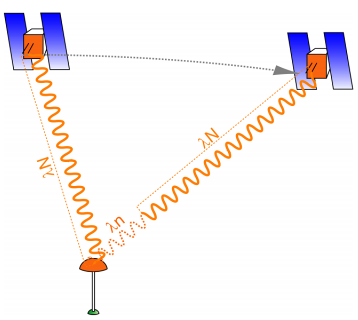
$$c\tau_R^S = \int_{\rho_R^S} \frac{cdx}{v(\mathbf{x})} = \int_{\rho_R^S} n(\mathbf{x})dx = \int_{\rho_R^S} dx + \int_{\rho_R^S} (n(\mathbf{x}) - 1)dx = \rho_R^S + \Delta_R^S$$

$n(\mathbf{x})$: refraction index in $\mathbf{x}$, $n(\mathbf{x}) = c / v(\mathbf{x})$,

ρ_R^S: path / distance between satellite and receiver,

$\Delta_R^S = c\tau_R^S - \rho_R^S = \int_{\rho_R^S} (n(\mathbf{x}) - 1)dx$: atmospheric effect, in length units. $\Delta_R^S = T_R^S + I_R^S$

Initial ambiguities



If no loss of lock happens during (t_1, t_2), the receiver can count and record $n_R^S(t_1, t_2)$

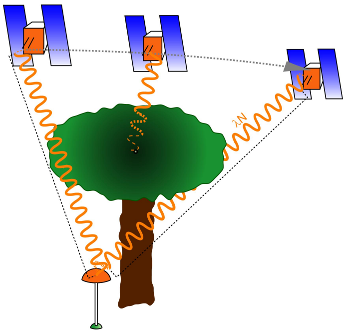
The integer part of the ambiguity at t_2 is given by
$N_R^S(t_2) = N_R^S(t_1) + n_R^S(t_1, t_2)$
$n_R^S(t_1, t_2)$: integer number of cycles passed during (t_1, t_2).

$$\lambda\eta_R^S(t_2) = N_R^S(t_2) + \varphi_{0R} - \varphi_0^S = N_R^S(t_1) + \varphi_{0R} - \varphi_0^S + n_R^S(t_1, t_2) = \lambda\eta_R^S(t_1) + n_R^S(t_1, t_2)$$

For a satellite-receiver couple, a unique unknown initial ambiguity $\eta_R^S(t_1)$ exists, related to the first observation epoch.

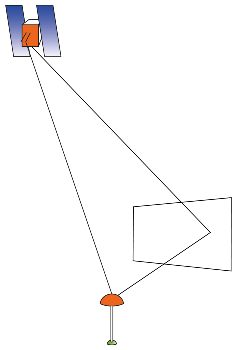
The initial ambiguity is unknown but constant in time

Cycle slips



A cycle slip occurs when an obstacle (such as a wall, a building ..) blocks the signal going from satellite to receiver.

Multipath



Typical in urban scenario.

Due to reflection of the GNSS signal when approaching the receiver antenna.
由于在接近接收机天线时对GNSS信号的反射

Final codes and phases observation equations

Final code observation equation

$$P_R^S(t) = c\tau_R^S(t) + c(dt_R(t) - dt^S(t))$$
$$= \rho_R^S(t) + c(dt_R(t) - dt^S(t)) + I_R^S(t) + T_R^S(t)$$

Satellite position $[X^S, Y^S, Z^S]$ (in ρ): known from the ephemerides
receiver position $[X_R, Y_R, Z_R]$ (in ρ): unknown,
satellite clock offset dt^S, known from the navigation message,
receiver clock offset dt_R unknown,
ionospheric and tropospheric effects I, T: computable by standard models.

The equation contains 4 receiver related unknowns $[X_R, Y_R, Z_R, dt_R]$.
4 satellites in view guarantee their estimation in single epoch!

Final carrier phase observation equation

$$L_R^S(t) = \rho_R^S(t) + c(dt_R(t) - dt^S(t)) - I_R^S(t) + T_R^S(t) + \lambda\eta_R^S(t)$$

$$\eta_R^S(t_1) = N_R^S(t_1) + \phi_{0R} - \phi_0^S$$

ϕ_{R0}, ϕ_0^S are fractional values, constant in time: represent the initial phases of receiver and satellite respectively.

$N_R^S(t_1)$ is an integer, time varying, value: represents the integer part of the distance between satellite and receiver.

Ionospheric free combination

The final observation equation is

$$P3_R^S(t) = \rho_R^S(t) + c(dt_R(t) - dt^S(t)) + T_R^S(t)$$

The final IF observation equation is given by

$$L3_R^S(t) = \rho_R^S(t) + c(dt_R(t) - dt^S(t)) + T_R^S(t) + \lambda_3(N3_R^S(t) + \phi3_R - \phi3^S)$$

Real time point positioning by Least Squares and final error budget

The solution: input

$\tilde{X}_R, \tilde{Y}_R, \tilde{Z}_R$: approximate receiver coordinates
$\tilde{I}_R^S, \tilde{T}_R^S$: ionospheric and tropospheric effects from standard atmospheric models

To be estimated

RESULT $\rightarrow \begin{bmatrix} X_R \\ Y_R \\ Z_R \\ dt_R \end{bmatrix}$

$\tilde{\mathbf{x}}^S, d\tilde{t}^S$: satellites coordinates and clock offset from navigational message

1. 观测方程线性化 linearization

$$\tilde{e}_R^S = \frac{1}{\tilde{\rho}_R^S} \begin{vmatrix} \tilde{X}_R - \tilde{X}^S \\ \tilde{Y}_R - \tilde{Y}^S \\ \tilde{Z}_R - \tilde{Z}^S \end{vmatrix}$$ 单位向量

$$\tilde{e}_R^{S\,T} \begin{vmatrix} X_R - \tilde{X}_R \\ Y_R - \tilde{Y}_R \\ Z_R - \tilde{Z}_R \end{vmatrix} = \rho_R^S - \tilde{\rho}_R^S$$

2. $$P_R^S = \rho_R^S + c[dt_R - d\tilde{t}^S] + \tilde{I}_R^S + \tilde{T}_R^S + \varepsilon(X^S, dt^S, I_R^S, T_R^S) + V_R^S$$

$$\frac{\tilde{X}_R - \tilde{X}^S}{\tilde{\rho}_R^S}(X_R - \tilde{X}_R) + \frac{\tilde{Y}_R - \tilde{Y}^S}{\tilde{\rho}_R^S}(Y_R - \tilde{Y}_R) + \frac{\tilde{Z}_R - \tilde{Z}^S}{\tilde{\rho}_R^S}(Z_R - \tilde{Z}_R) + c\,dt_R$$

$$+ \underbrace{(\tilde{\rho}_R^S - c\,d\tilde{t}^S + \tilde{T}_R^S + \tilde{I}_R^S)}_{\text{known term} \; = b_R^S} + \underbrace{\varepsilon(X^S, dt^S, I_R^S, T_R^S)}_{\text{model error due to mis-modeling of atmosphere, satelite coordinates and clock offset}} + V_R^S$$

Reduced code observations ($\delta \to \Delta$ 差别) correction

$$\delta P_R^S = P_R^S - b_R^S = \hat{e}_R^S \cdot \delta x_R + c\,dt_R + \varepsilon_R^S + v_R^S$$

写成矩阵形式 . with m satellites

$$\delta P = \begin{bmatrix} E & 1 \end{bmatrix} \begin{bmatrix} \delta x_R \\ c\,dt_R \end{bmatrix} + e$$

$$\begin{bmatrix} \delta P_R^1 \\ \delta P_R^2 \\ \vdots \\ \delta P_R^m \end{bmatrix} \quad \begin{bmatrix} e_{R,x}^1 & e_{R,y}^1 & e_{R,z}^1 & 1 \\ e_{R,x}^2 & e_{R,y}^2 & e_{R,z}^2 & 1 \\ \vdots & & & \\ e_{R,x}^m & e_{R,y}^m & e_{R,z}^m & 1 \end{bmatrix} \quad \begin{bmatrix} \varepsilon_R^1 + v_R^1 \\ \varepsilon_R^2 + v_R^2 \\ \vdots \\ \varepsilon_R^m + v_R^m \end{bmatrix}$$

IF $m > 4$, $\begin{bmatrix} \delta x_R \\ c\,dt_R \end{bmatrix}$ can be solved by LS ($y = A\boxed{x} + v$)

Error budget in a single epoch to a single satellite.

Measurement noise

P(Y): 10-20 cm

C/A: 20-200 cm (depending on the receiver quality)

Model errors and their variability		In time	Between satellites
broadcast ephemerides	≅ 1 m	slow	uncorrelated
broadcast dt ≅ 3 ns:	≅ 1 m	slow	uncorrelated
residual ionospheric	≅ 0-20 m	slow	unpredictable
residual tropospheric	≤ 1 m	slow	similar
multipath: (codes)	≅ 1-150 m, periodic with period of		
about 20 minutes			
(carriers)	≅ 1-5 cm, unpredictable correlation		
between satellites			

PDOP and HDOP: definition and extraction

The final LS covariance matrix of the estimated coordinates and clock offset is

$$\mathbf{C}_{\xi\xi} = \hat{\sigma}_0^2 (\mathbf{A}^T \mathbf{Q}^{-1} \mathbf{A})^{-1}$$

$\hat{\sigma}_0^2$ will depend on the observations accuracy and can predicted just as order of magnitude.

$$= \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 & \sigma_{zt} \\ \sigma_{xt} & \sigma_{yt} & \sigma_{zt} & \sigma_t^2 \end{bmatrix}$$

PDOP (Positional Dilution of Precision) : $PDOP = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} = \sqrt{q_{EE} + q_{NN} + q_{hh}}$

HDOP (Horizontal DOP) : horizontal coordinates accuracy $HDOP = \sqrt{q_{EE} + q_{NN}}$

GNSS Relative positioning

What does it mean a baseline estimation in GNSS.

Differential processing by combining (double differencing) simultaneous observations of two or more receivers

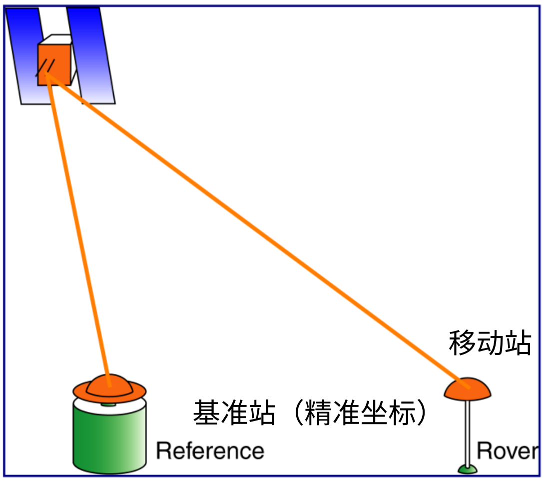
通过结合两个或多个接收器的同时观测进行差分处理

Single differences and double differences: observation equations

Single differences (SD)

Differences between two simultaneous observations from two receivers R1 and R2 to the same satellite S.

从两个接收器R1和R2到同一颗卫星S的两个同时观测结果之间的差异。



On codes

$$P^{S}_{R1,R2}(t)=P^{S}_{R1}(t)-P^{S}_{R2}(t)=\rho^{S}_{R1,R2}(t)+I^{S}_{R1,R2}(t)+T^{S}_{R1,R2}(t)+c(dt_{R1}(t)-dt_{R2}(t))$$

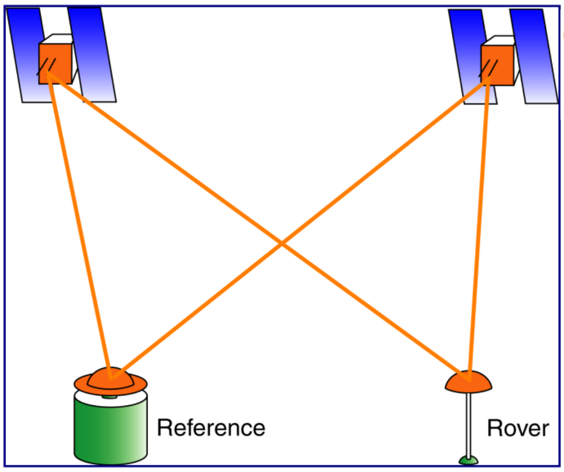
On phases

$$L^{IJ}_{R1,R2}(t)=L^{I}_{R1,R2}(t)-L^{J}_{R1,R2}(t)=\rho^{IJ}_{R1,R2}(t)+T^{IJ}_{R1,R2}(t)+\lambda N^{IJ}_{R1,R2}(t)-I^{IJ}_{R1,R2}(t)+c(dt_{R1}(t)-dt_{R2}(t))$$

Clock offsets and initial phases of the satellite vanish, tropospheric and ionospheric effects diminish.

Double differences (DD)

Differences of two simultaneous single differences to two satellites I and J.



On codes

$$P^{IJ}_{R1,R2}(t)=P^{I}_{R1,R2}(t)-P^{J}_{R1,R2}(t)=\rho^{IJ}_{R1,R2}(t)+I^{IJ}_{R1,R2}(t)+T^{IJ}_{R1,R2}(t)$$

On phases

$$L^{IJ}_{R1,R2}(t)=L^{I}_{R1,R2}(t)-L^{J}_{R1,R2}(t)=\rho^{IJ}_{R1,R2}(t)+T^{IJ}_{R1,R2}(t)+\lambda N^{IJ}_{R1,R2}(t)-I^{IJ}_{R1,R2}(t)$$

DD contain only geometric and integer ambiguities parameters plus reduced atmospheric effects.

The geometric content for short baselines. R1坐标确定，将估计基线与 R1 坐标相加，得到 R2 估计坐标。

On Code

$$\delta P^{IJ}_{R1,R2O}(t)=P^{IJ}_{R1,R2O}(t)-b^{IJ}_{R1,R2}(t)$$
$$=\tilde{\mathbf{e}}^{JI}_{R2}(t)\cdot\delta\Delta\mathbf{x}_{R1,R2}(t)+\varepsilon^{IJ}_{P\,R1,R2}(t)$$

$$b^{IJ}_{R1,R2}(t)=\tilde{\rho}^{IJ}_{R1,R2}(t)+\tilde{T}^{IJ}_{R1,R2}(t)+\tilde{I}^{IJ}_{R1,R2}(t),$$

On Phase

$$\delta L^{IJ}_{R1,R2O}(t)=L^{IJ}_{i\,R1,R2O}(t)-b^{IJ}_{R1,R2}(t)$$
$$=\tilde{\mathbf{e}}^{JI}_{R2}(t)\cdot\delta\Delta\mathbf{x}_{R1,R2}(t)+\lambda N^{IJ}_{R1,R2}(t)+\varepsilon^{IJ}_{R1,R2}(t)$$

$$b^{IJ}_{R1,R2}(t)=\tilde{\rho}^{IJ}_{R1,R2}(t)+\tilde{T}^{IJ}_{R1,R2}(t)-\tilde{I}^{IJ}_{R1,R2}(t),$$

$$\tilde{\mathbf{e}}^{JI}_{R2}(t)=\tilde{\mathbf{e}}^{J}_{R2}(t)-\tilde{\mathbf{e}}^{I}_{R2}(t)$$

$$\varepsilon^{IJ}_{R1,R2}(t)=\varepsilon(\mathbf{x}_{R1},\mathbf{x}^{I}(t),\mathbf{x}^{J}(t),I^{IJ}_{R1,R2}(t),T^{IJ}_{R1,R2}(t))+v_{DD}$$

model error:

- satellite ephemerides,
- ionospheric and tropospheric models; (in P3/L3 no iono)
- multipath,

+ electronic noise of DD

$\delta\Delta\mathbf{x}_{R1,R2}(t)$ contains the unknowns baseline components,

$\lambda N^{IJ}_{R1,R2}(t)$ contains the unknowns integer ambiguities (constancy has to be checked against cycle slips).

Order of magnitudes of the error terms in DD in single epoch

$$\varepsilon^{i,j}_{R1,R2} = f(\varepsilon(\tilde{\mathbf{x}}^i),\varepsilon(\tilde{\mathbf{x}}^j),\varepsilon(\tilde{I}),\varepsilon(\tilde{T})) = \varepsilon^i_{R1} - \varepsilon^i_{R2} - (\varepsilon^j_{R1} - \varepsilon^j_{R2})$$

DD error is smaller than the undifferenced errors, but it is still significant.

$\varepsilon^{IJ}_{P\,R1,R2}(t)$ is caused by all the modelling errors

Empirically, DD error increases with the baseline length.

Effect of ephemerides error

$$\varepsilon(\mathbf{x}^S) = \boldsymbol{\varepsilon}^S \frac{\Delta\mathbf{x}_{R1,R2}}{\bar{\rho}^S}$$

$$\boldsymbol{\varepsilon}^S = 1m \Rightarrow \varepsilon(\mathbf{x}^S) = 0.5\cdot 10^{-7}\Delta\mathbf{x}_{R1,R2}$$

$$\boldsymbol{\varepsilon}^S = 5cm \Rightarrow \varepsilon(\mathbf{x}^S) = 0.25\cdot 10^{-8}\Delta\mathbf{x}_{R1,R2}$$

Effect of tropospheric model error

$$\varepsilon(T^S_R) = 0.0 - 2.0\cdot 10^{-6}$$

Effect of ionospheric model error (no effect in P3/L3)

$$\varepsilon(I^S_R) = 0.0 - 8.0\cdot 10^{-6}$$

The single epoch estimation for code DD

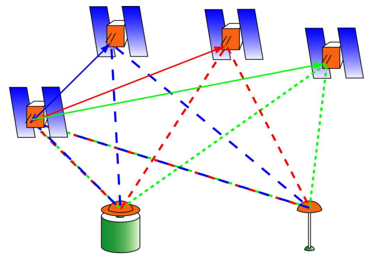
one epoch, 2 reciever, m satellites → m-1 indepent DD

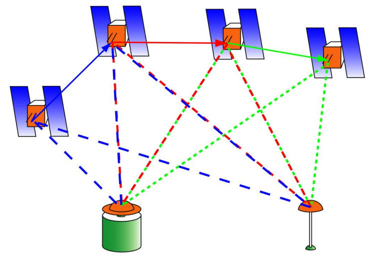
m-1 independent DD: either

$$O^{S1,S2}_{R1,R2}(t), O^{S1,S3}_{R1,R2}(t), \ldots O^{S1,Sm}_{R1,R2}(t)$$

or

$$O^{S1,S2}_{R1,R2}(t), O^{S2,S3}_{R1,R2}(t), \ldots O^{S(m-1),Sm}_{R1,R2}(t)$$





$$\delta\mathbf{P}_O(t) = \delta\mathbf{P}(t) + \varepsilon(t)$$

$$\delta\mathbf{P}(t) = \mathbf{P}(t) - \mathbf{b}(t) = \mathbf{E}(t)\delta\Delta\mathbf{x}_{R1R2}(t)$$

where

$$\mathbf{P}(t) = \begin{bmatrix} P^{12}_{12}(t) \\ P^{13}_{12}(t) \\ \ldots \\ P^{1m}_{12}(t) \end{bmatrix} - \mathbf{b}(t) = \begin{bmatrix} \tilde{b}^{12}_{12}(t) \\ \tilde{b}^{13}_{12}(t) \\ \ldots \\ \tilde{b}^{1m}_{12}(t) \end{bmatrix} = \mathbf{E}(t) = \begin{bmatrix} \tilde{e}^{21}_{2,x}(t) & \tilde{e}^{21}_{2,y}(t) & \tilde{e}^{21}_{2,z}(t) \\ \tilde{e}^{31}_{2,x}(t) & \tilde{e}^{31}_{2,y}(t) & \tilde{e}^{31}_{2,z}(t) \\ & \ldots & \\ \tilde{e}^{m1}_{2,x}(t) & \tilde{e}^{m1}_{2,y}(t) & \tilde{e}^{m1}_{2,z}(t) \end{bmatrix} \delta\Delta\mathbf{x}_{R1R2}(t)$$

未知数 需求解

With at least 4 satellites the system can be solved wrt the baseline.

A single epoch system of **phase** DD can't be solved with respect to the baseline, because both baseline and ambiguities are unknown.

The processing steps of phase DD

Two main cases:

Static solutions: both reference and rover receivers occupy the same points for the whole session. One baseline is estimated.基准站和移动站在过程中保持不动

Kinematic solution: the reference is static, the rover is moving. One baseline per epoch (the rover trajectory) is estimated.基准值不动，移动站移动，每个epoch计算一条基线

Steps:

1.Preprocessing: data preparation and cleaning

DD code observables are adjusted by LS to obtain an approximate estimate of the baseline. The results are used to identify and repair possible cycle slips in phase observations.

Static repair: Given a time series of residual double differences

1. $\Delta\delta L^{S1S2}_{R1,R2}(t_2)$ is computed

2. if

$$\left|\Delta\delta L^{S1S2}_{R1,R2}(t_2)\right| \le \varepsilon_{\lim}$$

no cycle slip between t_1 and t_2

3. otherwise a cycle slip has been identified,

$$x = \frac{\Delta\delta L^{S1S2}_{R1,R2}(t_2)}{\lambda}$$

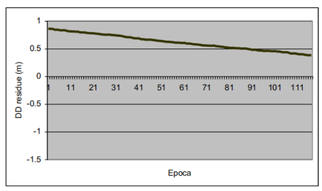
is computed.

4. let n be the nearest integer to x; if

$$\lambda\left|n - x\right| \le \varepsilon_{\lim}$$

n is the value of the cycle slip and is subtracted to all the following DD observations:

$$\hat{L}^{S1S2}_{R1,R2}(t_k) = L^{S1S2}_{R1,R2}(t_k) - \lambda n$$



2.Processing: parameters estimate

Float solution

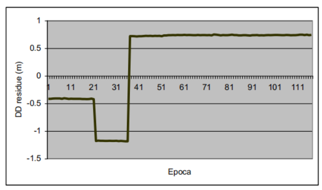
A LS system on all the epochs is built by phase DD and solved with respect to static baseline/kinematic trajectory and ambiguities

Ambiguities fixing

Real valued, estimated ambiguities are fixed to integer values.

Fixed solution

A LS system is solved with respect to static baseline/kinematic trajectory (and tropospheric delays for long sessions).



Order of magnitudes of final errors in baseline estimates as function of baseline length and survey time

基线测量中极限长度和观测时间的最终误差数量级

Typical final accuracies in baselines estimates

Time\ Baseline	1 km	10 km	50 km	1000 km
1 epoch	2.5 cm	4.0 cm	ne	ne
1-2 minutes	2.0 cm	2.5 cm	?	ne
10 minutes	1.5 cm	2.5 cm	5 cm	ne
1 hour	1.0 cm	1.5 cm	2 cm	ne
6 hours	0.5 cm	1.0 cm	1.5 cm	ne
24 hours	0.3 cm	0.5 cm	0.5 cm	1.0 cm (*)
1 week	0.1 cm	0.1 cm	0.3 cm	<1 cm (*)

Kinematic. Fast static. Static. Long static (permanent stations)

ne: not estimable; (*) multibase networks (not single baseline) processing

Given a survey time, the accuracy decreases with the baseline length.
给定测量时间，精度随着基线长度的增加而降低。

Given a baseline the accuracy increases with the survey time.
在给定基线的情况下，准确性会随着测量时间的推移而增加。

Not identified cycle slips can produce unpredictable errors.
未识别的周跳可能会产生不可预测的错误。

Indoor positioning

Introduction to applications and user requirements.

Applications

- **private homes**: ambient assistant living (AAL) systems (环境辅助生活系统)
- **hospitals**
  - location of medical personnel or patients
  - navigation of unmanned vehicles (carrying medicines, ...)
  - robotic assistance during surgery
- **security** (police or firefighters)
  - instantaneous detection of theft/burglary
  - location of stolen products
  - location of injured people
  - location of firmen
- **industries**
  - robotic guidance
  - robot cooperation
  - tagged tools finding
- **museum**: piece finding
- **navigation to parking slots**
- **surveying and monitoring of buildings**

User requirements

- **accuracy**: confidence region at 95%
  - from mm to decameter
- **availability**: percentage of time in which the system is available
  - low < 95%; regular > 99%; high > 99.9%
- **coverage area**: from mm level (robot arm), rom level (navigation in building) to global level (blind navigation)
  - **scalable system**: can increase the coverage area (in case by adding new components) without decreasing accuracy/availability
- **output positions**
  - 3D: complete goereferencing
  - 2D: floor location
  - symbolic: proximity to an object (e.g. artwork in an museum)
- **update rate**
  - on event: sensor passes a door
  - on request: where am I?
  - periodic: periodic monitoring
- **latency**
  - real time: no perceivable delay
  - sooner or later: as soon as possible
  - **STB with an upper limit**: a threshold is set
  - proprecessing: no specific requirements
- **privacy**
  - active techniques: user sends signals
  - passive techniques: user receives signals
  - **server side computation** (estimate of the positions are computed by a central server) or **user side computation** (each user autonomously estimates its position)
- **number of users**
  - single user: for example, surveying
  - restricted number of users: active / passive mobile sensors
  - unlimited users: passive mobile sensors
- **costs**: robotic industry can afford expensive systems, individual citizens do not
  - system set up, users devices, maintenance

Measuring principles

RSSI Received Signal Strength Indicator

Received Signal Strength averaged over a certain period. Usually specified as power in Decibels (dB).

Physical attenuation general model(物理衰减模型)

$$P_R = kP_T \frac{G_T G_R}{4\pi d_{TR}^p}$$

P_R, P_T : received and transmitted signal strengths
G_T, G_R: transmitter, receiver antenna gains
d_{TR} : distance between transmitter and receiver
p : path loss exponent

In open space $p = 2$. In indoor typically $4 \le p \le 6$ accordingly to the geometry, but in narrow corridors (waveguide), $p < 2$!

Time of Arrival(TOA)

Measurement based on the travel time from transmitter to receiver (zero differences in GNSS).

$$ToA_u^{bs} = \tau_u^{bs} + \delta_u - \delta^{bs}$$

τ_u^{bs}: travel time from the Base Station to the User
δ_u: clock offset of the user; δ^{bs}: clock offset of the base station

Typically the base stations are connected and are considered synchronized

$$ToA_u^{bs} = \tau_u^{bs} + \delta_u$$

Time Difference of Arrival (TDoA)

The difference in arrival time from transmitter to receiver

Round Trip Time (RTT)

Time needed by the signal to travel from a transmitter to a receiver and back.

Angle of Arrival (AoA)

The angular direction of incoming signals can be measured by a directional antenna in the receiver.

Processing algorithms

CoO(Cell of Origin): The mobile is located in the position of the most "probable" known node

Centroid determination：The mobile is located in the centroid(weighted mean) of the sensed known nodes

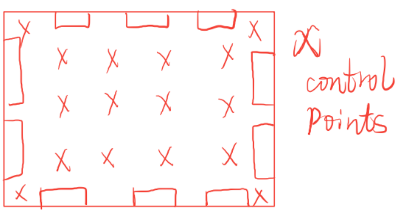
$$\mathbf{x}_u = \sum_{i=1}^{N} w_i \mathbf{x}_i$$

(*) weighted mean, $\sum_{i=1}^{N} w_i = 1$

RSSI observations: weights proportional to observations,
ToA observations: weights inversely proportional to observations

Multilateration：多点定位
Multiple distances derived from RSSI or ToA or TDoA are adjusted to estimate positions / paths.

Fingerprinting：
Step 1: Calibration
Suppose N known nodes serve the interest area with either transmitters or receivers, M control points are chosen in the interest area, which coordinates are accurately estimated.



For each control point, the signal(RSSI, ToA, TDoA...) are recorded:
a) from all the transmitter on known nodes to the user receiver
b) from the user transmitter to all the known nodes receivers

Step 2: Usage
At one epoch in an unknown position, the mobile is registered and compared with all the contorl points in database.
The coordinates of the mobile is the coordinates of the control point whose vector of observation is the "nearest" to the vector of observation of the mobile.

Technologies: WLAN / WI-FI, Radio Frequency Identification (RFID)
WLAN(Wireless Local Area Networks): Active beacons that create networks through wireless techniques.
Range up to 50-100 m.

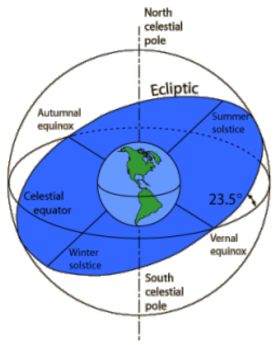
Radio Frequency Identification(RFID): A (cheap) technique to identify objects carrying RFID sensors.
Two kinds of sensors:
·Readers: read signal from tags
·Tags, send their identification codes(passive or active)

Inertial positioning

Connections to reference frames

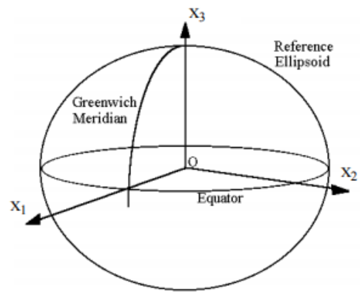
Pseudo inertial system

重心
- origin in the Earth barycentre;
- x_3 axis: oriented towards a celestial North Pole;
- x_1 axis: intersection between ecliptic and celestial equatorial plane;
- x_2 axis: to complete the right-handed triad.



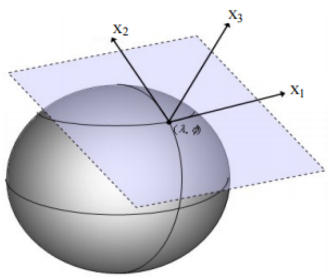
Earth-fixed system

- origin in the Earth barycentre;
- x_3 axis: oriented towards a conventional North Pole;
- x_1 axis: intersection between Greenwich meridian plane and terrestrial equatorial plane;
- x_2 axis: to complete the right-handed triad.



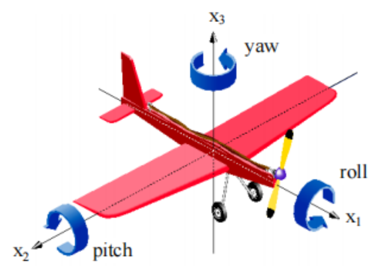
Navigation system (Local cartesian)

- origin in a generic point P;
- x_1 axis: oriented in the East direction;
- x_2 axis: oriented in the North direction;
- x_3 axis: oriented as the normal to reference ellipsoid, Up direction.



Body system

- origin in a generic point P;
- x_1 axis: oriented in the motion direction;
- x_3 axis: perpendicular to the vehicle plane and in the up direction;
- x_2 axis: to complete the right-handed triad.



The Inertial Measurement Units and their observations: accelerometers and gyroscopes

Inertial navigation system (INS):
- 3 accelerometers
- 3 gyroscopes
- the hardware collecting data
- the software for real time processing

→ Each component is called IMU (Inertial Measurement Unit)

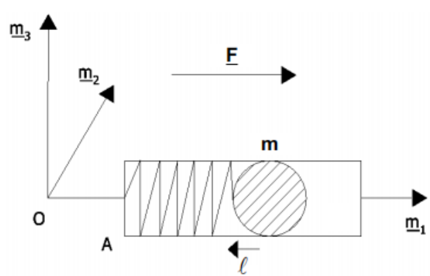
Accelerometers(加速度针): measure the forces acting on a proof mass.
2 Types:
·open loop(e.g. spring based accelerometers)
measure the displacement of the proof mass resulting from external forces acting on the sensor.
·closed loop(e.g. pendulous or electrostatic accelerometers)
keep the proof mass in a state of equilibrium by generating a force that is opposite to the applied force.

Gyroscopes(陀螺仪): measure the angular rate of the sensor rotation with respect to an inertial reference system.
2 Types:
·mechanical
·optical

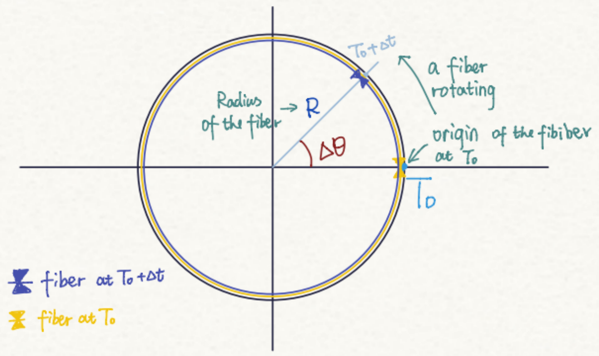
The spring accelerometer and the optical gyroscope

- In case of F=*const*, the following law holds: $-k_e\ell = F$
(after the initial oscillations have ceased)

- Knowing the proof mass, the acceleration *a* along the spring axis can be derived: $F = ma$



- optical gyroscope: based on **"Sagnac" effect**
  - An observer moving with fiber sees the light to cover exactly one revolution: **2π** (pai)
  - An inertial observer sees the light to cover an angle: **2π + △θ**
    - For a light emitted on the opposite direction of the rotation: **2π - △θ**



$$\Delta\theta = \omega\cdot\frac{2\pi R}{c}$$

$$\rightarrow \begin{cases} \Delta_+ L = \Delta\theta R = 2\pi R^2 \frac{\omega}{c} = 2\frac{\omega A}{c} \\ \Delta_- L = -2\frac{\omega A}{c} \end{cases}$$

$$\rightarrow \Delta = \Delta_+ L - \Delta_- L = 4\frac{\omega A}{c}$$

$$\rightarrow \omega = \frac{\Delta \times c}{4A}$$

Typical error budget of IMU and Inertial Navigation Systems

Accelerometer: $\delta a = \underset{\substack{\text{bias} \\ 25\,mGal}}{b} + \underset{\substack{\text{scale factor} \\ 5\times10^{-5}}}{\lambda a} + \underset{\substack{\text{thermal constant} \\ \text{depending on temperature } T \\ 0.5\,mGal/^\circ C}}{C_T(T-T_0)} + \underset{\substack{\text{measurement noise} \\ 40\,mGal/\sqrt{Hz}}}{v}$ 注: 1 Gal = 1 cm/s²

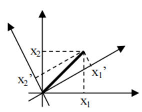
Gyroscope error: $\delta\omega = b + \lambda\omega + C_T(T-T_0) + v$

10^{-3}°/ora 2×10^{-6} 5×10^{-5}°/ora°C 6×10^{-7} rad/s $\sqrt{Hz}$

The basic rules of rotations

2D

$$\begin{cases} x_1' = +x_1\cos\alpha + x_2\sin\alpha \\ x_2' = -x_1\sin\alpha + x_2\cos\alpha \end{cases}$$



$$\underline{x}' = R_A^B\underline{x} \longrightarrow R_A^B = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

3D

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix} \quad R_2 = \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix} \quad R_3 = \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = R_3(\alpha_1)\,R_2(\alpha_2)\,R_1(\alpha_3) =$$

$$= \begin{bmatrix} \cos\alpha_3\cos\alpha_2 & \sin\alpha_3\cos\alpha_1 + \cos\alpha_3\sin\alpha_2\sin\alpha_1 & \sin\alpha_3\sin\alpha_1 - \cos\alpha_3\sin\alpha_2\cos\alpha_1 \\ -\sin\alpha_3\cos\alpha_2 & \cos\alpha_3\cos\alpha_1 - \sin\alpha_3\sin\alpha_2\sin\alpha_1 & \cos\alpha_3\sin\alpha_1 + \sin\alpha_3\sin\alpha_2\cos\alpha_1 \\ \sin\alpha_2 & -\cos\alpha_2\sin\alpha_1 & \cos\alpha_2\cos\alpha_1 \end{bmatrix}$$

The differential equations for rotations

2D $\quad \underline{x}' = R_A^B\underline{x} \longrightarrow R_A^B = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

3D Time derivative of the rotation matrix

↓

$$\dot{R}_A^B = -\Omega_A^B R_A^B \rightarrow$$ rotation from A to B at epoch t0

↓

matrix of angular velocities $\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$

The navigation equations in IRF and ERF

Navigation equations establish a link between the unknowns(namely position, velocity and attitude of the vehicle) and the observations of the accelerometers and of the gyroscopes.
导航方程建立了未知数（即车辆的位置、速度和姿态）与加速度计和陀螺仪的观测结果之间的联系
2 cases:
1. In inertial reference system(IRF)

牛顿第二定律

$F_{tot}^i = m\ddot{x}^i$ $\longrightarrow i$ stay for inertial RF $= F^i + m\underline{g}^i$ $\underline{g}$: acceleration produced by the gravitational field
重力场产生的加速度

$\underline{\ddot{x}}^i = \underline{f}^i + \underline{g}^i$
acceleration of the body

2. In Earth-fixed reference system(ERF)

$$\ddot{\underline{x}}^e = \underline{f}^e + \underline{g}^e - 2\underbrace{\Omega_i^e}_{\text{matrix of angular velocities of earth with inertial}}\dot{\underline{x}}^e$$

Location Based Services

The general problem and typical application

problem:

·**users** are central to LBS, that must be designed on a user centered view:needs and behaviors profiles.

·**location** Mathematically: 3D coordinates estimate.However, users perceive a location in completely different ways, rarely coordinates, typically (addresses +) directions.

·**data modeling:**

-The geographic (referenced) model: Locations and objects are points, lines, areas and volumes consistently georeferenced in a given reference system.

-The symbolic (conceptual概念) mode: Locations are classes with a sets of attributes.

The database of routes and connections: topological rules

roads: A road is a sequence of connections between couples of intersections.

nodes: Intersections between roads and terminal points of roads.

Stored data:

·General attributes: name, kind, surface, slope, width...

·Topological attributes: connections.

For each way:

rights: allowed yes/not

cost: at least 3 possible definitions: length, fee, time

Examples: database of connections

ID	Name	Node_S	Node_E	S_E	E_S	C1_S_E	C1_T_F	C2_S_E
1 univocal identifier, nothing to do with official name!	Official name of the road, can be shared by several connections	Starting node of the road	Ending node of the road	Allowed way (Y/N)	Allowed way (Y/N)	Cost 1 (for example time) from S to E	Cost 1 from E to S	Cost 2…
2	…							

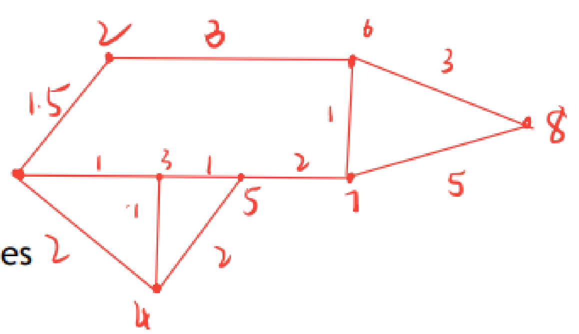
Database of nodes: a simple list of coordinates

ID	X	Y	N
1 univocal identifier	X coordinate	Y coordinate	Number of connected connections
2	…		

Database of turns on nodes

Node ID	Road_1	Road_2	1_2	2_1	C1_1_2	C1_2_1	C2_1_2
1 Node identifier	First connected road	Second connected road	Allowed turn (Y/N)	Allowed turn (Y/N)	Cost 1 (for example time) from 1 to 2	Cost 1 from 2 to 1	Cost 2…
1 for each node, many records as the couples of connections …	…						

Path optimization: the Dijkstra algorithm



1^{st} it explore node
$C_2 = 1.5$
~~$C_3 = 1$~~
$C_4 = 2$

2^{nd} EN: 3
$C_4 = 2$
$C_5 = 2$

3^{rd} EN 2
$C_6 = 4.5$

4th EN = 4

5th EN = 5
$C_7 = 4$

6^{th} EN = 7
~~$C_6 = 5$~~
$C_8 = 9$

7^{th} It EN = 6
$C_8 = 4.5 + 3 = 7.5$

Database of estimated nodes

× $C_2 = 1.5$ 1→2
× $C_3 = 1$ 1→3
× $C_4 = 2$ 1→4
× $C_5 = 2$ 1→3→5
$C_6 = 4.5$ 1→2→6
× $C_7 = 4$ 1→3→5→7
~~$C_8 = 9$ 1→3→5→7~~→8
$C_8 = 7.5$ 1→2→6→8

DB of already explored node

1
3
2
4
5
7
6