EECS 127 Lec 1

How to approach optimization problems

- Choose an optimal loss function
  - Minimize squared error between prediction and true output for regression
    - Maybe choose L1 loss?
  - Minimize # wrong labels if classifier
    - Maybe minimize probability of wrong labels

"Real World" Example

Oil Production

- 10,000 barrels of crude
- Can produce either
  - gasoline : $.20/unit
  - jet fuel : $.30/unit
- 1 barrel of crude → 0.6 barrel jet fuel or 0.7 barrel gas
- Constraints:

  ≥ 1000 units of jet fuel

  ≥ 2000 units of gasoline

X_J, X_G

maximize $0.3X_J + 0.2X_G$

$$\frac{X_J}{0.6} + \frac{X_G}{0.7} \leq 1000$$

$X_J \geq 1000$

$X_G \geq 2000$ (Linear programming)

General Optimization Problem

minimize $f_0(\vec{x})$ s.t. $f_i(\vec{x}) \leq b_i$ for $i = 1, 2, \ldots, m$

x_*, $\vec{x}^*$ denotes the optimizer

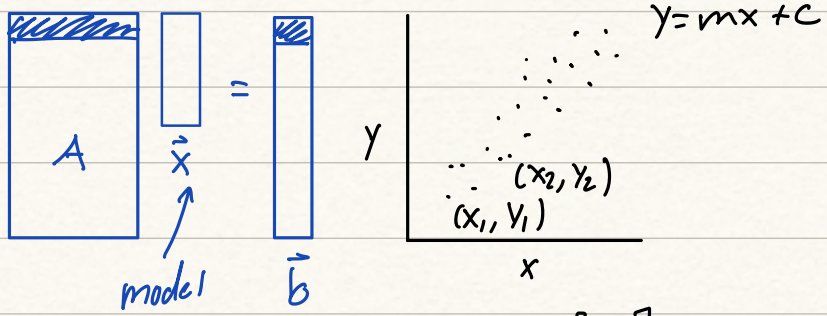
argmin: $\vec{x}$ that realizes the minimum value

Favorite Optimization Problem

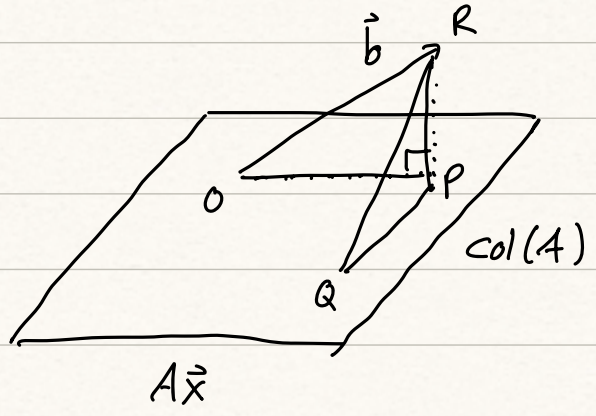
Least Squares

A matrix, $\vec{b}$ vector

- $\min_{\vec{x}} \|A\vec{x} - \vec{b}\|_2^2$
- $(A^TA)^{-1}A^T\vec{b} = \vec{x}_*$ (argmin)



$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$



Claim: Proj of $\vec{b}$ onto col(A) is closest point to $\vec{b}$