

**Exercises 1A:  $\mathbb{R}^n$  and  $\mathbb{C}^n$** *Linear Algebra Done Right, 4th ed.*

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**Exercise 1.** Suppose  $a$  and  $b$  are real numbers, not both 0. Find real numbers  $c$  and  $d$  such that

$$\frac{1}{a+bi} = c + di.$$

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**Answer:** \_\_\_\_\_

**Exercise 2.** Show that

$$\frac{-1 + \sqrt{3}i}{2}$$

is a cube root of 1 (meaning that its cube equals 1).

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**Exercise 3.** Find two distinct square roots of  $i$ .

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**Answer:** \_\_\_\_\_

**Exercise 4.** Show that  $\alpha + \beta = \beta + \alpha$  for all  $\alpha, \beta \in \mathbb{C}$ .

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**Exercise 5.** Show that  $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$  for all  $\alpha, \beta, \lambda \in \mathbb{C}$ .

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**Exercise 6.** Show that  $(\alpha\beta)\lambda = \alpha(\beta\lambda)$  for all  $\alpha, \beta, \lambda \in \mathbb{C}$ .

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**Exercise 7.** Show that for every  $\alpha \in \mathbb{C}$ , there exists a unique  $\beta \in \mathbb{C}$  such that  $\alpha + \beta = 0$ .

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**Exercise 8.** Show that for every  $\alpha \in \mathbb{C}$  with  $\alpha \neq 0$ , there exists a unique  $\beta \in \mathbb{C}$  such that  $\alpha\beta = 1$ .

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**Exercise 9.** Find  $x \in \mathbb{R}^4$  such that

$$(4, -3, 1, 7) + 2x = (5, 9, -6, 8).$$

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**Answer:** \_\_\_\_\_

**Exercise 10.** Explain why there does not exist  $\lambda \in \mathbb{C}$  such that

$$\lambda(2 - 3i, 5 + 4i, -6 + 7i) = (12 - 5i, 7 + 22i, -32 - 9i).$$

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**Exercise 11.** Show that  $(x + y) + z = x + (y + z)$  for all  $x, y, z \in \mathbb{F}^n$ .

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**Exercise 12.** Show that  $(ab)x = a(bx)$  for all  $x \in \mathbb{F}^n$  and all  $a, b \in \mathbb{F}$ .

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**Exercise 13.** Show that  $1x = x$  for all  $x \in \mathbb{F}^n$ .

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**Exercise 14.** Show that  $\lambda(x + y) = \lambda x + \lambda y$  for all  $\lambda \in \mathbb{F}$  and all  $x, y \in \mathbb{F}^n$ .

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**Exercise 15.** Show that  $(a + b)x = ax + bx$  for all  $a, b \in \mathbb{F}$  and all  $x \in \mathbb{F}^n$ .

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