# Exercises 1A: $\mathbb{R}^n$ and $\mathbb{C}^n$

*Linear Algebra Done Right, 4th ed.*

**Exercise 1.** Show that $\alpha + \beta = \beta + \alpha$ for all $\alpha, \beta \in \mathbb{C}$.

**Exercise 2.** Show that $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.

**Exercise 3.** Show that $(\alpha\beta)\lambda = \alpha(\beta\lambda)$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.

**Exercise 4.** Show that $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$ for all $\lambda, \alpha, \beta \in \mathbb{C}$.

**Exercise 5.** Show that for every $\alpha \in \mathbb{C}$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha + \beta = 0$.

**Exercise 6.** Show that for every $\alpha \in \mathbb{C}$ with $\alpha \neq 0$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha\beta = 1$.

**Exercise 7.** Show that

$$\frac{-1+\sqrt{3}\,i}{2}$$

is a cube root of 1 (meaning that its cube equals 1).

**Exercise 8.** Find two distinct square roots of $i$.

**Answer:** ______________________________

**Exercise 9.** Find $x \in \mathbb{R}^4$ such that

$$(4, -3, 1, 7) + 2x = (5, 9, -6, 8).$$

**Answer:** ____________________

**Exercise 10.** Explain why there does not exist $\lambda \in \mathbb{C}$ such that

$$\lambda(2 - 3i, 5 + 4i, -6 + 7i) = (12 - 5i, 7 + 22i, -32 - 9i).$$

**Exercise 11.** Show that $(x+y)+z = x+(y+z)$ for all $x, y, z \in \mathbb{F}^n$.

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**Exercise 12.** Show that $(ab)x = a(bx)$ for all $x \in \mathbb{F}^n$ and all $a, b \in \mathbb{F}$.

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**Exercise 13.** Show that $1x = x$ for all $x \in \mathbb{F}^n$.

**Exercise 14.** Show that $\lambda(x + y) = \lambda x + \lambda y$ for all $\lambda \in \mathbb{F}$ and all $x, y \in \mathbb{F}^n$.

**Exercise 15.** Show that $(a + b)x = ax + bx$ for all $a, b \in \mathbb{F}$ and all $x \in \mathbb{F}^n$.