Upper Division Tutoring Program Content Review

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Drop-In: Check Drop-In Schedule

Preparing for Math 110 Finals

Section I: Review Strategies and Problem Solving Strategies

These strategies can be useful for reviewed content for exams

- Concept Mapping
- Make a Cheat Sheet
- Take a Timed Practice Exam
- Self Monitoring
- Check Which Sections Are Most Problematic
- Practice Problems

Along with these broad strategies for review, when specifically solving practice problems, here are some tips along with questions to practice them with.

- Example Checking
- Definition and Theorem Clarifying
- Proof Sketching
- Self Explaining

We will now have you all work on example problems with specific strategies in mind

1. Self Explaining
   Gomez Midterm 2 #2: Let u, v be eigenvector of T such that $u + v$ is also an eigenvector or T. Prove that u and v have the same eigenvalue. (Hint: Contradiction)
2. Proof Sketching
   Gomez Final #2: Let T be a normal operator on a inner product space V. If T has eigenvalues 1,2 and 3, prove that there exists a vector v such that $||v|| = \sqrt{3}$ and $||Tv|| = \sqrt{14}$.
3. (Axler 8.B.2) Suppose $T \in \mathcal{L}(V)$ is invertible. Prove that $G(\lambda, T) = G\left(\frac{1}{\lambda}, T^{-1}\right)$ for every $\lambda \in \mathbb{F}$ with $\lambda \neq 0$.
4. (Axler 7.A.20) Suppose T is a normal operator on V. Suppose also that $v, w \in V$ satisfy the equations
   $$\|v\| = \|w\| = 2, \quad Tv = 3v, \quad Tw = 4w.$$
   Show that $\|T(v + w)\| = 10$.

Section II: Exam Taking Strategies

- Skim the Entire Exam
- Skip Around
- Write Down What you Know
- If you get stuck on a problem, move on to a different one for a bit

Here are some questions to practice these skills. You should write out the definitions of each object in the theorem statement and each assumption provided.

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5. Let the matrix representation of an operator $T \in \mathcal{L}(\mathbb{C}^3)$ be

$$M(T) = \begin{bmatrix} 2 & 4 & -3 \\ 0 & 2 & 0 \\ 0 & 3 & -1 \end{bmatrix}.$$

   Compute a basis for $M(T)$ such that the matrix representation is in Jordan Normal Form.

6. (Axler 6.C.12) Find a polynomial $p \in \mathbb{P}_3(\mathbb{R})$ such that $p(0) = 0$, $p'(0) = 0$, and

$$\int_0^1 \left(2 + 3x - p(x)\right)^2 \, dx$$

   is as small as possible.