Finals Review

Linear Algebra Done Right, 4th ed.

Exercise 1. (Gomez Midterm 2 #2) Let u, v be eigenvectors of T such that $u + v$ is also an eigenvector of T. Prove that u and v have the same eigenvalue. *(Hint: Contradiction)*

Exercise 2. (Gomez Final #2) Let T be a normal operator on an inner product space V. If T has eigenvalues $1, 2$, and 3, prove that there exists a vector v such that $\|v\| = \sqrt{3}$ and $\|Tv\| = \sqrt{14}$.

Exercise 3. (Axler 8.B.2) Suppose $T \in \mathcal{L}(V)$ is invertible. Prove that $G(\lambda, T) = G\left(\frac{1}{\lambda}, T^{-1}\right)$ for every $\lambda \in \mathbb{F}$ with $\lambda \neq 0$.

Exercise 4. (Axler 7.A.20) Suppose T is a normal operator on V. Suppose also that $v, w \in V$ satisfy the equations

$$\|v\| = \|w\| = 2, \quad Tv = 3v, \quad Tw = 4w.$$

Show that $\|T(v + w)\| = 10$.

Exercise 5. Let the matrix representation of an operator $T \in \mathcal{L}(\mathbb{C}^3)$ be

$$M(T) = \begin{bmatrix} 2 & 4 & -3 \\ 0 & 2 & 0 \\ 0 & 3 & -1 \end{bmatrix}.$$

Compute a basis for $\mathbb{C}^3$ such that the matrix representation is in Jordan Normal Form.

Answer: ____

Exercise 6. (Axler 6.C.12) Find a polynomial $p \in \mathcal{P}_3(\mathbb{R})$ such that $p(0) = 0$, $p'(0) = 0$, and

$$\int_0^1 (2 + 3x - p(x))^2 \, dx$$

is as small as possible.

Answer: ______________________________