

Exercises 1A: \mathbb{R}^n and \mathbb{C}^n *Linear Algebra Done Right, 4th ed.*

Exercise 1. Suppose a and b are real numbers, not both 0. Find real numbers c and d such that

$$\frac{1}{a+bi} = c+di.$$

Answer: _____

Exercise 2. Show that

$$\frac{-1+\sqrt{3}i}{2}$$

is a cube root of 1 (meaning that its cube equals 1).

Exercise 3. Find two distinct square roots of i .

Answer: _____

Exercise 4. Show that $\alpha + \beta = \beta + \alpha$ for all $\alpha, \beta \in \mathbb{C}$.

Exercise 5. Show that $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.

Exercise 6. Show that $(\alpha\beta)\lambda = \alpha(\beta\lambda)$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.

Exercise 7. Show that for every $\alpha \in \mathbb{C}$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha + \beta = 0$.

Exercise 8. Show that for every $\alpha \in \mathbb{C}$ with $\alpha \neq 0$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha\beta = 1$.

Exercise 9. Find $x \in \mathbb{R}^4$ such that

$$(4, -3, 1, 7) + 2x = (5, 9, -6, 8).$$

Answer:

Exercise 10. Explain why there does not exist $\lambda \in \mathbb{C}$ such that

$$\lambda(2 - 3i, 5 + 4i, -6 + 7i) = (12 - 5i, 7 + 22i, -32 - 9i).$$

Exercise 11. Show that $(x + y) + z = x + (y + z)$ for all $x, y, z \in \mathbb{F}^n$.

Exercise 12. Show that $(ab)x = a(bx)$ for all $x \in \mathbb{F}^n$ and all $a, b \in \mathbb{F}$.

Exercise 13. Show that $1x = x$ for all $x \in \mathbb{F}^n$.

Exercise 14. Show that $\lambda(x + y) = \lambda x + \lambda y$ for all $\lambda \in \mathbb{F}$ and all $x, y \in \mathbb{F}^n$.

Exercise 15. Show that $(a + b)x = ax + bx$ for all $a, b \in \mathbb{F}$ and all $x \in \mathbb{F}^n$.
