Signals & Systems, S0004E, Lab 3: Sampling

Due: May 22, 23:59

The purpose of this computer exercise is to familiarize ourselves with sampling of continuous-time signals, and the reconstruction of signals from their samples. Suppose that we have a continuous-time signal, $x(t)$, given as

$$x(t) = \left[\frac{3}{2} + \frac{3}{10}\sin(2\pi t) + \sin\left(\frac{2\pi}{3}t\right) - \sin\left(\frac{2\pi}{10}t\right)\right]\operatorname{sinc}(t);$$

where the sinc-functions is defined as

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}.$$

Exercise 1: In Matlab, plot the function $x(t)$ in the interval $-5 \leq t \leq 5$, using the `plot` function. Label the axes carefully. Please note that continuous-time waveforms are not possible to represent in Matlab. Instead use a very small sampling interval, the plot function will illustrate linear interpolation between the specified samples. Hint: the sinc function is implemented in Matlab, for more information run `help sinc`.

Exercise 2: Our intention is to sample the signal $x(t)$, to obtain the discrete-time signal

$$x[n] = x(nT_s), \qquad n = 0, \pm 1, \pm 2, \ldots$$

In order to avoid aliasing, and to surely enable reconstruction of the original signal, the sampling theorem has to be fulfilled. What is the bandwidth of $x(t)$ and what requirements do we have on the sampling interval T_s? Hint: if $y(t) = x_1(t)x_2(t)$, how can we express the bandwidth of $y(t)$ in terms of the bandwidth of $x_1(t)$ and $x_2(t)$.

Exercise 3: Construct the discrete-time sequence $x[n]$ and use `stem` to plot it. Choose suitable values of n. The value of T_s should be chosen close to its upper limit, while still fulfilling the sampling theorem.

Exercise 4: Argue/prove analytically that the signal $x(t)$ can be reconstructed from its samples $x[n]$ (assuming that the sampling theorem has been fulfilled) through the reconstruction formula,

$$x(t) = \sum_{n=-\infty}^{\infty} x[n]\operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right).$$

Exercise 5: Now, we would like to reconstruct the continuous-time signal, $x(t)$, using the formula derived above. As continuous-time signals cannot be represented in Matlab, reconstruct the signal at the same time instances as the ones used to illustrate $x(t)$ in exercise 1. We would like to study how the reconstruction evolves with the number of terms, plot the following

a) $x_0(t) = x[0]\operatorname{sinc}\left(\frac{t}{T_s}\right)$

b) $x_5(t) = \sum_{n=-2}^{2} x[n]\operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right)$

c) $x_{13}(t) = \sum_{n=-6}^{6} x[n]\operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right)$

d) $x_r(t) = \sum_{n=-k}^{k} x[n]\operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right)$, k large.

In each case, illustrate the reconstructed signal along with original signal $x(t)$, run `help hold`. Comments?