Lab 3

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1 Exercises

1.1 Exercise 1

Solved with the following two code blocks:
x(t):

```
function x = x(t)
    x = (3/2+3/10*sin(2*pi*t)+sin((2*pi)/3*t)-sin((2*pi)/10*t)).*sinc(t);
```

And for the plot:

```
function Ex1 = Ex1()
    t = -5:0.1:5;
    plot(t,x(t));
    xlabel('t');
    ylabel('x(t)');
```

See figure 1 for the result.

1.2 Exercise 2

We start by solving the spectrum for $x(t)$:

$$
\begin{aligned}
x(t) &= [\frac{3}{2}+\frac{3}{10}sin(2\pi t)+sin(\frac{2\pi}{3}t)-sin(\frac{2\pi}{10}t))]sinc(t)\\
&= [\frac{3}{2}+\frac{3}{10}\frac{1}{2j}(e^{j2\pi t}-e^{-j2\pi t})+\frac{1}{2j}(e^{j\frac{2\pi}{3}t}-e^{-j\frac{2\pi}{3}t})+\frac{1}{2j}(e^{j\frac{2\pi}{10}t}-e^{-j\frac{2\pi}{10}t}))]\frac{1}{\pi t}\frac{1}{2j}(e^{j\pi t}-e^{-j\pi t})\ \textit{(Euler's Formula)}\\
&= [\frac{3}{2}+\frac{3}{20j}(e^{j2\pi t}-e^{-j2\pi t})+\frac{1}{2j}(e^{j\frac{2}{3}\pi t}-e^{-j\frac{2}{3}\pi t})-\frac{1}{2j}(e^{j\frac{1}{5}\pi t}-e^{-j\frac{1}{5}\pi t})]\frac{1}{2j}(e^{j\pi t}-e^{-j\pi t})\\
&= \frac{3}{4j\pi t}(e^{j\pi t}-e^{-j\pi t})-\frac{3}{40\pi t}(e^{j\pi t}-e^{-j\pi t})(e^{j2\pi t}-e^{-j2\pi t})+\frac{1}{4\pi t}(e^{j\pi t}-e^{-j\pi t})(e^{j\frac{2}{3}\pi t}-e^{-j\frac{2}{3}\pi t})\\
&+\frac{1}{4\pi t}(e^{j\pi t}-e^{-j\pi t})(e^{j\frac{1}{5}\pi t}-e^{-j\frac{1}{5}\pi t})\\
&= \frac{3}{4j\pi t}(e^{j\pi t}-e^{-j\pi t})-\frac{3}{40\pi t}(e^{j\pi t}-e^{-j\pi t}+e^{3j\pi t}-e^{-3j\pi t}))+\frac{1}{4\pi t}(e^{j\pi t}-e^{-j\pi t}+e^{j\frac{5\pi}{3}t}-e^{-j\frac{5\pi}{3}t})\\
&+\frac{1}{4\pi t}(e^{j\pi t}-e^{-j\pi t}+e^{j\frac{6\pi}{5}t}-e^{-j\frac{6\pi}{5}t})
\end{aligned}
$$

From the last equation, we see that the maximum angular frequency is $w_{max} = 3\pi$ rad/s which, after converting, gives us$f_{max} = 1.5Hz$. The minimum angular frequency is $w_{min} = \frac{6}{5}\pi$ rad/s $\implies f_{min} = 0.6Hz$.
The bandwidth of $x(t)$ is then $f_{max} - f_{min} = 1.5 - 0.6 = 0.9Hz$.
The sampling theorem states that the sample rate must be atleast two times larger then our max frequency $1.5Hz$, hence sampling rate must be atleast $3Hz$. To achieve this rate of samples the time T_s must be $f = \frac{1}{T_s} \implies T_s = \frac{1}{f} = \frac{1}{3}$.

$\therefore T_s = \frac{1}{3}$s and the bandwidth $= 0.9Hz$

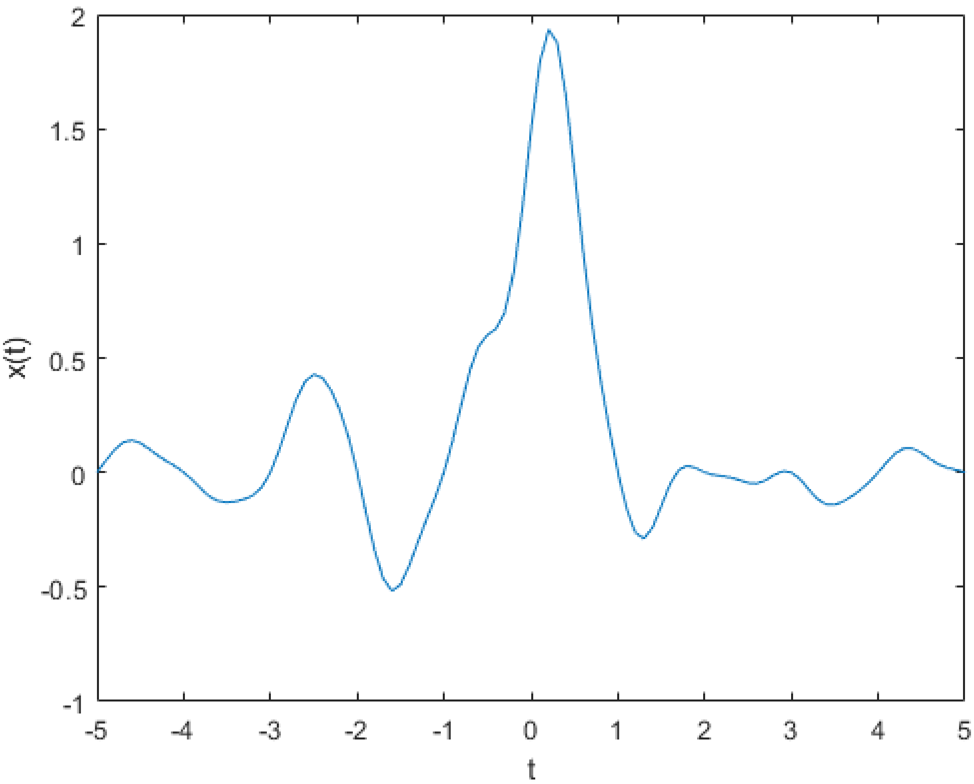

Figure 1: Exercise 1

1.3 Exercise 3

With the interval $T_s = \frac{1}{3}$ an almost identical graphical representation of the original signal is shown, see figure 2.

1.4 Exercise 4

The sampling theorem specifies a minimum-sampling rate where a properly sampled signal can be reconstructed from the samples. The rate $T_s = \frac{1}{4}$ that we solved in Exercise 2 will then be enough to reconstruct the signal.

1.5 Exercise 5

The command *hold* in Matlab helps with drawing these figures you are about to see. By allowing us to draw over a already present plot helps us showcase continuous-time signals by using two instances to plot multiple functions. We plot equation a, b, c, d in figure 3,4,5,6, respectively. We progressivly see a closer representation of $x(t)$ (Orange in figures 3 - 6) from exercise 1.For equation d in figure 6 we use the value $r = 1000$ which gives an almost exact representation of $x(t)$.

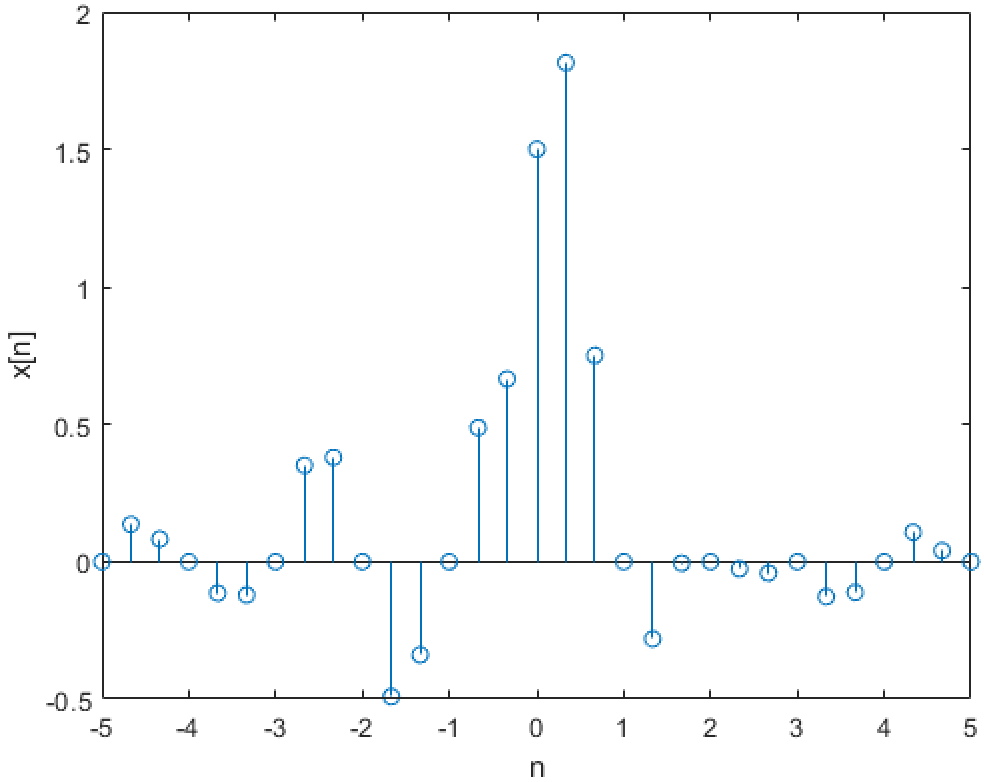

Figure 2: Exercise 3

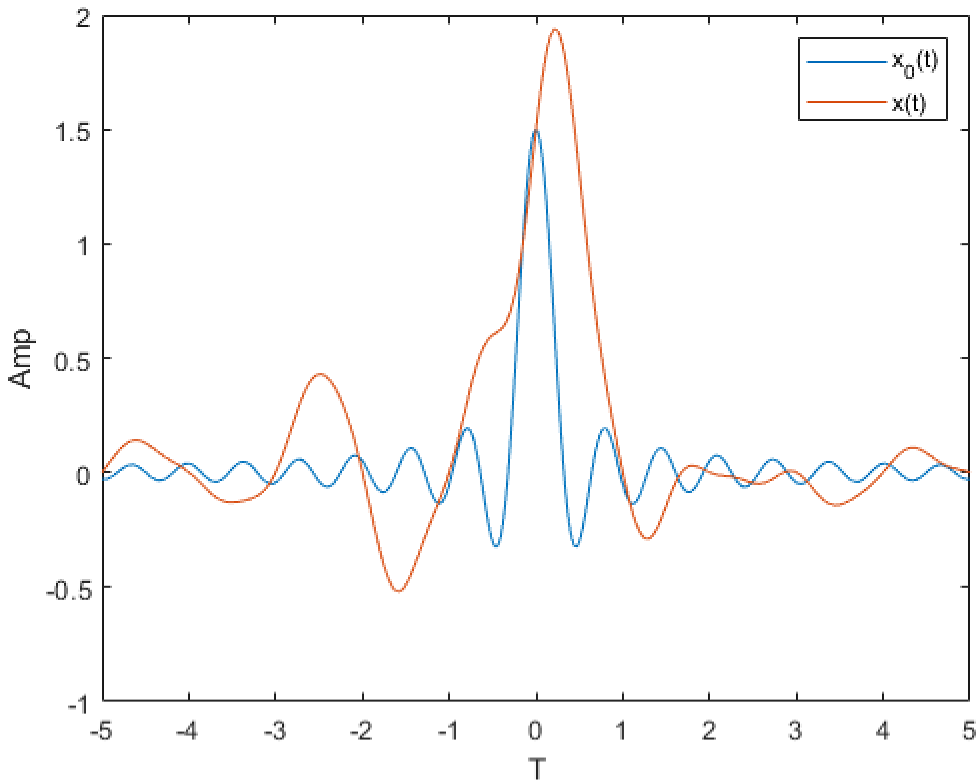

Figure 3: Exercise 5: a

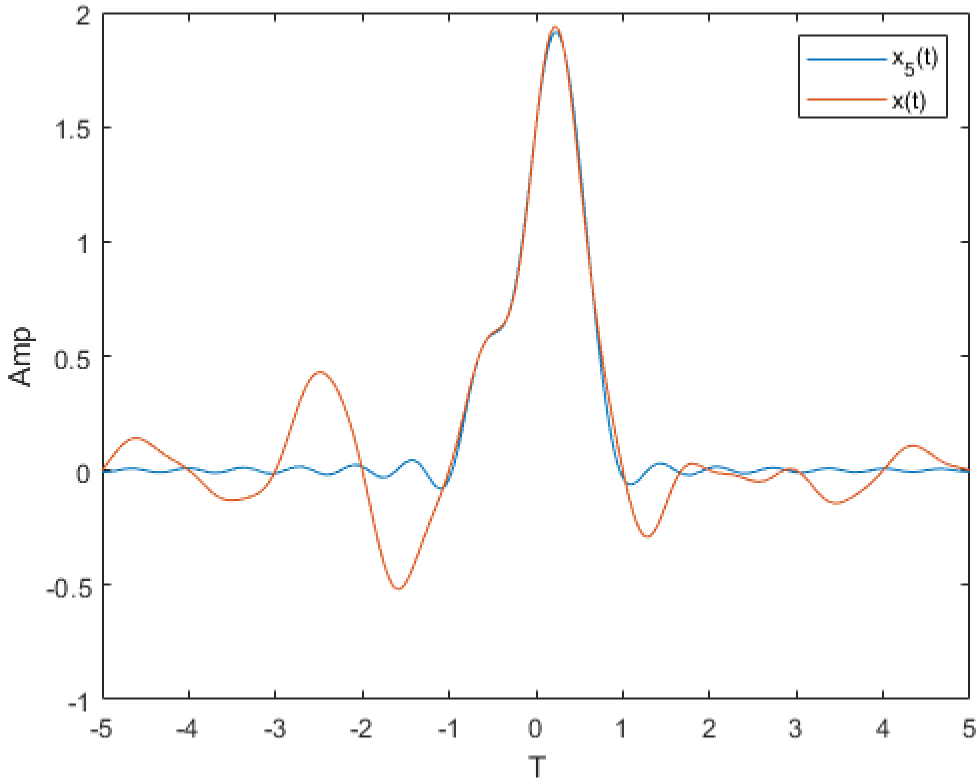

Figure 4: Exercise 5: b

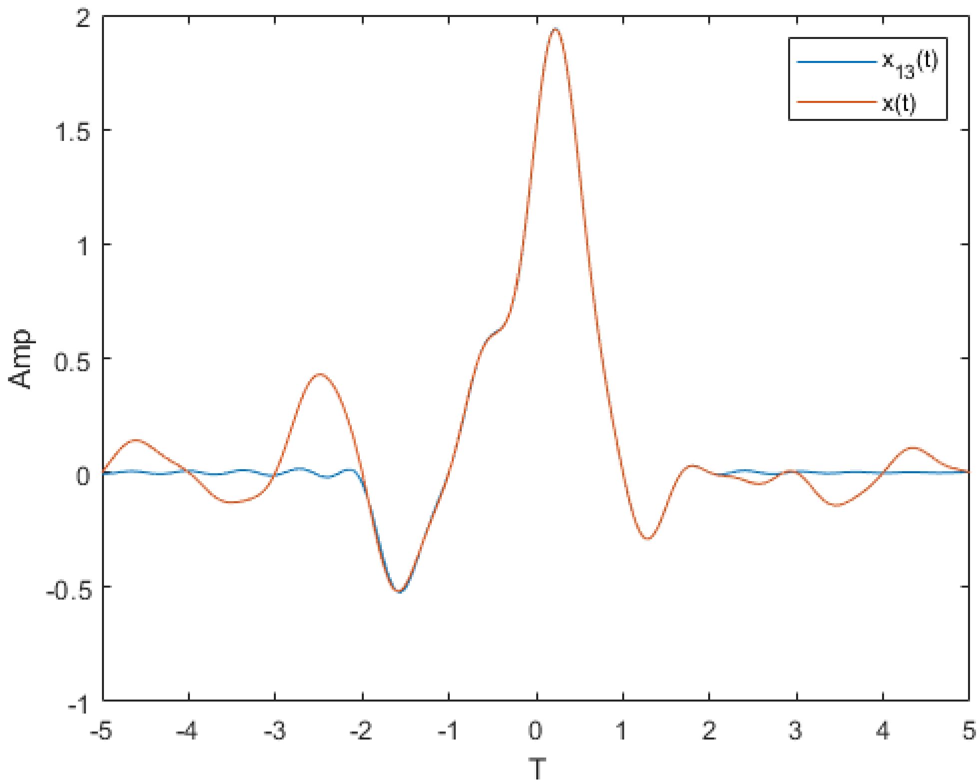

Figure 5: Exercise 5: c

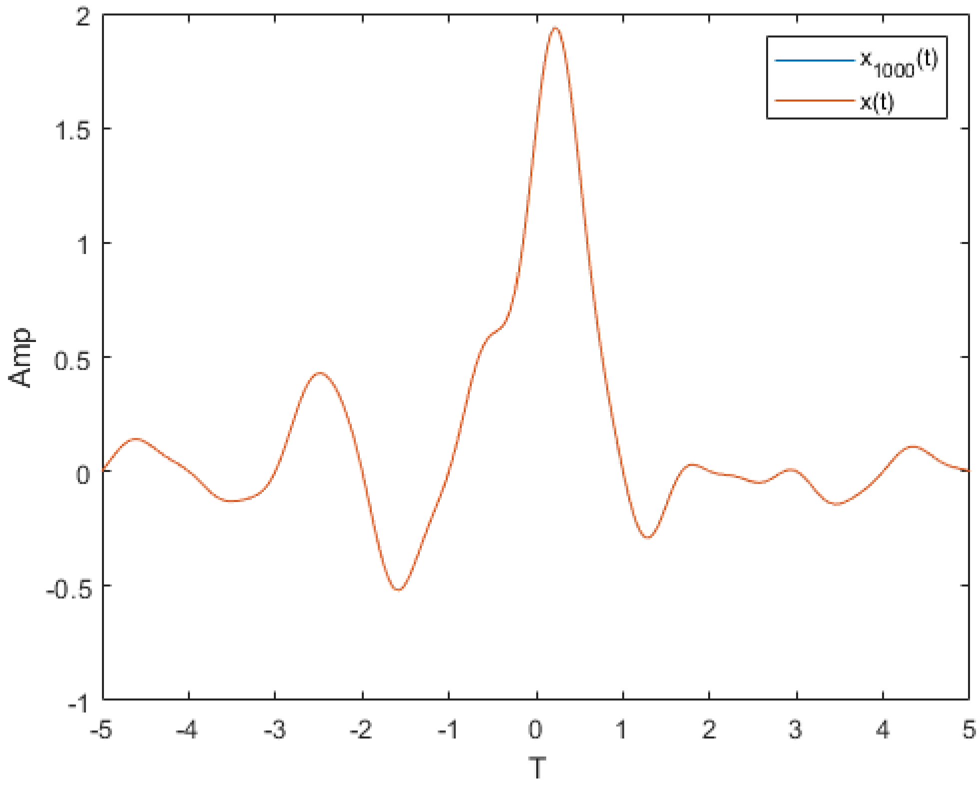

Figure 6: Exercise 5: d