Snowmass 白皮书:
如何定义量子场论?

作者: Mykola Dedushenko
arxiv: 2203.08053

Abstract

本文对七十年来严格定义和构造量子场论的文献进行了回顾。对现代物理文献中的思想和构造进行比较之后，可以得出结论：现有的量子场论公理系统均无法涵盖所有的物理情景。制定一个完整的量子场论定义是一个仍未解决的问题，对物理学家和数学家均具有重要意义。

Junkai-Wang@NJU

1 引言

量子场论这一学科即将迎来它的百年庆典。量子场论的起源可以追溯到文献 [1,2]，以及随后的 [3,4] 等许多论文。量子场论主要发源于 20 世纪初将年轻的狭义相对论与年轻的量子力学相结合的迫切需求，并在接下来的二十年里发展出了早期的微扰量子场论。有关量子场论早期岁月的有趣历史可以在 Weinberg 优秀教材的第一章 [5] 找到，而其必须克服的一大挑战是紫外发散问题，最终使得 Dyson、Feynman、Schwinger 和 Tomonaga 在四十年代末发展出了重整化技术 [6-19]。

作为一个全新领域，以及它奇特的重整化机制，量子场论迫切需要一套清晰的规则或公理，从而以逻辑推导的方式得到其他一切内容。这些规则将会将量子场论这个学科" 提升" 为数学的一个子领域，但这也是为了克服微扰拉格朗日量技术的局限性所必需的。因此从五十年代开始，各种量子场论公理系统开始涌现。

在本文中, 我们将以 Wightman 的公理体系 [20-23] 为起点展开讨论, 但这会无可避免地会忽略一些早期尝试，像是 Heisenberg 的 S 矩阵计划 [24]、与之有关的 Bogolyubov-Medvedev-Polivanov 的扩展（非在壳）S 矩阵方法 [25]，以及 Lehmann-Symanzik-Zimmermann 的公理体系 [26,27]（不过 LSZ 约化公式已成为标准量子场论形式体系的一部分)。其他一些方法如今仍在发展中。在这里, 我们想简述这些问题，并讨论各种公理系统的适用范围。我们要指出的一点是，目前存在的定义都没有涵盖物理学文献中出现的量子场论概念的全部范围。

作为物理学的一个子领域，实现量子场论的严格公理化具有两方面的哲学动机。一方面，这为数学研究提供了一个起点，从一团混乱的现实中提取出抽象真理。另一方面，其表明人们对该学科的物理理解已经足够成熟。实际上，大多数其他物理学子领域（所有的" 非量子" 物理学和非相对论量子力学）已经经历了这个过程。正如我们即将看到的，量子场论没有一个清晰而普适的公理集, 这一事实可能表明人们对量子场论的物理理解仍然不够。因此我们认为，对于物理学家和数学家来说，定义量子场论是一个不容忽视的挑战。在接下来的文章中，我们将简述现在已有的方案。

作者注: 欢迎指正！

2 现有的公理体系

2.1 着眼关联函数的进路 (Correlator-focused approaches)

Wightman 公理. 迄今为止仍受关注的早期公理系统之一是 Wightman 公理 [20-23]（另参阅书籍 [25,28,29])。这些公理将场视为算子值的有界分布, 并形式化了它们的乘积的期望值（"Wightman 函数"）。公理从假设 W0 开始，即相对论不变性（以及 Wigner 分类 [30][1]，也可参见 [31-34]）：物理希尔伯特空间是 Poincare 群的一个幺正表示。这个假设由谱条件（能量-动量谱位于上半封闭光锥内）以及真空态 $\Psi_0 \in \mathcal{H}$ 的唯一性和 Poincare 不变性来补充。公理 W1 规定存在一组由有限分

[1] Wigner 分类通常作为描述平直空间中满足庞加莱不变性的量子场论的微扰描述的良好起点，参见 [5]。

布[2]给出的场 $\varphi_i[f]$，其值为定义在希尔伯特空间 $\mathcal{H}$ 的一个稠密子集 D（其中包括真空态）上并且保持该稠密子集的算子。子集 D 被假定为 Poincare 不变的。然后，公理 W2 规定了场对 Poincare 群的协变性，W3 要求局域性（也称为微观因果性），即类空间隔场的（反）对易性。满足 W0-W3 的量子场论还要满足真空的循环条件 (cyclicity of the vaccum)：对于所有可能的 n 和 f_i，形如 $\varphi_1[f_1]\dots\varphi_n[f_n]\Psi_0$ 的向量张成的空间在 $\mathcal{H}$ 中是稠密的。后一个条件保证了理论中存在足够数量的场。

这些公理导出了一系列重要结果,例如 Wightman 函数 $\mathcal{W}(x_1,\dots,x_n)=\langle\Psi_0|\varphi_1(x_1)\dots\varphi_n(x_n)|\Psi_0\rangle$ 的各种性质，如相对论变换、谱条件、厄米性条件、定域对易性、正定性和集团分解，以及解析延拓 [35,36]。其中的大多数在 [20] 中得到证明，并在 [28] 中得到了回顾，还包括了从 Wightman 函数中重建 (W0)-(W3) 信息的重构定理 (reconstruction theorem)(仅适用于标量场的情形)。其他重要的参考文献（尤其是关于集团分解的文献。集团分解定理指出当点集 $\{x_1,\dots,x_k\}$ 和 $\{x_{k+1},\dots,x_n\}$ 无限远时，$\mathcal{W}(x_1,\dots,x_n)$ 趋于 $\mathcal{W}(x_1,\dots,x_k)\mathcal{W}(x_{k+1},\dots,x_n)$）包括 [37-42]，另参阅 [43] 中有关重构定理的内容。从这些公理还可以推导出 CPT 定理 [44,45] 和自旋-统计关系 [46,47] 等一些经典结果 [48-52]。与时空区域相关的可观测量的局域代数 $\mathfrak{A}(\Omega)$（这一点在下文的 Haag-Kastler 公理中将占据重要地位）的重要性在 Haag 的工作中已经与 Wightman 公理相关地进行了讨论 [53]。关于它们的进一步性质、不可约性问题，参阅 [40,54,55]。关于著名的 Reeh-Schlieder 定理可以参阅 [56]，这一定理断言了相对于 $\mathfrak{A}(\Omega)$ 而言的真空循环性。对于其他重要方面，如 Haag 定理、定域场的 Borchers 类、Haag-Ruelle 理论等，参阅 [57-64]。

欧氏场论公理. Osterwalder-Schrader (OS) 公理 [65-67] 以及被 Glimm-Jaffe 修改的公理版本 (GJ) [68] 和 Nelson 公理 [69],粗略地提供了 Wightman 公理的欧几里得版本[3]。和 Wightman 公理一样,OS 公理同样基于欧几里得情况下对关联函数进行形式化的想法，此时的关联函数称为 Schwinger 函数 $S_n(x_1,\dots,x_n)=\langle\phi_1(x_1)\dots\phi_n(x_n)\rangle$。OS 公理包括：OS0: S_n 作为分布的可调性 (temperedness)；OS1: 理论的欧几里得协变性；OS2: 反射正性 (Reflection positivity)；OS3: 对称性（在费米子情况下为反对称性）；OS4: 集团分解。增加一个微妙的线性增长条件的附加要求之后，OS 定理 [66]（有时称为 OS reconstruction theorem）说明 Schwinger 函数可以解析延拓到闵可夫斯基度规，以在延拓之后满足 Wightman 公理（另见 Zinoviev 的修改 [77]）。Glimm-Jaffe 公理以类似的方式形式化了生成泛函 $S[f]=\left\langle e^{\phi[f]}\right\rangle\equiv\int e^{\phi[f]}\mathrm{d}\mu$，其中 $\mathrm{d}\mu$ 是分布空间 ϕ 上的测度，被要求的解析性、正则性（以某种增长界形式的要求）、欧几里得协变性、反射正性和时间平移的遍历性。这些公理蕴含了具有增长条件的 OS 公理，从而也蕴含了 Wightman 公理。最后，Nelson 公理 [69,69,78-80] 以类似的方式认真考虑了测度论的方法，并且重要的是要求 Markov 性，这实质上要求局域性并暗示了系统在某个区域的状态被赋予该区域的边界（特别地，希尔伯特空间也被赋予边界）。Nelson 还要求遍历性，并证明了 Wightman 公理在解析延拓到闵可夫斯基空间后成立（“Nelson 的重构定理”，参见 B.Simon 的书 [81] 对此和其他主题的综述；另见 [82]）。还要注意 [83] 将 OS 重构扩展到平衡统计力学的结果（另见 [84]），以及 [85] 在欧几里得 QFT 的同一背景中研究闵可夫斯基群表示的

[2]这些分布定义在光滑的 Schwarz 函数 $f\in\mathcal{S}$ 上，这样可以避免紫外发散问题。

[3]关于欧几里得量子场论的源头，可以参考 [70-75] 以及 [76] 中的参考文献。

重构结果。最近的一项工作 [86] 还提供了 W 和 OS 公理（以及重构定理）的推广，据称适用于规范理论。

构造性场论 (Constructive Field Theory). 到目前为止，讨论的公理完全是非构造性的，并且需要额外的工作来提供具体例子。最初，只有自由场（包括在 [87] 中定义的广义自由场）和与自由场相关的各种可解模型已知满足 Wightman 公理（并且自然地满足其他公理系统）。这一点导致了在 20 世纪 60 年代出现了构造性场论（CQFT）这一研究领域 [88-92]，其主要目标是提供严格的相互作用量子场论的例子。关于这个领域的广泛综述（截至 1987 年）可以在书籍 [68] 中找到（重点关注欧几里得路径积分方法，另参阅其他有关该主题的书籍：[93-96]），也参阅 [97-101]。该领域的早期设想可以在 [102,103] 中找到，以及稍后的综述 [104]；一篇总结 CQFT 在 2000 年以来一些成功的综述文章在 [105] 中，以及稍微更详细的综述文章在 [106] 中。最近的综述包括 [107-109] 和一篇讲义 [110]。通过从六十年代末开始的大量工作，成功地在严格构造和研究 2 d 标量理论与任意多项式相互作用（即所谓的 $\mathcal{P}(\phi)_2$ 理论）[91,111-130]（另参阅书籍 [81]、最近的论文 [131,132] 和有关其他势能的论文 [133-136]），三维 $\lambda\phi^4$ 理论 [137-145]（另参阅较新文献 [146-150]），Gross-Neveu 理论 [151-156]，Thirring 模型 [157-161]（特别地，这些理论都被证明满足 Wightman 公理）；其他存在费米子的理论包括 2D 和 3D 的 Yukawa 模型 [65,162-186]（另参阅 [187]）和一些超对称模型 [188-194]。在欧几里得理论中，随机游走表示 (Random walks representation) 被引入 [73,75]，并在之后得到发展 [195-198] 和各种应用 [145,199]，其中最重要的是证明了 $d \geq 5$ 时 ϕ^4 理论的平凡性 [200-202]（参阅书籍 [203]）。四维情况更加微妙 [204-213]，直到最近才在 [214] 中得到解决（另参阅讲义 [215]），证实了 ϕ^4_4 模型的平凡性。[4]格点正规化在规范理论中发挥作用 [217-219]，例如参阅 [220] 及其中的许多引用文献，特别是 Balaban 的作品 [221-240]，也见 [241-243] 中 Balaban 对重整化群方法的重新审视（以 ϕ^4 理论作为示例）和 [244-246]。[232,234] 和 [247]（使用不同方法）的结果对于解决四维 Yang-Mills 问题[5]取得了重要进展，另参阅 [248,249] 中的讨论。

2.2 代数量子场论 (Algebraic Quantum Field Theory)

Haag-Kastler 公理 代数量子场论（Algebraic QFT, AQFT）是另一种对量子场论进行公理化的方法，它弱化了场的概念，而是首先在不指定任何希尔伯特空间的情况下形式化可观测量的代数。这一主题是从 Haag-Kastler（HK）公理的形成而开始的 [250]（其中的一些要素出现在早期作品中，如 [39,40,53,251-254]，另参阅 [250] 的参考文献 [2]）。另外也有许多关于 AQFT [255-261] 和算子代数 [262-280] 的书籍和专著，以供参阅。在更现代的文献中，我们推荐一本文集 [281]，一篇简明综述 [282]，书籍和专著 [283-285] 以及相关文献 [286]。关键点和文献也在 [287] 中得到总结。HK 公理（有时称为 Araki-Haag-Kastler 公理）涉及平坦闵可夫斯基时空中的相对论性定域幺正量子场

[4] “平凡” (trivial) 意味着“自由” (free) 或“高斯” (gaussian)，并且这一陈述是关于在某个特定积分维度下的 UV 完备模型 (即移除截断) 的。当然，这并没有阻止具有截断的模型成为非平凡的有效场论，而且 $d = 4 - \epsilon$[216] 并不包含在这些陈述中。

[5] 有趣的是，在 70 年代和 80 年代的快速进展之后，构造性场论这一领域已经远离主流；尽管它涉及理论物理中最著名的问题之一 (译者注: 即四维量子杨米尔斯理论)，如今许多年轻人甚至都不知道这个领域的存在。作者认为，随着越来越多的数学家开始重新思考量子场论，这种状况将在未来发生改变。

论。每个因果闭合 (casual closed)[6]可观测量子集 U 对应一个可观测量的 C^*-代数 $\mathfrak{A}(U)$。在包含关系 $U_1 \subset U_2$ 下，存在 C^*-代数[7]的包含关系 $\mathfrak{A}(U_1) \subset \mathfrak{A}(U_2)$（这个性质称为保序性 (isotony)），并且这个关系是函子性的，即保持组合关系（这一信息通常称为局域代数网；我们可以说 $\mathfrak{A}(\cdot)$ 是一个保序预余层 (isotonic pre-cosheaf)，除非它定义在因果闭合子集而不是开集上）。因果定域性的要求为: 当 U_1 和 U_2 类空分离时，$\mathfrak{A}(U_1)$ 和 $\mathfrak{A}(U_2)$ 彼此对易 (在 $\mathfrak{A}(U)$ 内，其中 $U_i \subset U$)。此外通常还要求保证庞加莱协变性，即对于任意庞加莱变换 p，存在一个态射 $\alpha(p) : \mathfrak{A}(U) \to \mathfrak{A}(pU)$。另一个要求是由于线性动力学的存在，即时间片公理 (time slice theorem)，它要求如果 $U_1 \subset U_2$，且 U_1 包含 U_2 的一个柯西面，那么 $\mathfrak{A}(U_1) \to \mathfrak{A}(U_2)$ 是一个同构映射。人们通常还要求能谱的正定性（即时间平移算符的正定性)。

在代数形式化体系下，量子态被理解为满足所有 $A \in \mathfrak{A}$ 的正定条件 $\omega\left(A^* A\right) \geq 0$ 的线性映射 $\omega : \mathfrak{A} \to \mathbb{C}$，其中 $\omega(A)$ 是 A 的期望值。可以通过 GNS 构造从态 ω 获得代数 $\mathfrak{A}$ 的有界算符 $\pi_\omega(\mathfrak{A}(U)) \subset \mathfrak{B}(\mathcal{H})$ 的忠实表示 π_ω[289,290]。代数 $\mathfrak{B}(\mathcal{H})$ 有两个有用的闭 $*$-子代数的概念：上述提到的 C^*-代数（在范数拓扑下是闭的），以及 von Neumann 代数 [291]（它在三个拓扑：强算符拓扑、弱 $*$ 拓扑和弱算符拓扑中都是闭的)，参阅 [255] 或其他上面提到的任一教材。有时 $\mathfrak{A}(U)$ 被视为一个 C^*-代数，但很常见的做法是关注 $\mathcal{R}(U) = (\mathfrak{A}(U))''$，它是包含 $\mathfrak{A}(U)$ 的最小冯·诺依曼代数，其中 $(\cdot)'$ 表示在 $\mathfrak{B}(\mathcal{H})$ 内的共轭转置[8]。人们经常谈论形成真空表示网 $\mathcal{R}(U)$，参见综述 [292]。这与 von Neumann 代数的丰富理论相关 [293-299]（参见文集 [300] 和之前引用的教材)，特别是诸如：分解 [299] 成 I 型、II 型、III 型 [293] 因子，取决于 H 中不变子空间上的投影算符维度谱是否是离散的、包含某个区间中的所有整数、连续的或仅由 0 和 ∞ 组成，并且一个深刻结果: 在 QFT 中处理的是 III 型因子 [301-303]；模理论 (modular theory) 或 Tomita-Takesaki 理论（由 Tomita [304] 引入并由 Takesaki [305] 解释，另参阅 [275] 以及 Borchers [306] 和 Summers [307] 的论述，或书籍 [255]）提供了 Type III 因子的结构理论，并与 KMS 态等其他主题有关（参见 [308-310])。最近人们对模理论的兴趣很浓厚，因为它与 QFT 中的纠缠性质有关 (参见 [311] 作为文献研究的切入点)。

和其他方法类似，可以在 AQFT 的公理体系下研究一般的结构属性，例如散射态的存在性 [40,252]、超选择定则[9][315,316]（另参阅，例如，[317-320])、自旋-统计定理和 CPT 定理（参见 [321-323]，也可以参见 [324] 中的拓扑版本)、Reeh-Schlieder 定理 [56]（之前已提到过，但传统上这个定理被视为 AQFT 技术的一部分)、Goldstone 定理 [325,326]。AQFT 与 Wightman 公理的一个区别应该是明显的：尽管 Wightman 公理不要求算符的有界性（例如，动量算符具有无界谱)，但这是一种理想模型。任何现实的实验都涉及到具有有限值范围的设备，因此任何结果都应该可以由仅含有界算符的理论以任意精度预测，就像在 AQFT 框架中一样[10]。在与 Wightman 公理的联

[6]关于因果闭合性 (casually closedness) 的讨论，请参见 [288]。

[7]C 代数由 Gelfand 和 Naimark 提出, 请参考上面的参考文献和关于 C^* 代数的文献 [256]。简而言之, C^* 代数由以下信息描述: 含有满足特定性质的对合 (involution) 操作 $*$ 的 $\mathbb{C}$-代数 (这构成一个 $*$-代数)；范数 $|A|$, 满足 $|AB| \leq |A||B|$ 和 $|A^*| = |A|$；相对于 $|\cdot|$ 诱导的拓扑是完备的（Banach 代数或 B^* 代数)；C^* 性质 $|A^* A| = |A|\,|A^*|$。

[8]在算子代数的领域中，“X 的共轭”意味着与 X 相互交换的所有元素，而在数学的其他领域中，这个概念被称为中心化子 (centralizer)。

[9]正如文献 [250] 中所示，超选择扇区 (superselection sector) 的概念 [312,313] 在 AQFT 的发展中一直很重要，也参阅综述 [314]。这一想法是不同的超选择扇区来自于同一个代数结构的不等价表示。

[10]将无界算子替换为有界算子以绕过量子力学里的技术性问题的想法，可以追溯到冯·诺依曼的工作，而 I.Segal 建议

系讨论中，有两个问题：其一是用紧支撑测试函数来涂抹 (smear out) 的 Wightman 场，是否总是可以找到对应的自伴有界算符，其二是从有界算符的代数网出发，是否可以通过将区域收缩到点极限的过程得到 Wightman 场（这些问题在例如文献 [327-333] 中进行了研究）。

微扰代数量子场论 (Perturbative AQFT). 微扰 AQFT 放弃了有界性、C^* 和 von Neumann 条件的要求，而是研究形式级数组成的 *-代数。相关综述包括文献 [284,334,335]，另参阅书籍 [336] 和综述 [337,338]。在这个背景下，与此相关的因果微扰理论的几个参考文献包括：[339-345]，书籍 [346-348] 和综述 [349]。关于微扰 AQFT 形式体系的最新一批论文是 [350-362]（包括 [353] 关于 $1/N$ 展开的论文）和 [363]（另参阅 [364] 中的评论）。另参阅 [365-369] 和 [284, 第 7 章] 关于 Batalin-Vilkovisky 形式体系的作用，以及 [370-371] 关于与形变量子化的关系的文献。

弯曲时空代数量子场论 (AQFT in curved space). AQFT 在弯曲空间中的应用将量子场论的适用性推向了极限[11]。它带来了新的物理现象（如粒子产生效应 [377]、霍金效应 [378]，以及前驱波 (early precursor)[379-381]、安鲁效应 (Fulling-Davies-Unruh effect)[382-384)]。这些现象通常源于庞加莱不变性的缺失，因此没有明确真空、没有粒子诠释、没有动量空间表示等。从闵可夫斯基度规到欧几里得度规的延拓也通常是不可行的，因此费曼传播子的选择也不唯一。所有这些微妙之处都使得传统的基于粒子的技术面临风险，并且很早人们就认识到[12]：将 AQFT 框架推广到一般的弯曲背景情形应当是正确的进路[13]。到了 80 年代，这种方法的某种版本已经出现 [386-391]，但它存在一些问题：只能描述自由场；在对复合算符（如能动张量 [394] 或一般的 Wick 多项式）进行重整化时存在减除奇性 (subtracting sigularity)[14]的问题，而对这一问题的解答依赖于参考准自由 (quasi-free) 的 Hadamard 态的选择。这阻碍了相互作用情形下的反作用 (backreaction) 研究和建立一致的微扰理论。要求局域性和协变性 [394] 最终将有助于解决这些问题。而真正的进展始于 90 年代，当时人们意识到，微定域 (microlocal) 分析对于分布的奇异性有更精细的控制，并以更系统的方式克服了这些问题 [395-397]。文献 [398-407] 研究了微定域这一方面，特别是：形式化了微定域的谱条件，发展了适当的（定域和协变的）Wick 多项式的概念，包括协变守恒的应变张量的构造，并将重整化的模糊性减少到由局部引力抵消项产生的模糊性。在经典背景上讨论的 QFT 中，引力抵消项（以及更一般的背景抵消项）是普遍存在的，它们导致基本的模糊性和正规化方案依赖性，稍后将提到。关于弯曲空间量子场论有许多书籍、专著和综述 [394,408-415]，特别是参考文献 [416] 中的最新简介（及后续文章 [417]）。在这个背景下，人们通常谈论全局双曲时空[15]（参考 [418] 以了解与此条件偏离的情形）。[400,419,420] 提出了全局双曲时空上的 AQFT 公理，另参阅综述 [414,421-425]，特别是 [422,423] 的历史简述，以及综述 [424] 和文集 [414] 中的更多技术细节。

在 QFT 的情形使用相同的方法 [253]。

[11] 这里我们将牛顿常数设为 0，因此背景是经典且非动力学的。

[12] 参见戴森著名的论文“错失的机会”的第 6 节 [385]。

[13] 另一种可行的进路，即欧几里得路径积分，将在稍后讨论。

[14] 两点函数的奇性结构在 [392,393] 中进行了研究。

[15] 一个伪闵可夫斯基时空 (洛伦兹指标) 在全局上是双曲的，如果它没有闭合的因果曲线，并且对于任意两点，任意一点的因果过去与另外任意一点的因果未来的交集是紧致的。

这些公理通常被称为定域协变量子场论 (locally covariant quantum field theory, LCQFT) 公理。它们与 Haag-Kastler 公理类似，但有重要的区别。再次，存在一个 C^* 代数网络，但不仅仅是在单个全局双曲时空 M 和时空的因果的全局双曲子集上；相反，它同时定义在所有全局双曲 d 维流形上，并具有对等距嵌入而言的自然定域协变性 (natural local covariance)[16]。显然，庞加莱协变性被舍弃了，而 Haag-Kastler 体系中的其他条件被它们的局部协变类似物所取代。[322,429,430] 研究了弯曲空间上的超选择定域和自旋-统计关系，可以进一步参见 [431-439]。[440] 介绍了一种在弯曲时空 AQFT 形式化 OPE 的替代方法。通过微扰论和重整化建立具体的相互作用模型的构造过程，可以参见原始文章和综述 [284, 334, 335, 363, 399, 401, 405, 406, 441]。[442] 探讨了如何将量子的杨-米尔斯理论构造为微扰 AQFT，另请参见 [366]。有关规范理论的其他参考文献包括 [443-448] 等。

动力学 C* 代数 (Dynamical C* algebras). 最近的一系列论文 [449-456] 正在开发一种新颖的 C^* 代数方法来研究 QFT。它基于场论的拉格朗日体系，可以称为" 构造性 AQFT"(Constructive AQFT)。给定一个拉格朗日量 L，这一方法产生一个具体的 C^* 代数 $\mathcal{A}_L$，被称为动力学 C^* 代数 (Dynamical C* algebra)。这一输出过程遵循 Haag-Kastler 公理，同时还吸纳了微扰 QFT 的思想。

同伦代数量子场论 (Homotopical AQFT). 微扰规范理论在拓扑平凡的扇区 (sector) 上存在。然而，拓扑效应，如瞬子，会破坏 LCQFT 的某些公理：违反保序性 (isotony)，无法从局域重构出全局代数[17]。这些事实在一些讲义中有解释，例如 [457-459]，另参阅 [445,460]。解决这个问题的一种方法是将规范轨道空间（构型空间）替换为相应的规范群胚 (gauge groupoid)（一个范畴，其对象是丛-联络构成的对，其态射是规范等价关系）给出的叠 (stack, 即 2-sheaf)。相应地，通常的" 场上的函数的量子代数" 典型进路被适当的同伦[18]微分分次代数 (dg-algebra) 取代。在这种推广方法中（实践者称之为同伦 LCQFT），我们得到的不再是局部协变的 C^*-代数网，而是它们的同伦微分版本。这些结构目前正在研究中，参见综述 [461,462]，专著 [463] 和发展该领域的论文 [464-476]（也见于 [477]）。

Haag 对偶和 DHR 方案. Doplicher、Haag 和 Roberts（DHR）研究了全局对称性及其在 AQFT（特别地，超选择扇区）中的作用 [315,316,478,479]。为了包含规范理论，他们提出了修改后的定域量子场论规则 [259,480,481]，建议不仅要考虑闵可夫斯基空间中的有界区域，还要考虑无限光锥 (infinite cone)。另一种方法在 [482-484] 中正在发展，这种方法的核心是 Haag 对偶性的破坏 $\mathfrak{A}(U) = \mathfrak{A}(U')'$ [255, 485]（这些作者还考虑了广义对称性和相关联的扩展算符 (extended operators)，它们使得 Haag 对偶性被破坏，也参阅 [320,486]）。

[16]关于“相同物理在所有时空中”的原则（Same Physics in All Spacetimes, SPASs），可以参考 [425-428]。

[17]我们没有在本综述中描述这一点，但大多数参考文献对此进行了讨论。

[18]在这些文本中，" 同伦" 一词意味着一系列关系，如可交换性 (commutativity) 或结合性 (associativity)，这些性质直到高阶同伦都成立。

因子化代数和欧氏微扰代数量子场论 (Factorization algebras and Euclidean perturbative AQFT). 因子化代数 (Factorization algebras, FA) 是另一种欧几里得微扰 QFT 的方案，它与 AQFT 的哲学框架具有类似的精神。因子化代数的概念可以追溯到 [487,488]。[489-491] 发展了 QFT 的微扰重整化，以及通过因子化代数进行形式化（另参阅 [492]）。表面上因子化代数的思想类似于 AQFT 中的代数网，而 [493] 对因子化代数方法和微扰 AQFT 进行了比较，得出了两者之间的密切关系——至少自由理论显示它们是等价的。相同的作者在稍后的一篇论文 [494] 将微扰 AQFT 和 FA 中的可观测量联系起来。[495] 的一个普遍结果在自然的前提假设下概要地展示了它们的等价性，在洛伦兹定向且时间定向的全局双曲空间 (Lorentzian oriented time-oriented globally hyperbolic spaces) 上考虑了因子化代数。因此，因子化代数方法也许提供了一种替代视角；从技术上来说，它是一种完全不同的构建具体模型的方法。使用这种框架的一些论文包括 [496-511]，以及 [512-515]。[516] 提议这种方法与 [517] 之间存在紧密联系。

2.3 Atiah-Segal 进路, 即函子场论 (Atiah-Segal-like approach, or Functorial Field Theory)

继 Atiyah-Segal 对 TQFT 的公理化 [518]（以及其许多成功应用，例如对完全扩展 (fully extended) 的 TQFT 的分类 [488]）以及早先在 CFT 中的工作 [519,520] 的思想，G.Segal 在一系列讲座中 [521] 提出了另一组公理，这些公理被认为定义了一般的欧几里德量子场论。类似的公理曾被 Stolz 和 Teichner 使用 [522,523]，而且显然也被 Kontsevich 考虑过（未发表）。这些想法有时被称为函子量子场论（Functorial Field Theory, FFT）[524]，尽管该名称有些误用，因为之前讨论的局部协变 AQFT 也被定义为适当的范畴之间（全局双曲空间和 C^*-代数）的函子。然而，为了明确起见，我们在这里仍然使用 FFT 这个术语，但值得指出的是一些作者 [525-527] 将其称为几何场论，因为它依赖于时空的一些几何数据，例如度规。持有这些观点的作者似乎已经认真地开始发展几何 FFT 的思想，并声称已经对完全扩展 (fully extended) 的几何 FFT 进行了定义甚至分类（类似于 [488] 中的配边 (cobordism) 假设）[526,527]。最近，Segal 和 Kontsevich 使用 FFT 框架 [528] 将 Riemann 流形上的 QFT 的定义扩展到“允许的”(allowable) 复度规, 作为欧几里德场论和洛伦兹场论之间的桥梁。总的来说，过去十年来，非拓扑量子场论的函子量子场论哲学已经蓬勃发展[19]，尽管致力于非拓扑函子场论的论文数量仍相对较少[20]。我们应该指出，[530-533] 开发了在 CW 复形（作为时空的离散）上进行 FFT 的方法。FFT 的主要思想是，量子场论是从几何协边范畴 (Geometric Bordism Category) 到拓扑线性空间范畴的一个函子（见 [528]）。这里的函子性编码了由定域性得出的粘合公理 (gluing axiom)，意味着时空可以由各组分粘合而成，组分之间只通过边界进行信息交换。换言之，每个组分上的 FFT 给出了一个态矢量 (或态余矢)，这些态矢生活在线性空间的张量积上，这些线性空间被指定给了其边界组分，而粘合（至少在没有角落 (corner) 的情况下）通过组合矢量和余矢量来完成。本质上讲，这正是 70 年代 Nelson 注意到的欧几里德路径

[19]参阅 [529, Section 2]，这篇文章的作者从 FFT 的范式来思考 QFT。在文献中可以找到更多类似的例子。

[20]另一方面，拓扑 FFT 领域（通常称为 TQFT 或 Atiyah-Segal TQFT）正在蓬勃发展，所以我们在上面跳过了对它的讨论，出于同样的原因，我们不得不跳过 CFT：这个子领域实在太庞大，以至于值得一篇单独的综述。不幸的是，与 CFT 话题相关的上同调场论也必须跳过。

积分的 Markov 性，如前文所述（也参见 [534]）。[535] 提出了 FFT 与 AQFT 之间的关系, 以及 FFT 理论框架是如何蕴含 AQFT 的。还可以参考 [536] 中有关粘合公理的物理属性和形式化属性 (formal properties) 的讨论。

2.4 Conformal Field Theory (CFT)

CFT（共形场论）是 QFT 的一个特殊类别, 对应于重整化流 (RG flow) 的不动点。由于共形对称性的限制，它们的局部可观测量的算子乘积展开 (Operator Product Expansion, OPE) 比一般的 QFT 更具体可感。从公理上讲,我们可以从上述任何公理化方案中选择一个并将其专门应用于 CFT。在共形不变性（实际上尺度不变性就足够了）存在的情况下，Wightman 和 Osterwalder-Schrader 公理通过关联函数下的算子乘积展开关系提供了 CFT 的基础。历史上这一直是研究 CFT 的最流行方法，参见最近的研究 [537,538]，其中回顾了包括欧几里德 CFT 公理以及它们与 Wightman 和 Osterwalder-Schrader 公理的关系等内容。在存在二维共形对称性的情况下，函子 QFT 启发了 Segal 对共形场论的定义 [519, 520]，实际上这甚至早于函子 QFT 公理体系的建立。最后，二维共形不变性提供的 AQFT 公理启发了共形网的概念 [539-546]（共形网与顶点算符代数（VOAs）相关，参见 [546-549]，其中最后两篇论文还提到与 FQFT 的关系），参见一系列论文 [550 – 555], [556] 和 [557]。

关于 CFT 的大部分结果出现在物理文献中，但它们通常在数学上是严格的，或者在概念上没有严格化的障碍。CFT 通常通过局域可观测量的算子谱和它们的 OPE 信息来表征。在二维时空中，增强的 Virasoro 对称性 (enhanced Virasoro Symmetry) 通常能够提供精确解 [558]，这与顶点算子代数（Vertex Operator Algebra, VOAs）理论相关。高维 CFT 是一个活跃的子领域，一篇独立的 Snowmass 论文 [559] 对其进行了回顾，还有一篇回顾了 VOAs 的一些方面 [560]。在这里，我们只是简单地涉及了这个领域的浅表，主要是因为 CFT 的文献实在是太广泛了，在本综述中无法全面介绍。

3 Discussion

正如本文所述，以及不可避免但仍不完整的庞大参考文献列表，我们为理解量子场论投入了巨大的智力资源。尽管如此，我们仍然缺乏一个令人满意的一致观点。可以说，Brunetti-Fredenhagen-Verch-Fewster 的 LCQFT 公理和上一节的 FFT 公理是对量子场论进行公理化的最一般和最前沿的尝试，但即使是最古老的 Wightman 公理在现代仍然发挥其作用（例如，参见文献 [537,538]）。然而，这些公理存在一些明显的问题:

- LCQFT 在规范理论中面临困难，因此不得不被同伦 AQFT 代替，这一事实暗示了一些东西。在过去的几十年中，我们了解了场论中的对偶性，并理解到“规范理论”并不是量子场论的固有属性，而只是一种构造。实际上，已知有些规范理论允许存在一个对偶的非规范形式。因此，像 AQFT 这样的模型无关的形式化方法 (formalism) 不应将规范理论单独对待。事实上,

拓扑效应不仅出现在规范理论中。这表明，也许同伦 AQFT 是一般 AQFT 机制的正确舞台，而不仅仅是在规范理论的领域。与这一趋势一致的是，它所导出的数学工具在物理学中发挥着越来越重要的作用，这也许起源于 Batalin-Vilkovisky 形式化方法 (BV formalism)。另一方面，[482,483] 提到规范理论的方面正在取得的一些进展。

- 关于函子场论 (FFT) 方法，人们可以提出一些显著的问题。它非常基本地将态空间的概念根植于公理当中，与之相对的，AQFT 范式强调态的希尔伯特空间是公理的一部分。此外，正如最近在 [416] 中强调的，如果空间切片是非紧的，则希尔伯特空间甚至不必存在。当然，函子场论可以通过只允许紧空间切片来克服这个问题，但这个情况似乎有点尴尬。
- 函子场论 (FFT) 框架似乎没有解决的另一个问题是模糊性 (Ambiguity)。正如我们在文本中提到的，由于背景反项 (background counterterm) 的存在，这导致配分函数依赖于正规化，因此一般背景中的 QFT 存在模糊性。在某些情况下，比如存在额外对称性的情况下，这种模糊性性使得配分函数在纤维丛上取值，例如二维 $(2,2)$ 超共形场论中的 S^2 配分函数，它的值在 Kähler 丛 [561] 上取值。在更一般的情况下（例如具有 $(1,1)$ 超对称性或更少超对称的 2 D 理论），这种解释丢失了，配分函数变得完全模糊。因此，认为函子场论中的函子总是产生唯一解，特别地，总为封闭时空赋予一个复数值，这个想法可能过于天真了。然而，这可能只是一个归一化问题 (normalization issue)。
- 当前可用的非拓扑 QFT 的公理化方法没有严肃地考虑扩展算符 (extended operators) 和缺陷 (defect)。这并不是说不可能做到：可以在局部可观测量构成的网中包含扩展算符，并且可以通过修改赋予和缺陷相交区域的代数来包含缺陷在内[21]。还可以通过切除可观测量的一个管状邻域并将相应的态分配给边界来在 FFT 形式化方法中纳入各种（扩展或非扩展）观测量。然而，这些问题似乎并没有得到特别深入的研究。对于角落 (corner)（即，在非拓扑情况下将理论扩展到余维 ≥ 2 的情形），我们了解得更少，因此也更加神秘。

此外，我们应该注意到，在凝聚态文献中存在一种哲学，即量子场论描述了某个格点体系、多体体系或其他有限系统的临界点附近的小扰动。我们没有在正文中涵盖这一观点，因为它并不提供一套公理。目前还不清楚如何将这种哲学与我们现有的任何公理化方法，尤其是与 AQFT 联系起来。例如，格点系统通常具有明确定义的唯一的希尔伯特空间，而能够从中得出的量子场论的一般理论必须以某种方式失去这个特定属性。这些当然是老的问题，其中一些已经在具体模型的构造性场论项目中得到了部分回答。我们还注意到，QFT 中的格点方法引起了最近的越来越多的兴趣，特别是论文 [562-564] 中，其中的一个专门构造的连续极限被认为可以解决上述问题。

除了之前提到的问题，现代文献中研究的许多 QFT 并不符合当前可用的任何公理系统。QFT 最初被引入是为了将量子力学与狭义相对论结合起来，但今天我们知道这更多是历史的偶然。例如，存在于洛伦兹不变背景之外的 QFT，像是 Lifshitz 场论（参见综述 [565] 及其参考文献），Horava 引力 [566]（不过这是一个引力理论），以及许多其他理论。当这些理论放在弯曲空间上时，它们预计将遵循某种修改后的局部协变性 (如果有的话)。因此，它们不符合上述任何公理系统。

[21]感谢 O. Gwilliam 对这一点的讨论。

更一般地说，最近在高能物理社区 [567] 中人们认识到我们对 QFT 的理解是不完整的。标准技术非常有限，许多物理上可接受的理论并不符合传统的 QFT 模型。这些理论包括非交换空间上的场论、小弦理论 (little string theory) 以及像是 [568-571] 及其引用的各种奇特理论。综上所述，目前存在一个明显的问题：我们没有对 QFT 的一般定义。

虽然尚不知道如何推广 QFT 的概念，但有一个值得一提的想法。在 [572] 中，A.Losev 和 S.Hu 提出了一个大胆的建议，即应该修改定义 QFT 的几何结构。与其在普通流形上工作，如黎曼流形或洛伦兹流形，不如考虑捕捉用于构建 QFT 的代数运算的某种推广。[572] 的作者创造了"Feynmann 几何"的名字，并建议它应该由一个具有迹类操作的 A_∞-代数描述（这样的定义涵盖了许多 UV 正则化方法：动量截断、晶格、非交换性）。在这方面，还应该提到 Kontsevich-Soibelman 关于非交换几何的 A_∞ 方法的工作 [573]，这也许可以派上用场。如果这是正确的方法，那么"Feynman 几何"的正确想法可能需要更加一般化，以涵盖所有奇特的 QFT 实例。

参考文献

[1] M. Born, W. Heisenberg, and P. Jordan, ”Zur Quantenmechanik. II.,” Z. Phys. 35 no. 8-9, (1926)557 – 615.

[2] P. A. M. Dirac, ”Quantum theory of emission and absorption of radiation,” Proc. Roy. Soc. Lond. A 114(1927)243.

[3] W. Heisenberg and W. Pauli, ”On Quantum Field Theory. (In German),” Z. Phys. 56 (1929)1 – 61.

[4] W. Heisenberg and W. Pauli, ”On Quantum Field Theory. 2. (In German),” Z. Phys. 59 (1930)168 – 190.

[5] S. Weinberg, The Quantum theory of fields. Vol. 1: Foundations. Cambridge University Press, 6, 2005.

[6] J. S. Schwinger, ”On Quantum electrodynamics and the magnetic moment of the electron,” Phys. Rev. 73(1948)416 – 417.

[7] J. S. Schwinger, ”Quantum electrodynamics. 2. Vacuum polarization and selfenergy,” Phys. Rev. 75(1948)651

[8] J. S. Schwinger, ”Quantum electrodynamics. I A covariant formulation,” Phys. Rev. **74** (1948)1439

[9] J. S. Schwinger, ”Quantum electrodynamics. III: The electromagnetic properties of the electron: Radiative corrections to scattering,” Phys. Rev. 76 (1949) 790-817.

[10] R. P. Feynman, ”Space-time approach to nonrelativistic quantum mechanics,” Rev. Mod. Phys. 20 (1948) 367-387.

[11] R. P. Feynman, ”Relativistic cutoff for quantum electrodynamics,” Phys. Rev. 74 (1948) 1430 – 1438.

[12] R. P. Feynman, ”A Relativistic cutoff for classical electrodynamics,” Phys. Rev. 74 (1948) 939-946.

[13] S. Tomonaga, ”On a relativistically invariant formulation of the quantum theory of wave fields,” Prog. Theor. Phys. 1 (1946) 27-42.

[14] Z. Koba, T. Tati, and S. i. Tomonaga, ”On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields. II: Case of Interacting Electromagnetic and Electron Fields,” Prog. Theor. Phys. 2 no. 3, (1947) 101-116.

[15] Z. Koba, T. Tati, and S.-i. Tomonaga, ”On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields. III: Case of Interacting Electromagnetic and Electron Fields,” Progress of Theoretical Physics 2 no. 4, (12, 1947) 198-208.

[16] S. Kanesawa and S.-i. Tomonaga, ”On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields. IV: Case of Interacting Electromagnetic and Meson Fields,” Progress of Theoretical Physics 3 no. 1, (03, 1948) 1-13.

[17] Z. Koba and S.-i. Tomonaga, ”On Radiation Reactions in Collision Processes. I: Application of the ”Self-Consistent” Subtraction Method to the Elastic Scattering of an Electron*,” Progress of Theoretical Physics 3 no. 3, (09, 1948) 290-303.

[18] S.-I. Tomonaga and J. R. Oppenheimer, ”On Infinite Field Reactions in Quantum Field Theory,” Phys. Rev. 74 (1948) 224-225.

[19] F. J. Dyson, ”The Radiation theories of Tomonaga, Schwinger, and Feynman,” Phys. Rev. $75(1949)486-502$.

[20] A. S. Wightman, ”Quantum Field Theory in Terms of Vacuum Expectation Values,” Phys. Rev. 101 (1956) 860-866.

[21] A. S. Wightman, ”Quelques problèmes mathématique de la théorie quantique relativiste,” Colloq. Int. CNRS $75(1959)1-38$.

[22] A. S. Wightman and L. Garding, ”Fields as operator-valued distributions in relativistic quantum theory,” Arkiv Fys. . https://www.osti.gov/biblio/4606723.

[23] A. Wightman, ”Recent Achievements of Axiomatic Field Theory,” in Theoretical Physics, pp. 11-58. IAEA, Vienna, 1963.

[24] W. Heisenberg, ”Die ”beobachtbaren Größen” in der Theorie der Elementarteilchen,” Zeitschrift für Physik 120 (1943) 513-538.

[25] N. N. Bogolyubov, A. A. Logunov, A. I. Oksak, and I. T. Todorov, General principles of quantum field theory. Mathematical Physics and Applied Mathematics. Kluwer Academic Publishers, 1990.

[26] H. Lehmann, K. Symanzik, and W. Zimmermann, ”On the formulation of quantized field theories,” Nuovo Cim. 1 (1955) 205-225.

[27] H. Lehmann, K. Symanzik, and W. Zimmermann, ”On the formulation of quantized field theories. II,” Nuovo Cim. 6 (1957) 319-333. [28] R. F. Streater and A. S. Wightman, PCT, Spin and Statistics, and All That. Princeton University Press, 1964.

[29] N. Bogoliubov, A. Logunov, and I. Todorov, Introduction to Axiomatic Quantum Field Theory. Mathematical physics monograph series. W. A. Benjamin, Advanced Book Program, 1975.

[30] E. P. Wigner, ”On Unitary Representations of the Inhomogeneous Lorentz Group,” Annals Math. $40(1939)149-204$.

[31] V. Bargmann, ”Irreducible Unitary Representations of the Lorentz Group,” Annals of Mathematics 48 no. 3, (1947) 568-640.

[32] V. Bargmann, ”On Unitary Ray Representations of Continuous Groups,” Annals of Mathematics 59 no. 1, (1954) 1-46. http://ww.jstor.org/stable/1969831.

[33] Representations of the Rotation and Lorentz Groups and Their Applications.

[34] Y. Ohnuki, Unitary representations of the Poincare group and relativistic wave equations. World Scientific Pub Co Inc, 1988.

[35] G. Kallen, ”Properties of Vacuum Expectation Values of Field Operators,” Les Houches Lect. Notes 10 (1960) 387-454.

[36] A. Wightman, ”Quantum Field Theory and Analytic Functions of several Complex Variables,” J Indian Math. Soc. 24 no. 3-4, (1960) 625

[37] H. Araki, ”On asymptotic behavior of vacuum expectation values at large space-like separation,” Annals of Physics 11 no. 2, (1960) 260-274.

[38] K. Hepp, R. Jost, D. Ruelle, and O. Steinmann, ”Necessary restriction on Wightman functions,” Helv. Phys. Acta 34 no. V, (1961) 542-544.

[39] R. Haag and B. Schroer, ”Postulates of Quantum Field Theory,” Journal of Mathematical Physics 3 no. 2, (1962) 248-256.

[40] D. Ruelle, ”On the asymptotic condition in quantum field theory,” Helv. Phys. Acta 35 no. III, (1962) 147-163.

[41] R. Jost and K. Hepp, ”Über die Matrixelemente des Translationsoperators,” Helv. Phys. Acta 35 no. I, (1962) 34-46.

[42] H. Araki, K. Hepp, and D. Ruelle, ”On the asymptotic behaviour of Wightman functions in space-like directions,” Helv. Phys. Acta 35 no. III, (1962) 164-174.

[43] W. Schmidt and K. Baumann, ”Quantentheorie der felder als distributionstheorie,” Il Nuovo Cimento (1955-1965) $4(1956)860 - 886$.

[44] G. Luders, ”On the Equivalence of Invariance under Time Reversal and under Particle-Antiparticle Conjugation for Relativistic Field Theories,” Kong. Dan. Vid. Sel. Mat. Fys. Med. 28N5 no. 5, (1954) $1 - 17$.

[45] W. Pauli, Exclusion Principle, Lorentz Group and Reflection of Space-Time and Charge. Pergamon Press, New York, 1955.

[46] M. Fierz, ”Force-free particles with any spin,” Helv. Phys. Acta 12 (1939) 3-37.

[47] W. Pauli, ”The Connection Between Spin and Statistics,” Phys. Rev. 58 (1940) 716-722.

[48] R. Jost, ”A remark on the C.T.P. theorem,” Helv. Phys. Acta 30 (1957) 409-416.

[49] F. J. Dyson, ”Connection between local commutativity and regularity of Wightman functions,” Phys. Rev. 110 (1958) 579-581.

[50] G. Luders and B. Zumino, ”Connection between Spin and Statistics,” Phys. Rev. 110

(1958) 1450-1453.

[51] ”On the connection of spin with statistics,” Nuovo Cimento 8 (1958) 607-609.

[52] G. Dell’Antonio, ”On the connection between spin and statistics,” Annals of Physics **16** no. 2, $(1961) 153-157$

[53] R. Haag, Discussion des ”axiomes” et des propriétés asymptotiques d’une théorie des champs locales avec particules composées. Colloque Internationaux du CNRS LXXV (Lille 1957). CNRS Paris, 1959.

[54] R. Jost, ”Properties of wightman functions,” in Lectures on the Many-body Problems, E. CAIANIELLO, ed., pp. 127-145. Academic Press, 1961.

[55] H. J. Borchers, ”On structure of the algebra of field operators,” Nuovo Cim. 24 no. 2, $(1962) 214-236$.

[56] H. Reeh and S. Schlieder, ”Bemerkungen zur unitäräquivalenz von lorentzinvarianten feldern,” Nuovo Cim. 22 no. 5, (1961) 1051-1068.

[57] R. Haag, ”On quantum field theories,” Kong. Dan. Vid. Sel. Mat. Fys. Med. 29 N12 (1955) $1-37$.

[58] D. Hall and A. Wightman, ”A Theorem on invariant analytic functions with applications to relativistic quantum field theory,” Mat. Fys. Medd. Dan. Vid. Selsk. 31 (1957). [59] O. W. Greenberg, ”Haag’s Theorem and Clothed Operators,” Phys. Rev. 115 (1959) $706-710$.

[60] P. G. Federbush and K. A. Johnson, ”Uniqueness Property of the Twofold Vacuum Expectation Value,” Phys. Rev. 120 (1960) 1926-1926.

[61] H.-J. Borchers, ”Über die mannigfaltigkeit der interpolierenden felder zu einer kausalen S Matrix,” Il Nuovo Cimento 15 (03, 1960) 784-794.

[62] S. Kamefuchi, L. O’Raifeartaigh, and A. Salam, ”Change of variables and equivalence theorems in quantum field theories,” Nucl. Phys. 28 (1961) 529-549.

[63] H. Epstein, ”On the borchers class of a free field,” Il Nuovo Cimento 27 (1963) 886-893.

[64] R. Jost, The General Theory of Quantized Fields. Lectures in Applied Mathematics. American Mathematical Society, Providence, Rhode Island.

[65] K. Osterwalder and R. Schrader, ”Axioms for Euclidean Green’s Functions,” Commun. Math. Phys. 31 (1973) 83-112.

[66] K. Osterwalder and R. Schrader, ”Axioms for Euclidean Green’s Functions. 2.,” Commun. Math. Phys. 42(1975)281.

[67] V. Glaser, ”On the equivalence of the Euclidean and Wightman formulation of field theory,” Commun. Math. Phys. 37 (1974) 257-272.

[68] J. Glimm and A. M. Jaffe, Quantum Physics. A Functional Integral Point of View.

Springer, 1987.

[69] E. Nelson, ”Construction of quantum fields from Markoff fields,” Journal of Functional Analysis 12 no. 1, (1973) 97-112.

[70] J. Schwinger, ”On the Euclidean Structure of Relativistic Field Theory,” Proc. Nat. Acad. Sci. 44 no. 9, (1958) 956-965.

[71] J. Schwinger, ”Euclidean quantum electrodynamics,” Phys. Rev. 115 (Aug, 1959) 721-731.

[72] T. Nakano, ”Quantum Field Theory in Terms of Euclidean Parameters,” Progress of Theoretical Physics 21 no. 2, (02, 1959) 241-259.

[73] K. Symanzik, ”A Modified Model of Euclidean Quantum Field Theory,”. .

[74] K. Symanzik, ”Euclidean quantum field theory. i. equations for a scalar model,” Journal of Mathematical Physics 7 no. 3, (1966) 510-525. [75] K. Symanzik, ”Euclidean quantum field theory,” in Local quantum theory (Varenna, 1968), R. Jost, ed. Academic Press, New York, 1969.

[76] A. Jaffe, ”Euclidean quantum field theory,” Nuclear Physics B 254 (1985) 31-43.

[77] Y. M. Zinoviev, ”Equivalence of Euclidean and Wightman field theories,” Communications in Mathematical Physics 174 no. 1, (1995) $1-27$.

[78] E. Nelson, ”Quantum fields and Markoff fields,” in Partial Differential Equations, pp. 413-420. American Mathematical Society, Providence, R.I., 1973.

[79] E. Nelson, ”The free Markoff field,” Journal of Functional Analysis 12 no. 2, (1973) 211-227.

[80] E. Nelson, ”Probability theory and euclidean field theory,” in International School of Mathematical Physics, Ettore Majorana: 1st course: Constructive Quantum Field Theory, pp. 94-124. 1973.

[81] B. Simon, The $P(\phi)_2$ Euclidean (Quantum) Field Theory. Princeton Univ. Press, 1974.

[82] G. C. Hegerfeldt, ”From Euclidean to relativistic fields and on the notion of Markoff fields,” Communications in Mathematical Physics 35 (1974) 155-171.

[83] Fr ”Unbounded, symmetric semigroups on a separable Hilbert space are essentially self-adjoint,” Advances in Applied Mathematics 1 no. 3, (1980) 237-256.

[85] J. Frohlich, J. Osterwalder, and E. Seiler, ”On Virtual Representations of Symmetric Spaces and Their Analytic Continuation,” Annals of Mathematics 118 no. 3, (1983) 461-489. http://www.jstor.org/stable/2006979.

[86] M. C. Lee and J. Glimm, ”Axioms for Quantum Gauge Fields,” arxiv:2112.08575 [math-ph].

[87] O. W. Greenberg, ”Generalized Free Fields and Models of Local Field Theory,” Annals Phys. $16(1961)158-176$.

[88] A. Jaffe, ”Existence Theorems for a Cut-off $\lambda\varphi^4$ Field Theory,” in Conference on the Mathematical Theory of Elementary Particles. MIT Press, Cambridge, Massachusetts, 1966.

[89] O. Lanford, Construction of quantum fields interacting by a cutoff Yukawa coupling. PhD thesis, 1966. [90] E. Nelson, ”A quadratic interaction in two dimensions,” in Conference on the Mathematical Theory of Elementary Particles. MIT Press, Cambridge, Massachusetts, 1966.

[91] J. Glimm and A. M. Jaffe, ”A $\lambda\varphi^4$ quantum field theory without cutoffs. 1,” Phys. Rev. 176 (1968) 1945-1951.

[92] A. M. Jaffe, O. E. Lanford, and A. S. Wightman, ”A General class of cut-off model field theories,” Commun. Math. Phys. 15 (1969) 47-68.

[93] B. Simon, Functional Integration and Quantum Physics: Second Edition. AMS Chelsea Publishing, 2nd ed., 2005.

[94] J. R. Klauder, A Modern Approach to Functional Integration. Applied and Numerical Harmonic Analysis. Birkhäuser Basel, 1 ed., 2011.

[95] S. Albeverio, R. Høegh-Krohn, and S. Mazzucchi, Mathematical Theory of Feynman Path Integrals. Lecture Notes in Mathematics. Springer, Berlin, Heidelberg, 2nd ed., 2008.

[96] S. Mazzucchi, Mathematical Feynman Path Integrals and Their Applications. WORLD SCIENTIFIC, 2nd ed., 2021.

[97] A. Jaffe and G. Ritter, ”Quantum field theory on curved backgrounds. I. The Euclidean functional integral,” Commun. Math. Phys. 270 (2007) 545-572, arxiv:hep-th/0609003.

[98] A. Jaffe and G. Ritter, ”Quantum field theory on curved backgrounds. II. Spacetime symmetries,” arxiv:0704.0052 [hep-th].

[99] V. Rivasseau, From Perturbative to Constructive Renormalization. Princeton Series in Physics. Princeton University Press, 2014.

[100] J. C. Baez, I. E. Segal, and Z. Zhou, Introduction to Algebraic and Constructive Quantum Field Theory. Princeton Series in Physics. Princeton University Press, 1992.

[101] ”Constructive Physics Results in Field Theory, Statistical Mechanics and Condensed Matter Physics,” Lecture Notes in Physics. Springer, 1995.

[102] A. S. Wightman, ”Constructive field theory - introduction to the problems,” Stud. Nat. Sci. $3(1973)1-85$.

[103] ”Constructive Quantum Field Theory,” Lect Notes Phys. Springer, Berlin, Heidelberg, 1973.

[104] T. Balaban and A. M. Jaffe, ”Constructive Gauge Theory,” NATO Sci. Ser. B 141 (1986) $207-263$. [105] A. M. Jaffe, ”Constructive quantum field theory,” in Mathematical physics 2000, A. Fokas, A. Grigorian, T. Kibble, and B. Zegarlinski, eds., pp. 111-127. 2000.

[106] V. Rivasseau, "Constructive field theory and applications: Perspectives and open problems," Journal of Mathematical Physics 41 no. 6, (Jun, 2000) 3764-3775, math-ph/0006017.

[107] A. Jaffe, "Quantum Theory and Relativity," 2007.

[108] S. J. Summers, "A Perspective on Constructive Quantum Field Theory," 2016.

[109] S. Summers, "Constructive Quantum Field Theory."

[110] A. Jaffe, "Is relativity compatible with quantum theory?," December 2, 2020. CMSA/YMSC literature talk.

[111] J. Glimm and A. M. Jaffe, "A $\lambda\varphi^4$ quantum field theory without cutoffs, II. The field operators and the approximate vacuum," Ann. of Math. 91 no. 2, (1970) 362-401.

[112] J. Glimm and A. Jaffe, "The $\lambda\left(\varphi^4\right)_2$ quantum field theory without cutoffs, III. The physical vacuum," Acta Mathematica 125 no. none, (1970) 203 - 267.

[113] J. Glimm and A. M. Jaffe, "The $\lambda\phi^4$ in Two-dimensions Quantum Field Theory Without Cutoffs. 4. Perturbations of the Hamiltonian," J. Math. Phys. 13 (1972) 1568-1584.

[114] J. T. Cannon and A. M. Jaffe, "Lorentz covariance of the $\lambda\left(\phi^4\right)_2$ quantum field theory," Commun. Math. Phys. 17 (1970) 261-321.

[115] F. Guerra, "Uniqueness of the vacuum energy density and van hove phenomenon in the infinite-volume limit for two-dimensional self-coupled bose fields," Phys. Rev. Lett. 28 (May, 1972) $1213 - 1215$

[116] F. Guerra, L. Rosen, and B. Simon, "The $p(\phi)_2$ euclidean quantum field theory as classical statistical mechanics," Annals of Mathematics 101 no. 1, (1975) 111-189..

[117] J. Glimm, A. Jaffe, and T. Spencer, "The particle structure of the weakly coupled $P(\phi)_2$ model and other applications of high temperature expansions, Part II: The cluster expansion.," in Constructive Quantum Field Theory, vol. 25 of Springer Lecture Notes in Physics. Springer, 1973.

[118] J. Glimm and A. M. Jaffe, "Absolute Bounds on Vertices and Couplings," Ann. Inst. H. Poincare Phys. Theor. A 22(1975)97.

[119] J. Glimm, A. Jaffe, and T. Spencer, "The wightman axioms and particle structure in the $\mathcal{P}(\phi)_2$ quantum field model," Annals of Mathematics 100 no. 3, (1974) 585-632.

[120] T. Spencer, "The Absence of Even Bound States for $\lambda\left(\phi^4\right)$ in Two-Dimensions," Commun. Math. Phys. 39 (1974) 77-79.

[121] T. Spencer and F. Zirilli, "Scattering States and Bound States in $\lambda\wp(\varphi)_2$ in Two-Dimensions," Commun. Math. Phys. 49 (1976) 1.

[122] J. Fröhlich, "Verification of axioms for euclidean and relativistic fields and Haag's theorem in a class of $P(\varphi)_2$-models," Annales de l'I.H.P. Physique théorique 21 no. 4, (1974) 271-317. .

[123] J. Fröhlich, "Schwinger functions and their generating functionals I.," Helv. Phys. Acta

47 no. 3, (1974)265 – 306

[124] J. Dimock and J. P. Eckmann, "On the Bound State in Weakly Coupled $\lambda\left(\phi^6-\phi^4\right)$ in Two-Dimensions," Commun. Math. Phys. 51 (1976) 41-54.

[125] J. Glimm, A. Jaffe, and T. Spencer, "Existence of phase transitions for ϕ^4_2 quantum fields," in Mathematical Methods of Quantum Field Theory. CNRS, Paris, 1976.

[126] J. Frohlich and B. Simon, "Pure states for general $p(\phi)_2$ theories: Construction, regularity and variational equality," Annals of Mathematics 105 no. 3, (1977) 493-526.

[127] J. Glimm, A. M. Jaffe, and T. Spencer, "Phase Transitions for φ^4_2 Quantum Fields," Commun. Math. Phys. 45 (1975) 203.

[128] S. J. Summers, "A New Proof of the Asymptotic Nature of Perturbation Theory in $P(\phi)_2$ Models," Helv. Phys. Acta 53 (1980) 1.

[129] Z. Haba, "Fluctuations of $P(\phi)$ Random Fields," J. Math. Phys. 22 (1981) 1687.

[130] E. Dynkin, "Markov processes as a tool in field theory," Journal of Functional Analysis 50 no. 2, (1983) 167-187.

[131] H. Shen, R. Zhu, and X. Zhu, "An SPDE approach to perturbation theory of Φ^4_2 :asymptoticity and short distance behavior," arxiv:2108.11312 [math.PR].

[132] J. Fröhlich, A. Knowles, B. Schlein, and V. Sohinger, "The Euclidean ϕ^4_2 theory as a limit of an interacting Bose gas," arxiv:2201.07632 [math-ph].

[133] J. Frohlich and Y. M. Park, "Remarks on Exponential Interactions and the Quantum Sine-Gordon Equation in Two Space-Time Dimensions," Helv. Phys. Acta 50 (1977) 315 – 329.

[134] Y. M. Park, "Massless Quantum Sine-Gordon Equation in Two Space-Time Dimensions: Correlation Inequalities and Infinite Volume Limit," J. Math. Phys. 18 (1977) 2423-2426.

[135] E. P. Osipov, "Possible Approach to the Construction of the $:\exp\xi:_d$ quantum field theory in a finite volume," J. Math. Phys. 25 (1984) 633.

[136] N. Barashkov and F. C. De Vecchi, "Elliptic stochastic quantization of Sinh-Gordon QFT," arxiv:2108.12664 [math.PR]

[137] J. Glimm and A. M. Jaffe, "Positivity of the ϕ^4_3 Hamiltonian," Fortsch. Phys. 21 (1973) 327-376.

[138] J. S. Feldman and K. Osterwalder, "The Wightman Axioms and the Mass Gap for Weakly Coupled $\left(\phi^4\right)_3$ Quantum Field Theories," Annals Phys. 97 (1976) 80-135.

[139] J. Magnen and R. Seneor, "The Infinite Volume Limit of the ϕ^4_3 Model," Ann. Inst. H. Poincare Phys. Theor. 24 (1976) 95-159.

[140] J. Glimm, A. M. Jaffe, and T. Spencer, "A Convergent Expansion About Mean Field Theory. 1. the Expansion," Annals Phys. 101 (1976) 610.

[141] J. Glimm, A. M. Jaffe, and T. Spencer, ”A Convergent Expansion About Mean Field Theory. 2. Convergence of the Expansion,” Annals Phys. 101 (1976) 631-669.

[142] J. Frohlich, B. Simon, and T. Spencer, ”Infrared Bounds, Phase Transitions and Continuous Symmetry Breaking,” Commun. Math. Phys. 50 (1976) 79-95.

[143] J. Magnen and R. Seneor, ”Phase Space Cell Expansion and Borel Summability for the Euclidean ϕ_3^4 Theory,” Commun. Math. Phys. 56 (1977) 237.

[144] Y. M. Park, ”Convergence of Lattice Approximations and Infinite Volume Limit in the $\left(\lambda\phi^4 - \sigma\phi^2 - \tau\phi\right)_3$ Field Theory,” J. Math. Phys. 18 (1977) 354-366.

[145] D. C. Brydges, J. Fröhlich, and A. D. Sokal, ”A new proof of the existence and nontriviality of the continuum φ_2^4 and φ_3^4 quantum field theories,” Communications in Mathematical Physics 91(1983) 141-186.

[146] M. Gubinelli and M. Hofmanová, ”A PDE Construction of the Euclidean Φ_3^4 Quantum Field Theory,” Commun. Math. Phys. 384 no. 1, (2021) 1-75, arxiv:1810.01700 [math-ph].

[147] S. Albeverio and S. Kusuoka, ”Construction of a non-Gaussian and rotation-invariant Φ^4-measure and associated flow on $\mathbb{R}^3$ through stochastic quantization,” arxiv:2102.08040 [math.PR].

[148] M. Hairer and R. Steele, ”The Φ_3^4 Measure Has Sub-Gaussian Tails,” J. Statist. Phys. 186 no. 3, (2022) 38, arxiv:2102.11685 [math.PR].

[149] T. Oh, M. Okamoto, and L. Tolomeo, ”Stochastic quantization of the Φ_3^3-model,” arxiv:2108.06777 [math.PR].

[150] A. Jagannath and N. Perkowski, ”A simple construction of the dynamical Φ_3^4 model,” arxiv:2108.13335 [math.PR].

[151] K. Gawedzki and A. Kupiainen, ”Exact Renormalization for the Gross-Neveu Model of Quantum Fields,” Phys. Rev. Lett. 54 (1985) 2191-2194.

[152] K. Gawedzki and A. Kupiainen, ”Gross-Neveu Model Through Convergent Perturbation Expansions,” Commun. Math. Phys. 102 (1985) 1.

[153] J. Feldman, J. Magnen, V. Rivasseau, and R. Seneor, ”Massive Gross-Neveu Model: a Rigorous Perturbative Construction,” Phys. Rev. Lett. 54 (1985) 1479-1481.

[154] J. Feldman, J. Magnen, V. Rivasseau, and R. Seneor, ”A Renormalizable Field Theory: The Massive Gross-Neveu Model in Two-dimensions,” Commun. Math. Phys. 103 (1986) 67-103

[155] M. Disertori and V. Rivasseau, ”Continuous constructive fermionic renormalization,” Annales Henri Poincare 1 (2000) 1-57, arxiv:hep-th/9802145.

[156] M. Salmhofer and C. Wieczerkowski, Construction of the renormalized $GN_{2-\epsilon}$ trajectory, pp. 1-19. 2002.

[157] J. Frohlich and E. Seiler, ”The Massive Thirring-Schwinger Model (QED in Two-Dimensions):

Convergence of Perturbation Theory and Particle Structure," Helv. Phys. Acta 49 (1976) 889.

[158] G. Benfatto, P. Falco, and V. Mastropietro, "Functional Integral Construction of the Thirring model: Axioms verification and massless limit," Commun. Math. Phys. 273 (2007) $67 - 118, arxiv : hep - th/0606177$.

[159] G. Benfatto, P. Falco, and V. Mastropietro, "Massless sine-Gordon and massive Thirring models: Proof of the Coleman's equivalence," Commun. Math. Phys. 285 (2009) 713-762, arxiv:0711.5010 [hep-th].

[160] V. Mastropietro, "Schwinger functions in Thirring and Luttinger models," Nuovo Cim. B 108(1993) 1095-1107.

[161] P. Falco, Rigorous construction of the Thirring model: Ward-Takahashi Identities, Schwinger-Dyson Equations and New Anomalies. PhD thesis, Rome U., La Sapienza, 2005. arxiv:hep-th/0703274.

[162] K. Osterwalder and R. Schrader, "Feynman-kac formula for euclidean fermi and bose fields," Phys. Rev. Lett. 29 (1972) 1423-1425.

[163] R. Schrader, "Yukawa quantum field theory in two space-time dimensions without cut-offs," Annals Phys. 70 (1972) 412-457.

[164] K. Osterwalder and R. Schrader, "Euclidean fermi fields and a feynman-kac formula for boson-fermion models," Helv. Phys. Acta 46 (1973) 277-302.

[165] J. Glimm and A. M. Jaffe, "Quantum field theory models," in Les Houches Summer School of Theoretical Physics: Statistical mechanics and quantum field theory, pp. 1-108. 1971.

[166] J. Glimm and A. Jaffe, "The Yukawa, quantum field theory without cutoffs," J. Funct. Anal. 7 no.2,(1971) 323-357

[167] E. Seiler, "Schwinger Functions for the Yukawa Model in Two-Dimensions with Space-Time Cutoff," Commun. Math. Phys. 42 (1975) 163.

[168] E. Seiler and B. Simon, "On Finite Mass Renormalizations in the Two-Dimensional Yukawa Model," J. Math. Phys. 16 (1975) 2289.

[169] E. Seiler and B. Simon, "Bounds in the Yukawa in Two-Dimensions Quantum Field Theory: Upper Bound on the Pressure, Hamiltonian Bound and Linear Lower Bound," Commun. Math. Phys. 45(1975) 99.

[170] E. Seiler and B. Simon, "Nelson's Symmetry and All That in the Yukawa-2 and ϕ_3^4 Field Theories," Annals Phys. 97 (1976) 470-518.

[171] O. A. McBryan, "Higher Order Estimates for the Yukawa Two-Dimensional Quantum Field Theory," Commun. Math. Phys. 42 (1975) 1.

[172] O. A. Mcbryan, "Recent Progress on the Yukawa-2 Quantum Field Theory," in Sympo-

sium on Mathematical Problems of Quantum Dynamics - Models and Mathematics. 1975.

[173] O. A. McBryan, "Volume Dependence of Schwinger Functions in the Yukawa-2 Quantum Field Theory," Commun. Math. Phys. 45 (1975) 279.

[174] O. A. McBryan, "Finite Mass Renormalizations in the Euclidean Yukawa-2 Field Theory," Commun. Math. Phys. 44(1975)237.

[175] O. A. McBryan, "Convergence of the Vacuum Energy Density, phi-Bounds and Existence of Wightman Functions for the Yukawa-2 Model," in International Colloquium on Mathematical Methods of Quantum Field Theory, pp. 237-252. 1975.

[176] O. A. McBryan and Y. M. Park, "Lorentz covariance of the yukawa2 quantum field theory," Journal of Mathematical Physics 16 no. 1, (1975) 104-110.

[177] J. Magnen and R. Seneor, "The Wightman Axioms for the Weakly Coupled Yukawa Model in Two-Dimensions," Commun. Math. Phys. 51 (1976) 297-313.

[178] A. Cooper and L. Rosen, "The Weakly Coupled Yukawa, Field Theory: Cluster Expansion and Wightman Axioms," Trans. Am. Math. Soc. 234 no. 1, (1977) 1-88.

[179] E. P. Osipov, "The Yukawa-2 Quantum Field Theory: Linear n (tau) Bound, Locally Fock Property," Ann. Inst. H. Poincare Phys. Theor. 30 (1979) 159-192.

[180] E. P. Osipov, "The Yukawa-2 Quantum Field Theory: Lorentz Invariance," Annals Phys. $125(1980)53-66$

[181] P. Renouard, "Analyticity and Borel Summability of Schwinger Functions in the Two-Dimensional Yukawa Model. 1. Finite Volume Approximation," Ann. Inst. H. Poincare Phys. Theor. 27 (1977) 237-277.

[182] P. Renouard, "Analyticity and Borel Summability of Schwinger Functions in the Two-Dimensional Yukawa Model. II. The 'Adiabatic Limit'. (In French)," Ann. Inst. H. Poincare Phys. Theor. 31 (1979) 235-318.

[183] J. Magnen and R. Seneor, "Yukawa Quantum Field Theory in Three Dimensions $(Y_3)^*$, ," Annals of the New York Academy of Sciences 337 no. 1, (1980) 13-43.

[184] T. Balaban and K. Gawedzki, "A Low Temperature Expansion for the Pseudoscalar Yukawa Model of Quantum Fields in Two Space-time Dimensions," Ann. Inst. H. Poincare Phys. Theor. 36 (1982) 271.

[185] A. Lesniewski, "Effective Action for the Yukawa(2) Quantum Field Theory," Commun. Math. Phys. 108 (1987) 437-467. [186] J. Frohlich and K. Osterwalder, "Is There a Euclidean Field Theory for Fermions," Helv. Phys. Acta 47(1975)781.

[187] J. Frohlich and P. A. Marchetti, "Bosonization, Topological Solitons and Fractional Charges in Two-dimensional Quantum Field Theory," Commun. Math. Phys. 116 (1988) 127.

[188] H. Nicolai, ”A Possible Constructive Approach to Super ϕ^3 in Four-dimensions. 2. Regularization of the Model,” Nucl. Phys. B 156 (1979) 157.

[189] H. Nicolai, ”On the Normalization of Schwinger Functions in the Euclidean Wess-Zumino Model,” other thesis, 10, 1978.

[190] A. M. Jaffe, A. Lesniewski, and J. Weitsman, ”The Two-dimensional $N = 2$ Wess-Zumino Model on a Cylinder,” Commun. Math. Phys. 114 (1988) 147.

[191] A. M. Jaffe and A. Lesniewski, ”A Priori Estimates for $N = 2$ Wess-Zumino Models on a Cylinder,” Commun. Math. Phys. 114 (1988) 553-575.

[192] A. M. Jaffe and A. Lesniewski, ”Supersymmetric Quantum Fields and Infinite Dimensional Analysis,” in NATO Advanced Summer Institute on Nonperturbative Quantum Field Theory (Cargese Summer Institute). 12, 1987. https://lib-extopc.kek.jp/preprints/PDF/2000/0031/0031810.pdf

[193] A. M. Jaffe, A. Lesniewski, and J. Weitsman, ”The Loop Space S(1) $\to$ R and Supersymmetric Quantum Fields,” Annals Phys. 183 (1988) 337.

[194] S. A. Janowsky and J. Weitsman, ”The Phase structure of the two-dimensional N=2 Wess-Zumino model,” Commun. Math. Phys. 142 (1991) 25-66.

[195] D. Brydges, J. Fröhlich, and T. Spencer, ”The random walk representation of classical spin systems and correlation inequalities,” Communications in Mathematical Physics 83 no. 1, (1982) 123-150.

[196] D. C. Brydges, J. Fröhlich, and A. D. Sokal, ”The random-walk representation of classical spin systems and correlation inequalities. II. The skeleton inequalities,” Communications in Mathematical Physics 91 no. 1, (1983) 117 - 139.

[197] J. Frohlich, ”Quantum Field Theory in Terms of Random Walks and Random Surfaces,” NATO Sci. Ser. B 115(1984) 169-233.

[198] M. Aizenman, The Intersection of Brownian Paths as a Case Study of a Renormalization Group Method for Quantum Field Theory, pp. 91-110. Springer Berlin Heidelberg, Berlin, Heidelberg, 1985.

[199] J. T. Chayes, L. Chayes, and J. Frohlich, ”The Low Temperature Behavior of Disordered Magnets,” Commun. Math. Phys. 100 (1985) 399.

[200] M. Aizenman, ”Proof of the Triviality of ϕ^4_D Field Theory and Some Mean Field Features of Ising Models for D>4,” Phys. Rev. Lett. 47 (1981) 1-4.

[201] M. Aizenman, ”Geometric Analysis of ϕ^4 Fields and Ising Models (Parts 1 & 2),” Commun. Math. Phys. 86(1982)1.

[202] J. Frohlich, ”On the Triviality of $\lambda\phi^4_d$ Theories and the Approach to the Critical Point in $d \geq 4$ Dimensions,” Nucl. Phys. B 200 (1982) 281-296.

[203] R. Fernandez, J. Fröhlich, and A. D. Sokal, Random Walks, Critical Phenomena, and Triviality in Quantum Field Theory. Texts and Monographs in Physics. Springer, Berlin, Heidelberg, 1992.

[204] A. D. Sokal, "An Alternate Constructive Approach to the ϕ^4_3 Quantum Field Theory, and a Possible Destructive Approach to ϕ^4_4," Ann. Inst. H. Poincare Phys. Theor. A 37 (1982) 317-398.

[205] K. Gawedzki and A. Kupiainen, "Triviality of ϕ^4_4 and All That in a Hierarchical Model Approximation," J. Statist. Phys. 29 (1982) 683-698.

[206] C. Aragao De Carvalho, S. Caracciolo, and J. Frohlich, "Polymers and $g\phi^4$ Theory in Four-Dimensions," Nucl. Phys. B 215 (1983) 209-248.

[207] M. Aizenman and R. Graham, "On the renormalized coupling constant and the susceptibility in ϕ^4_4 field theory and the Ising model in four dimensions," Nuclear Physics B 225 no. 2, (1983) 261-288.

[208] K. Gawedzki and A. Kupiainen, "Masless Lattice ϕ^4_4 Theory: a Nonperturbative Control of a Renormalizable Model," Phys. Rev. Lett. 54 (1985) 92-94.

[209] K. Gawedzki and A. Kupiainen, "Nontrivial Continuum Limit of a ϕ^4_4 Model With Negative Coupling Constant," Nucl. Phys. B 257 (1985) 474-504.

[210] K. Gawedzki and A. Kupiainen, "Massless lattice ϕ^4_4 theory: Rigorous control of a renormalizable asymptotically free model," Communications in Mathematical Physics 99 (1985)197−252.

[211] T. Hara and H. Tasaki, "A rigorous control of logarithmic corrections in four-dimensional φ^4 spin systems," Journal of Statistical Physics 47 (1987) 99-121. [212] J. Feldman, J. Magnen, V. Rivasseau, and R. Sénéor, "Construction and Borel summability of infrared Φ^4_4 by a phase space expansion," Communications in Mathematical Physics 109 no. 3, $(1987)437-480$.

[213] R. Bauerschmidt, D. C. Brydges, and G. Slade, "Scaling limits and critical behaviour of the 4-dimensional n-component $|\varphi|^4$ spin model," arxiv:1403.7424 [math-ph].

[214] M. Aizenman and H. Duminil-Copin, "Marginal triviality of the scaling limits of critical 4D Ising and ϕ^4_4 models," Annals Math. 194 no. 1, (2021), arxiv:1912.07973 [math-ph].

[215] M. Aizenman, "A geometric perspective on the scaling limits of critical Ising and φ^4_d models," 12, 2021. arxiv:2112.04248 [math-ph].

[216] K. G. Wilson and M. E. Fisher, "Critical exponents in 3.99 dimensions," Phys. Rev. Lett. 28 (1972) 240-243.

[217] K. G. Wilson, "Quarks and Strings on a Lattice," in 13th International School of Subnuclear Physics: New Phenomena in Subnuclear Physics. 11, 1975.

[218] T. L. Bell and K. G. Wilson, "Finite-lattice approximations to renormalization groups,"

Phys. Rev. $B11$ no. 9, (1975) 3431 .

[219] K. G. Wilson, ”Quantum Chromodynamics on a Lattice,” in Cargese Summer Institute: New Developments in Quantum Field Theory and Statistical Mechanics. 1, 1977.

[220] A. Brothier and A. Stottmeister, ”Canonical quantization of 1+1-dimensional Yang-Mills theory: An operator-algebraic approach,” arxiv:1907.05549 [math-ph].

[221] T. Balaban, ”(Higgs) 2,3 QUANTUM FIELDS IN A FINITE VOLUME. 1. A LOWER BOUND,” Commun. Math. Phys. 85 (1982) 603-636.

[222] T. Balaban, ”(Higgs $)_{2,3}$ QUANTUM FIELDS IN A FINITE VOLUME. 2. AN UPPER BOUND,” Commun. Math. Phys. 86 (1982) 555-594.

[223] T. Balaban, ”(Higgs) 2,3 Quantum Field in a Finite Volume. 3. Renormalization,” Commun. Math. Phys. 88 (1983) 411.

[224] T. Balaban, ”Propagators and Renormalization Transformations for Lattice Gauge Theories. I,” Commun. Math. Phys. 95 (1984) 17-40.

[225] T. Balaban, ”Regularity and Decay of Lattice Green’s Functions,” Commun. Math. Phys. 89(1983) 571.

[226] T. Balaban, ”Propagators and Renormalization Transformations for Lattice Gauge Theories. 2.,” Commun. Math. Phys. 96 (1984) 223

[227] T. Balaban, J. Imbrie, A. M. Jaffe, and D. Brydges, ”The Mass Gap for Higgs Models on a Unit Lattice,” Annals Phys. 158 (1984) 281.

[228] T. Balaban, ”Spaces of Regular Gauge Field Configurations on a Lattice and Gauge Fixing Conditions,” Commun. Math. Phys. 99 (1985) 75.

[229] T. Balaban, J. Imbrie, and A. M. Jaffe, ”Renormalization of the Higgs Model: Minimizers, Propagators and the Stability of Mean Field Theory,” Commun. Math. Phys. 97 (1985) 299

[230] T. Balaban, ”Averaging Operations for Lattice Gauge Theories,” Commun. Math. Phys. 98 (1985) 17-51.

[231] T. Balaban, ”Propagators for Lattice Gauge Theories in a Background Field,” Commun. Math. Phys. 99 (1985) 389.

[232] T. Balaban, ”Ultraviolet Stability of Three-Dimensional Lattice Pure Gauge Field Theories,” Commun. Math. Phys. 102 (1985) 255.

[233] T. Balaban, ”The Variational Problem and Background Fields in Renormalization Group Method for Lattice Gauge Theories,” Commun. Math. Phys. 102 (1985) 277.

[234] T. Balaban, ”Renormalization Group Approach to Lattice Gauge Fields Theories. 1. Generation of Effective Actions in a Small Fields Approximation and a Coupling Constant Renormalization in Four-dimensions,” Commun. Math. Phys. 109 (1987) 249.

[235] T. Balaban, J. Z. Imbrie, and A. M. Jaffe, ”Effective Action and Cluster Properties of the Abelian Higgs Model,” Commun. Math. Phys. 114 (1988) 257.

[236] T. Balaban, ”Renormalization group approach to lattice gauge field theories. II. Cluster expansions,” Communications in Mathematical Physics 116 no. 1, (1988) 1 - 22 .

[237] T. Balaban, ”Convergent Renormalization Expansions for Lattice Gauge Theories,” Commun. Math. Phys. 119 (1988) 243-285.

[238] T. Balaban, M. O’Carroll, and R. Schor, ”Block Averaging Renormalization Group for Lattice and Continuum Euclidean Fermions: Expected and Unexpected Results,” Lett. Math. Phys. 17 (1989) 209-214.

[239] T. Balaban, ”Large Field Renormalization. 1: The Basic Step of the R Operation,” Commun. Math. Phys. 122 (1989) 175-202.

[240] T. Balaban, ”Large Field Renormalization. 2: Localization, Exponentiation, and Bounds for the R Operation,” Commun. Math. Phys. 122 (1989) 355-392.

[241] J. Dimock, ”The Renormalization Group According to Balaban, I. Small fields,” Rev. Math. Phys. 25 no. 7, (2013) 1330010, arxiv:1108.1335 [math-ph].

[242] J. Dimock, ”The Renormalization Group According to Balaban - II. Large fields,” J. Math. Phys. 54 no. 9, (2013) 092301, arxiv:1212.5562 [math-ph].

[243] J. Dimock, ”The Renormalization Group According to Balaban III. Convergence,” Annales Henri Poincare 15 no. 11, (2014) 2133-2175, arxiv:1304.0705 [math-ph].

[244] J. Dimock, ”Nonperturbative renormalization of scalar quantum electrodynamics in d=3,” J. Math. Phys. 56 no. 10, (2015) 102304, arxiv:1502.02946 [math-ph]

[245] J. Dimock, ”Ultraviolet regularity for QED in d=3,” J. Math. Phys. 59 no. 1, (2018) 012301, arxiv:1512.04373 [math-ph].

[246] J. Dimock, ”Multiscale block averaging for QED in $d = 3$,” J. Math. Phys. 61 no. 3, (2020) 032302 , arxiv:1712.10029 [math-ph].

[247] J. Magnen, V. Rivasseau, and R. Seneor, ”Construction of YM(4) with an infrared cutoff,” Commun. Math. Phys. 155 (1993) 325-384.

[248] A. M. Jaffe and E. Witten, ”Quantum Yang-Mills theory,”.

[249] M. R. Douglas, ”Report on the Status of the Yang-Mills Millenium Prize Problem,”. http://ww.claymath.org/sites/default/files/ym2.pdf.

[250] R. Haag and D. Kastler, ”An Algebraic approach to quantum field theory,” J. Math. Phys. 5 (1964) 848-861.

[251] H. Araki, ”Einführung in die axiomatische quantenfeldtheorie.”.

[252] R. Haag, ”Quantum field theories with composite particles and asymptotic conditions,”

Phys. Rev. 112(1958)669 – 673.

[253] I. E. Segal, ”Postulates for General Quantum Mechanics,” Annals of Mathematics 48 no. 4 , (1947)930 – 948.

[254] I. Segal, ”Mathematical Problems of Relativistic Physics,” ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik **43** no. 12, (1963) 572 – 572. [255] R. Haag, Local Quantum Physics: Fields, Particles, Algebras. Texts and Monographs in Physics. Springer, Berlin, Heidelberg, 1996.

[256] H. Araki, Mathematical theory of quantum fields., vol. 101. Oxford: Oxford University Press, 1999 .

[257] H. Baumgaertel and M. Wollenberg, Causal nets of operator algebras: Mathematical aspects of algebraic quantum field theory. Akademie Verlag, 1992.

[258] H. Baumgaertel, Operator algebraic methods in quantum field theory: A Series of lectures. Akademie Verlag, 1995 .

[259] S. Horuzhy, Introduction to Algebraic Quantum Field Theory. Kluwer Academic Publishers, 1990.

[260] G. G. Emch, Algebraic Methods in Statistical Mechanics and Quantum Field Theory. Interscience Monographs and Texts in Physics and Astronomy (Edited by R.E. Marshak). Wiley-Interscience, 1972.

[261] H. J. Borchers, Translation group and particle representations in quantum field theory. vol. 40. 1996 .

[262] O. Bratteli and D. Robinson, Operator algebras and quantum statistical mechanics, I. Springer, 1979.

[263] O. Bratteli and D. Robinson, Operator algebras and quantum statistical mechanics II. Springer, 1981.

[264] J. Dixmier, Von Neumann Algebras. North Holland, 1981, 2011.

[265] J. Dixmier, C^*-Algebras. North Holland, 2011.

[266] R. V. Kadison and J. R. Ringrose, Fundamentals of the Theory of Operator Algebras. American Mathematical Society, Volumes I-IV, 1997.

[267] M. A. Naimark, Normed Algebras. Springer Netherlands, 1 ed., 1972.

[268] G. K. Pedersen, C*-algebras and their automorphism groups. Academic Press, 2 ed., 2018.

[269] S. Sakai, C^*-Algebras and W^*-Algebras. Classics in Mathematics. Springer, springer ed., 1997.

[270] S. Sakai, Operator algebras in dynamical systems. Encyclopedia of Mathematics and its

Applications. Cambridge University Press, 1991. [271] V. Sunder, An invitation to von Neumann algebras. Universitext. Springer-Verlag, 1 ed., 1987.

[272] S. V. Stratila and L. Zsido, Lectures on von Neumann Algebras. Cambridge IISc Series. Cambridge University Press, 2 ed., 2019.

[273] S. V. Stratila, Modular Theory in Operator Algebras. Cambridge Univ Press US, 2 ed., 2021.

[274] M. Takesaki, Theory of operator algebras I. Encyclopaedia of mathematical sciences, Operator algebras and non-commutative geometry. Springer, 2001.

[275] M. Takesaki, Theory of Operator Algebras II. Enc.Math.Sci.125. Springer, 1 ed., 2002.

[276] M. Takesaki, Theory of Operator Algebras III. Enc.Math.Sci.127. Springer, 1 ed., 2002.

[277] V. F. Jones, Von Neumann Algebras. 2009.

[278] D. E. Evans and M. Takesaki, Operator Algebras and Applications: Volume 1, Structure Theory; K-theory, Geometry and Topology. London Mathematical Society Lecture Note Series. Cambridge University Press, 1989.

[279] D. E. Evans and M. Takesaki, Operator Algebras and Applications: Volume 2. London Mathematical Society Lecture Note Series. Cambridge University Press, 1989.

[280] A. Inoue, Tomita-Takesaki Theory in Algebras of Unbounded Operators. Lecture Notes in Mathematics. Springer-Verlag Berlin Heidelberg, 1 ed., 1998.

[281] ”Advances in Algebraic Quantum Field Theory,” Mathematical Physics Studies. Springer International Publishing, 2015.

[282] C. J. Fewster and K. Rejzner, ”Algebraic Quantum Field Theory - an introduction,” arxiv:1904.04051 [hep-th].

[283] H. Halvorson and M. Muger, ”Algebraic quantum field theory,” in Philosophy of physics, J. Butterfield and J. Earman, eds., pp. 731-864. 2007. arxiv:math-ph/0602036

[284] K. Rejzner, Perturbative Algebraic Quantum Field Theory: An Introduction for Mathematicians. Mathematical Physics Studies. Springer International Publishing, 2016.

[285] M. Dütsch, ”From Classical Field Theory to Perturbative Quantum Field Theory,”.

[286] S. Hollands and K. Sanders, ”Entanglement Measures and Their Properties in Quantum Field Theory,”.

[287] ”Haag-kastler axioms.” http://ncatlab.org/nlab/show/Haag-Kastler+axioms

[288] ”Causal complement.” http://ncatlab.org/nlab/show/causal+complement.

[289] I. Gelfand and M. Neumark, ”On the imbedding of normed rings into the ring of operators in Hilbert space,” Rec. Math. [Mat. Sbornik] N.S. 12(54) (1943) 197-217. http://mi.mathnet.ru/msb6155.

[290] I. Segal, ”Irreducible representations of operator algebras,” Bull. Amer. Math. Soc. 53 (1947),

[291] J. von Neumann, ”Zur Algebra der Funktionaloperationen und Theorie der normalen Operatoren,” Mathematische Annalen 102 no. 1, (1930) 370-427.

[292] ”Haag-kastler vacuum representation.”.

[293] F. J. Murray and J. von Neumann, ”On Rings of Operators,” Annals of Mathematics 37 no. 1, (1936) 116-229.

[294] F. J. Murray and J. von Neumann, ”On rings of operators. II,” Trans. Amer. Math. Soc. 41 (1937)$208-248$.

[295] J. von Neumann, ”On infinite direct products,” Compositio Mathematica 6 (1939) 1-77. http://eudml.org/doc/88704.

[296] J. von Neumann, ”On Rings of Operators. III,” Annals of Mathematics 41 no. 1, (1940) $94-161$.

[297] F. J. Murray and J. von Neumann, ”On Rings of Operators. IV,” Annals of Mathematics 44 no. $4,(1943)716-808$.

[298] J. von Neumann, ”On Some Algebraical Properties of Operator Rings,” Annals of Mathematics 44 no. 4, (1943) 709-715.

[299] J. V. Neumann, ”On Rings of Operators. Reduction Theory,” Annals of Mathematics 50 no. $2,(1949)401-485$.

[300] J. Von Neumann, ”Collected Works, Volume III: Rings of Operators,” Pergamon Press, 1961.

[301] H. Araki, ”Type of von Neumann Algebra Associated with Free Field,” Progress of Theoretical Physics 32 no. 6, (12, 1964) 956-965. [302] R. Longo, ”Algebraic And Modular Structure of Von Neumann Algebras of Physics,” vol. 38. 1982.

[303] K. Fredenhagen, ”On the modular structure of local algebras of observables,” Communications in Mathematical Physics 97 no. 1-2, (1985) 79 - 89.

[304] M. Tomita, ”Quasi-standard von neumann algebras,” 1967.

[305] M. Takesaki, Tomita’s Theory of Modular Hilbert Algebras and its Applications. Lecture Notes in Mathematics. Springer-Verlag, 1970.

[306] H. J. Borchers, ”On revolutionizing quantum field theory with Tomita’s modular theory,” J. Math. Phys. 41 (2000) 3604-3673. https://www.mat.univie.ac.at/ esiprpr/esi773.pdf

[307] S. J. Summers, ”Tomita-Takesaki modular theory,” arxiv:math-ph/0511034.

[308] R. Haag, N. M. Hugenholtz, and M. Winnink, ”On the Equilibrium states in quantum statistical mechanics,” Commun. Math. Phys. 5 (1967) 215-236.

[309] M. Takesaki and M. Winnink, "Local normality in quantum statistical mechanics," Commun. Math. Phys. 30 (1973) 129-152.

[310] H. J. Borchers and J. Yngvason, "Modular groups of quantum fields in thermal states," J. Math. Phys. 40 (1999) 601-624, arxiv:math-ph/9805013.

[311] E. Witten, "APS Medal for Exceptional Achievement in Research: Invited article on entanglement properties of quantum field theory," Rev. Mod. Phys. 90 no. 4, (2018) 045003, arxiv:1803.04993 [hep-th].

[312] G. C. Wick, A. S. Wightman, and E. P. Wigner, "The intrinsic parity of elementary particles," Phys. Rev. 88 (1952) 101-105.

[313] G. C. Wick, A. S. Wightman, and E. P. Wigner, "Superselection rule for charge," Phys. Rev. D 1 (1970) 3267-3269.

[314] R. F. Streater, "Outline of Axiomatic Relativistic Quantum Field Theory," Rept. Prog. Phys. $38(1975)771-846$

[315] S. Doplicher, R. Haag, and J. E. Roberts, "Local observables and particle statistics. 1," Commun. Math. Phys. 23 (1971) 199-230.

[316] S. Doplicher, R. Haag, and J. E. Roberts, "Local observables and particle statistics. 2," Commun. Math. Phys. 35 (1974) 49-85. [317] N. P. Landsman, "Quantization and Superselection Sectors. 1. Transformation Group C* Algebras," Rev. Math. Phys. 2 (1990) 45-72.

[318] N. P. Landsman, "Quantization and Superselection Sectors. 2. Dirac Monopole and Aharonov-Bohm Effect," Rev. Math. Phys. 2 (1990) 73-104.

[319] N. P. Landsman, "Algebraic theory of superselection sectors and the measurement problem in quantum mechanics," Int. J. Mod. Phys. A 6 (1991) 5349-5372.

[320] H. Casini, M. Huerta, J. M. Magán, and D. Pontello, "Entanglement entropy and superselection sectors. Part I. Global symmetries," JHEP 02 (2020) 014 arxiv : $1905.10487[hep-th]$

[321] D. Guido and R. Longo, "An Algebraic spin and statistics theorem," Commun. Math. Phys. 172(1995)517, arxiv:funct-an/9406005.

[322] R. Verch, "A spin statistics theorem for quantum fields on curved space-time manifolds in a generally covariant framework," Commun. Math. Phys. 223 (2001) 261-288, arxiv:math-ph/0102035.

[323] H. J. Borchers and J. Yngvason, "On the PCT theorem in the theory of local observables," Fields Inst. Commun. 30 (2001) 39-64, arxiv:math-ph/0012020.

[324] T. Johnson-Freyd, "Spin, statistics, orientations, unitarity," Algebr. Geom. Topol. 17 no. 2 , (2017) 917-956, arxiv:1507.06297 [math-ph].

[325] D. Kastler, D. W. Robinson, and A. Swieca, "Conserved currents and associated symme-

tries; Goldstone's theorem," Commun. Math. Phys. 2 no. 1, (1966) 108-120.

[326] H. Ezawa and J. A. Swieca, "Spontaneous Breakdown of Symmetries and Zero-Mass States," Commun. Math. Phys. 5 (1967) 330-336.

[327] H. J. Borchers and W. Zimmermann, "On the self-adjointness of field operators," Il Nuovo Cimento 31 (1964) 1047-1059.

[328] W. Driessler, S. J. Summers, and E. H. Wichmann, "On the Connection Between Quantum Fields and Von Neumann Algebras of Local Operators," Commun. Math. Phys. 105 (1986) $49-84$

[329] H. J. Borchers and J. Yngvason, "Positivity of Wightman Functionals and the Existence of Local Nets," Commun. Math. Phys. 127 (1990) 607.

[330] H. J. Borchers and J. Yngvason, "Local nets and selfadjoint extensions of quantum field operators," Lett. Math. Phys. 21 (1991) 151-155. [331] H. J. Borchers and J. Yngvason, "From quantum fields to local von Neumann algebras," Rev. Math. Phys. 4 no. spec01, (1992) 15-47.

[332] K. Fredenhagen and J. Hertel, "Local Algebras of Observables and Point - Like Localized Fields," Commun. Math. Phys. 80 (1981) 555.

[333] S. J. Summers, "From Algebras of Local Observables to Quantum Fields: Generalized H Bonds," Helv. Phys. Acta 60 (1987) 1004-1023

[334] K. Fredenhagen and K. Rejzner, "Perturbative algebraic quantum field theory," in Winter School in Mathematical Physics: Mathematical Aspects of Quantum Field Theory, pp. 17-55. Springer, 8, 2012. arxiv:1208.1428 [math-ph].

[335] K. Fredenhagen and K. Rejzner, Perturbative Construction of Models of Algebraic Quantum Field Theory, pp. 31-74. 3, 2015. arxiv:1503.07814 [math-ph].

[336] M. Dütsch, From Classical Field Theory to Perturbative Quantum Field Theory, vol. 74 of Progress in Mathematical Physics. Springer, 2019.

[337] U. Schreiber, "Mathematical Quantum Field Theory," 2017.

[338] "Perturbative algebraic quantum field theory." http: //ncatlab.org/nlab/show/perturbative algebraic quantum field theory.

[339] E. C. G. Stueckelberg, "Relativistic quantum theory for finite time intervals," Phys. Rev., II. Ser. $81(1951)130-133$.

[340] E. C. G. Stueckelberg and D. Rivier, "Causalite et structure de la matrice S," Helv. Phys. Acta 23 (1950) 215-222.

[341] E. C. G. Stueckelberg and A. Petermann, "Normalization of constants in the quanta theory," Helv. Phys. Acta 26 (1953) 499-520.

[342] A. Salam, "Overlapping divergences and the S-matrix," Phys. Rev., II. Ser. 82 (1951)

217 – 227.

[343] N. N. Bogoliubow and O. S. Parasiuk, ”Über die Multiplikation der Kausalfunktionen in der Quantentheorie der Felder,” Acta Math. 97 (1957) 227-266.

[344] H. Epstein and V. Glaser, ”The role of locality in perturbation theory,” Annales de l’I.H.P. Physique théorique 19 no. 3, (1973) 211-295.

[345] V. A. Ilyin and D. A. Slavnov, ”Observable Algebras in the S Matrix Approach,” Teor. Mat. Fiz. 36 (1978) 32-41.

[346] G. Scharf, Finite quantum electrodynamics. Texts and monographs in physics. Springer-Verlag, 1989.

[347] G. Scharf, Finite Quantum Electrodynamics: The Causal Approach. Dover, 3 ed., 2014.

[348] G. Scharf, Gauge field theories : spin one and spin two : 100 years after general relativity. Dover Publications, 2016.

[349] A. Aste, C. von Arx, and G. Scharf, ”Regularization in quantum field theory from the causal point of view,” Prog. Part. Nucl. Phys. 64 (2010) 61-119, arxiv:0906.1952 [hep-th].

[350] M. Dütsch and K. Fredenhagen, ”A Local (perturbative) construction of observables in gauge theories: The Example of QED,” Commun. Math. Phys. 203 (1999) 71-105, arxiv:hep-th/9807078

[351] M. Dütsch and K. Fredenhagen, ”Algebraic quantum field theory, perturbation theory, and the loop expansion,” Commun. Math. Phys. 219 (2001) 5-30, arxiv:hep-th/0001129.

[352] M. Düetsch and K. Fredenhagen, ”The Master Ward Identity and generalized Schwinger-Dyson equation in classical field theory,” Commun. Math. Phys. 243 (2003) 275 – 314, arxiv:hep-th/0211242.

[353] S. Hollands, ”Algebraic Approach to the 1/N Expansion in Quantum Field Theory,” Reviews in Mathematical Physics 16 no. 04, (2003) 509-558.

[354] R. Brunetti and K. Fredenhagen, ”Algebraic approach to quantum field theory,” arxiv:math-ph/0411072.

[355] M. Dütsch and K. Fredenhagen, ”Causal perturbation theory in terms of retarded products, and a proof of the action Ward identity,” Rev. Math. Phys. 16 (2004) 1291-1348 arxiv: hep-th/0403213

[356] M. Dütsch and K. Fredenhagen, ”Action Ward identity and the Stuckelberg-Petermann renormalization group,” Prog. Math. 251 (2007) 113-124, arxiv:hep-th/0501228.

[357] F. Brennecke and M. Dütsch, ”Removal of violations of the Master Ward Identity in perturbative QFT,” Rev. Math. Phys. 20 (2008) 119-172, arxiv:0705.3160 [hep-th].

[358] K. J. Keller, Dimensional Regularization in Position Space and a Forest Formula for

Regularized Epstein-Glaser Renormalization. PhD thesis, Hamburg U., 2010. arxiv:1006.2148 [math-ph]. [359] K. Rejzner, ”Fermionic fields in the functional approach to classical field theory,” Rev. Math. Phys. 23 (2011) 1009-1033, arxiv:1101.5126 [math-ph].

[360] M. Duetsch, K. Fredenhagen, K. J. Keller, and K. Rejzner, ”Dimensional Regularization in Position Space, and a Forest Formula for Epstein-Glaser Renormalization,” J. Math. Phys. 55 (2014) 122303, arxiv:1311.5424 [hep-th].

[361] S. Crawford, K. Rejzner, and B. Vicedo, ”Lorentzian 2D CFT from the pAQFT Perspective,” Annales Henri Poincare 23 no. 10, (2022) 3525-3585, arxiv:2107.12347 [math-ph].

[362] S. Crawford, K. Rejzner, and B. Vicedo, ”Chirality in 2d pAQFT,” arxiv:2205.01003 [math-ph].

[363] R. Brunetti, M. Dütsch, and K. Fredenhagen, ”Perturbative Algebraic Quantum Field Theory and the Renormalization Groups,” Adv. Theor. Math. Phys. 13 no. 5, (2009) 1541-1599, arxiv:0901.2038 [math-ph].

[364] R. Stora, ”Renormalized Perturbation Theory: A Missing Chapter,” Int. J. Geom. Meth. Mod. Phys. 5 (2008) 1345-1360, arxiv:0901.3426 [hep-th].

[365] K. Fredenhagen and K. Rejzner, ”Batalin-Vilkovisky formalism in the functional approach to classical field theory,” Commun. Math. Phys. 314 (2012) 93-127, arxiv:1101.5112 [math-ph].

[366] K. Fredenhagen and K. Rejzner, ”Batalin-Vilkovisky Formalism in Perturbative Algebraic Quantum Field Theory,” Communications in Mathematical Physics 317 (2011) 697-725, 1110.5232.

[367] K. A. Rejzner, Batalin-Vilkovisky formalism in locally covariant field theory. PhD thesis, Hamburg U., 2011. arxiv:1111.5130 [math-ph].

[368] K. Rejzner, ”Remarks on Local Symmetry Invariance in Perturbative Algebraic Quantum Field Theory,” Annales Henri Poincare 16 no. 1, (2015) 205-238, arxiv:1301.7037 [math-ph].

[369] K. Rejzner, ”BV quantization in perturbative algebraic QFT: Fundamental concepts and perspectives,” 4, 2020. arxiv:2004.14272 [math-ph].

[370] I. A. Batalin and G. A. Vilkovisky, ”Gauge Algebra and Quantization,” Phys. Lett. B 102 (1981) 27-31

[371] I. A. Batalin and G. A. Vilkovisky, ”Quantization of Gauge Theories with Linearly Dependent Generators,” Phys. Rev. D 28 (1983) 2567-2582. [Erratum: Phys.Rev.D 30, 508 (1984)]

[372] J. Dito, ”Star product approach to quantum field theory: The free scalar field,” Lett. Math. Phys. 20 (1990) 125-134.

[373] M. Dütsch and K. Fredenhagen, ”Perturbative algebraic field theory, and deformation quantization,” Fields Inst. Commun. 30 (2001) 151-160, arxiv:hep-th/0101079.

[374] A. C. Hirshfeld and P. Henselder, ”Star products and perturbative quantum field theory,” Annals Phys. 298 (2002) 382-393, arxiv:hep-th/0208194.

[375] G. Collini, ”Fedosov Quantization and Perturbative Quantum Field Theory,” arxiv: 1603.09626[math-ph].

[376] E. Hawkins and K. Rejzner, ”The star product in interacting quantum field theory,” Letters in Mathematical Physics 110 (2016) 1257-1313, 1612.09157.

[377] L. Parker, ”Quantized fields and particle creation in expanding universes. 1.,” Phys. Rev. $183(1969)1057-1068$.

[378] S. W. Hawking, ”Particle Creation by Black Holes,” Commun. Math. Phys. 43 (1975) 199-220, [Erratum: Commun.Math.Phys. 46, 206 (1976)].

[379] Y. B. Zeldovich and A. A. Starobinsky, ”Particle production and vacuum polarization in an anisotropic gravitational field,” Zh. Eksp. Teor. Fiz. 61 (1971) 2161-2175.

[380] A. A. Starobinsky, ”Amplification of waves reflected from a rotating ”black hole”.,” Sov. Phys. JETP 37 no. 1, (1973) 28-32.

[381] W. G. Unruh, ”Second quantization in the Kerr metric,” Phys. Rev. D 10 (1974) 3194-3205.

[382] S. A. Fulling, ”Nonuniqueness of canonical field quantization in Riemannian space-time,” Phys. Rev. D 7 (1973) 2850-2862.

[383] P. C. W. Davies, ”Scalar particle production in Schwarzschild and Rindler metrics,” J. Phys. A 8 (1975) 609-616.

[384] W. G. Unruh, ”Notes on black hole evaporation,” Phys. Rev. D 14 (1976) 870.

[385] F. J. Dyson, ”Missed opportunities,” Bull. Am. Math. Soc. 78 (1972) 635-639.

[386] A. Ashtekar and A. Magnon, ”Quantum Fields in Curved Space-Times,” Proc. Roy. Soc. Lond. A 346 (1975) 375-394. [387] J. Dimock, ”Algebras of local observables on a manifold,” Communications in Mathematical Physics 77(1980) 219-228.

[388] J. Dimock, ”Dirac quantum fields on a manifold,” Trans. Amer. Math. Soc. 269 (1982) $133-147$.

[389] G. L. Sewell, ”Quantum fields on manifolds: PCT and gravitationally induced thermal states,” Annals Phys. 141 (1982) 201-224.

[390] B. S. Kay, ”Linear Spin 0 Quantum Fields in External Gravitational and Scalar Fields. 1. A One Particle Structure for the Stationary Case,” Commun. Math. Phys. 62 (1978) 55-70.

[391] B. S. Kay, ”The Double Wedge Algebra for Quantum Fields on Schwarzschild and Minkowski Space-times,” Commun. Math. Phys. 100 (1985) 57.

[392] S. A. Fulling, M. Sweeny, and R. M. Wald, ”Singularity Structure of the Two Point

Function in Quantum Field Theory in Curved Space-Time," Commun. Math. Phys. 63 (1978)257−264.

[393] S. A. Fulling, F. J. Narcowich, and R. M. Wald, "Singularity Structure of the Two Point Function in Quantum Field Theory in Curved Space-time. II," Annals Phys. 136 (1981) 243 − 272,

[394] R. M. Wald, Quantum field theory in curved spacetime and black hole thermodynamics. Chicago, IL: Univ. of Chicago Press, 1994.

[395] M. Radzikowski, The Hadamard condition and Kay's conjecture in (axiomatic) quantum field theory on curved space-time. PhD thesis, Princeton University, 1992.

[396] M. J. Radzikowski, "Micro-local approach to the Hadamard condition in quantum field theory on curved space-time," Commun. Math. Phys. 179 (1996) 529-553.

[397] M. J. Radzikowski and R. Verch, "A Local to global singularity theorem for quantum field theory on curved space-time," Commun. Math. Phys. 180 (1996) 1-22.

[398] R. Brunetti, K. Fredenhagen, and M. Kohler, "The Microlocal spectrum condition and Wick polynomials of free fields on curved space-times," Commun. Math. Phys. 180 (1996) 633−652, arxiv:gr-qc/9510056

[399] R. Brunetti and K. Fredenhagen, "Microlocal analysis and interacting quantum field theories: Renormalization on physical backgrounds," Commun. Math. Phys. 208 (2000) 623-661, arxiv:math-ph/9903028.

[400] R. Brunetti, K. Fredenhagen, and R. Verch, "The Generally covariant locality principle: A New paradigm for local quantum field theory," Commun. Math. Phys. 237 (2003) 31-68, arxiv:math-ph/0112041.

[401] S. Hollands and R. M. Wald, "Local Wick polynomials and time ordered products of quantum fields in curved space-time," Commun. Math. Phys. 223 (2001) 289-326, arxiv:gr-qc/0103074.

[402] S. Hollands and R. M. Wald, "Existence of local covariant time ordered products of quantum fields in curved space-time," Commun. Math. Phys. 231 (2002) 309-345, arxiv:gr-qc/0111108.

[403] H. Sahlmann and R. Verch, "Microlocal spectrum condition and Hadamard form for vector valued quantum fields in curved space-time," Rev. Math. Phys. 13 (2001) 1203-1246, arxiv:math-ph/0008029.

[404] A. Strohmaier, R. Verch, and M. Wollenberg, "Microlocal analysis of quantum fields on curved space-times: Analytic wavefront sets and Reeh-Schlieder theorems," J. Math. Phys. 43 (2002) 5514-5530, arxiv:math-ph/0202003.

[405] S. Hollands and R. M. Wald, "On the renormalization group in curved space-time,"

Commun. Math. Phys. 237 (2003) 123-160, arxiv:gr-qc/0209029.

[406] S. Hollands and R. M. Wald, ”Conservation of the stress tensor in perturbative interacting quantum field theory in curved spacetimes,” Rev. Math. Phys. 17 (2005) 227-312 arxiv:gr-qc/0404074

[407] K. Sanders, ”Equivalence of the (Generalised) Hadamard and Microlocal Spectrum Condition for (Generalised) Free Fields in Curved Spacetime,” Communications in Mathematical Physics 295 (2010) 485-501, 0903.1021.

[408] B. S. DeWitt, ”Quantum Field Theory in Curved Space-Time,” Phys. Rept. 19 (1975) 295-357.

[409] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space. Cambridge Monographs on Mathematical Physics. Cambridge Univ. Press, Cambridge, UK, 2, 1984.

[410] S. A. Fulling, Aspects of Quantum Field Theory in Curved Space-time, vol. 17. Cambridge University Press, 1989.

[411] I. L. Buchbinder, S. D. Odintsov, and I. L. Shapiro, Effective Action in Quantum Gravity. Institute of Physics Publishing, 1992. [412] L. Ford, Quantum field theory in curved spacetime. lectures ed., 1997.

[413] L. Parker and D. Toms, Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2009.

[414] ”Quantum Field Theory on Curved Spacetimes: Concepts and Mathematical Foundations,” Lecture Notes in Physics. Springer-Verlag, 2009.

[415] S. Hollands and R. M. Wald, ”Quantum fields in curved spacetime,” Phys. Rept. 574 (2015) 1-35, arxiv:1401.2026 [gr-qc].

[416] E. Witten, ”Why Does Quantum Field Theory In Curved Spacetime Make Sense? And What Happens To The Algebra of Observables In The Thermodynamic Limit?,” arxiv:2112.11614 [hep-th].

[417] E. Witten, ”Gravity and the Crossed Product,” arxiv:2112.12828 [hep-th].

[418] B. S. Kay, ”The Principle of locality and quantum field theory on (nonglobally hyperbolic) curved space-times,” Rev. Math. Phys. 4 no. spec01, (1992) 167-195.

[419] R. Brunetti and K. Fredenhagen, ”Quantum Field Theory on Curved Backgrounds,” Lect. Notes Phys. 786 (2009) 129, arxiv:0901.2063 [gr-qc].

[420] C. J. Fewster and R. Verch, ”Algebraic Quantum Field Theory in Curved Spacetimes,” Advances in Algebraic Quantum Field Theory. Mathematical Physics Studies., pp. 125-189. Springer, 2015. 1504.00586 .

[421] K. Fredenhagen, ”Locally covariant quantum field theory,” in 14th International Congress

on Mathematical Physics, pp. 29-37. 3, 2004. arxiv:hep-th/0403007.

[422] R. M. Wald, ”The History and Present Status of Quantum Field Theory in Curved Spacetime,” Einstein Stud. 12 (2012) 317-331, arxiv:gr-qc/0608018.

[423] R. M. Wald, ”The Formulation of Quantum Field Theory in Curved Spacetime,” Einstein Stud. 14 (2018) 439-449, arxiv:0907.0416 [gr-qc].

[424] K. Fredenhagen and K. Rejzner, ”Quantum field theory on curved spacetimes: Axiomatic framework and examples,” J. Math. Phys. 57 no. 3, (2016) 031101, arxiv:1412.5125 [math-ph].

[425] C. J. Fewster, ”Locally covariant quantum field theory and the problem of formulating the same physics in all space-times,” Phil. Trans. Roy. Soc. Lond. A 373 (2015) 20140238, arxiv:1502.04642 [gr-qc]. [426] C. J. Fewster and R. Verch, ”Dynamical locality and covariance: What makes a physical theory the same in all spacetimes?,” Annales Henri Poincare 13 (2012) 1613-1674, arxiv : 1106.4785[math-ph].

[427] C. J. Fewster and R. Verch, ”Dynamical locality of the free scalar field,” Annales Henri Poincare 13 (2012) 1675-1709, arxiv:1109.6732 [math-ph],

[428] C. J. Fewster, ”On the notion of ’the same physics in all spacetimes’,”] in Quantum Field Theory and Gravity: Conceptual and Mathematical Advances in the Search for a Unified Framework, pp. 207-227. 2012. arxiv:1105.6202 [math-ph].

[429] D. Guido, R. Longo, J. E. Roberts, and R. Verch, ”Charged sectors, spin and statistics in quantum field theory on curved space-times,” Rev. Math. Phys. 13 (2001) 125-198, arxiv:math-ph/9906019.

[430] R. Brunetti and G. Ruzzi, ”Superselection sectors and general covariance. I.,” Commun. Math. Phys. 270 (2007) 69-108, arxiv:gr-qc/0511118.

[431] C. J. Fewster, ”Endomorphisms and automorphisms of locally covariant quantum field theories,” Rev. Math. Phys. 25 (2013) 1350008, arxiv:1201.3295 [math-ph].

[432] C. J. Fewster and R. Verch, ”The Necessity of the Hadamard Condition,” Class. Quant. Grav. 30 (2013) 235027, arxiv:1307.5242 [gr-qc].

[433] C. J. Fewster and B. Lang, ”Pure quasifree states of the Dirac field from the fermionic projector,” Classical and Quantum Gravity (2015), 1408.1645.

[434] M. Brum and K. Fredenhagen, ”’Vacuum-like’ Hadamard states for quantum fields on curved spacetimes,” Class. Quant. Grav. 31 (2014) 025024, arxiv:1307.0482 [gr-qc].

[435] C. J. Fewster and A. Schenkel, ”Locally covariant quantum field theory with external sources,” Annales Henri Poincare 16 no. 10, (2015) 2303-2365, arxiv:1402.2436 [math-ph].

[436] C. Becker, A. Schenkel, and R. J. Szabo, ”Differential cohomology and locally covariant quantum field theory,” Rev. Math. Phys. 29 no. 01, (2016) 1750003, arxiv:1406.1514 [hep-th].

[437] C. J. Fewster, ”The Split Property for Locally Covariant Quantum Field Theories in Curved Spacetime,” Letters in Mathematical Physics 105 (2015) 1633-1661, 1501.02682.

[438] M. Ferguson, ”Dynamical locality of the nonminimally coupled scalar field and enlarged algebra of Wick polynomials,” Annales Henri Poincare 14 (2013) 853-892,

arxiv:1203.2151 [math-ph]. [439] K. Sanders, ”On the Reeh-Schlieder Property in Curved Spacetime,” Communications in Mathematical Physics 288 (2008) 271-285, 0801.4676.

[440] S. Hollands and R. M. Wald, ”Axiomatic quantum field theory in curved spacetime,” Commun. Math. Phys. 293 (2010) 85-125, arxiv:0803.2003 [gr-qc].

[441] I. Khavkine and V. Moretti, ”Analytic Dependence is an Unnecessary Requirement in Renormalization of Locally Covariant QFT,” Commun. Math. Phys. 344 no. 2, (2016) 581-620, 、 arxiv: 1411.1302[gr-qc].

[442] S. Hollands, ”Renormalized Quantum Yang-Mills Fields in Curved Spacetime,” Rev. Math. Phys. 20 (2008) 1033-1172, arxiv:0705.3340 [gr-qc].

[443] J. Dimock, ”Quantized electromagnetic field on a manifold,” Rev. Math. Phys. 4 (1992) 223-233.

[444] M. J. Pfenning, ”Quantization of the Maxwell field in curved spacetimes of arbitrary dimension,” Class.Quant.Grav. 26 (2009), 0902.4887.

[445] C. Dappiaggi and B. Lang, ”Quantization of Maxwell’s equations on curved backgrounds and general local covariance,” Lett. Math. Phys. 101 (2012) 265-287, arxiv:1104.1374 [gr-qc].

[446] C. Dappiaggi and D. Siemssen, ”Hadamard States for the Vector Potential on Asymptotically Flat Spacetimes,” Rev. Math. Phys. 25 (2013) 1350002, arxiv:1106.5575 [gr-qc].

[447] K. Sanders, C. Dappiaggi, and T.-P. Hack, ”Electromagnetism, Local Covariance, the Aharonov-Bohm Effect and Gauss’ Law,” Commun. Math. Phys. 328 (2014) 625-667, arxiv : 1211.6420[math-ph].

[448] T.-P. Hack and A. Schenkel, ”Linear bosonic and fermionic quantum gauge theories on curved spacetimes,” Gen. Rel. Grav. 45 (2013) 877-910, arxiv:1205.3484 [math-ph].

[449] D. Buchholz and K. Fredenhagen, ”Correction to: A C*-algebraic Approach to Interacting Quantum Field Theories [doi: 10.1007/s00220-020-03700-9],” Commun. Math. Phys. 377 no. 2, (2020) 947-969, arxiv:1902.06062 [math-ph].

[450] D. Buchholz and K. Fredenhagen, ”Classical dynamics, arrow of time, and genesis of the Heisenberg commutation relations,” Expositiones Mathematicae 38 no. 2, (2020) 150-167, arxiv:1905.02711 [quant-ph]. Special Issue in Honor of R.V. Kadison (1925-2018).

[451] D. Buchholz and K. Fredenhagen, ”From path integrals to dynamical algebras: a macroscopic view of quantum physics,” Found. Phys. 50 no. 7, (2020) 727-734, arxiv:1905.04250 [quant-

ph].

[452] D. Buchholz and K. Fredenhagen, ”Dynamical C*-algebras and kinetic perturbations,” Annales Henri Poincare 22 no. 3, (2021) 1001-1033, arxiv:2008.02034 [math-ph].

[453] R. Brunetti, M. Dütsch, K. Fredenhagen, and K. Rejzner, ” C^*-algebraic approach to interacting quantum field theory: Inclusion of Fermi fields,” arxiv:2103.05740 [math-ph].

[454] R. Brunetti, M. Dütsch, K. Fredenhagen, and K. Rejzner, ”The unitary Master Ward Identity: Time slice axiom, Noether's Theorem and Anomalies,” arxiv:2108.13336 [math-ph].

[455] R. Brunetti, M. Dütsch, K. Fredenhagen, and K. Rejzner, ”Unitary, anomalous Master Ward Identity and its connections to the Wess-Zumino condition, BV formalism and L_∞-algebras,” arxiv:2210.05908 [math-ph].

[456] R. Brunetti, K. Fredenhagen, and K. Rejzner, ”Locally covariant approach to effective quantum gravity,” arxiv:2212.07800 [gr-qc].

[457] ”On the problem of gauge theories in locally covariant qft,” Trento, 2014., talk at Operator and Geometric Analysis on Quantum Theory.

[458] ”Towards homotopical algebraic quantum field theory,” Mainz Institute for Theoretical Physics, 2017. http://aschenkel.eu/Mainz17.pdf. talk at Foundational and Structural Aspects of Gauge Theories.

[459] ”From fredenhagen's universal algebra to homotopy theory and operads,” Hamburg, 2017. http://aschenkel.eu/Hamburg17.pdf. talk at Quantum Physics meets Mathematics.

[460] C. J. Fewster and B. Lang, ”Dynamical Locality of the Free Maxwell Field,” Annales Henri Poincaré 17 (2016) 401-436, 1403.7083

[461] M. Benini and A. Schenkel, ”Higher Structures in Algebraic Quantum Field Theory,” Fortsch. Phys. 67 no. 8-9, (2019) 1910015, arxiv:1903.02878 [hep-th].

[462] S. Bruinsma, ”Coloring Operads for Algebraic Field Theory,” Fortsch. Phys. 67 no. 8-9, (2019) 1910004, arxiv:1903.02863 [hep-th].

[463] D. Yau, ”Homotopical Quantum Field Theory,” 1802.08101. [464] M. Benini, C. Dappiaggi, and A. Schenkel, ”Quantized Abelian principal connections on Lorentzian manifolds,” Commun. Math. Phys. 330 (2014) 123-152, arxiv:1303.2515 [math-ph].

[465] M. Benini, A. Schenkel, and R. J. Szabo, ”Homotopy colimits and global observables in Abelian gauge theory,” Lett. Math. Phys. 105 no. 9, (2015) 1193-1222, arxiv:1503.08839 [math-ph].

[466] M. Benini and A. Schenkel, ”Quantum field theories on categories fibered in groupoids,” Commun. Math. Phys. 356 no. 1, (2017) 19-64, arxiv:1610.06071 [math-ph].

[467] M. Benini, A. Schenkel, and U. Schreiber, ”The Stack of Yang-Mills Fields on Lorentzian

Manifolds," Commun. Math. Phys. 359 no. 2, (2018) 765-820, arxiv:1704.01378 [math-ph].

[468] M. Benini, A. Schenkel, and L. Woike, "Operads for algebraic quantum field theory," Commun. Contemp. Math. 23 no. 02, (2021) 2050007, arxiv:1709.08657 [math-ph].

[469] M. Benini, A. Schenkel, and L. Woike, "Homotopy theory of algebraic quantum field theories," Lett. Math. Phys. 109 no. 7, (2019) 1487-1532, arxiv:1805.08795 [math-ph].

[470] M. Benini, S. Bruinsma, and A. Schenkel, "Linear Yang-Mills Theory as a Homotopy AQFT," Commun. Math. Phys. 378 no. 1, (2019) 185-218, arxiv:1906.00999 [math-ph].

[471] M. Benini, M. Perin, A. Schenkel, and L. Woike, "Categorification of algebraic quantum field theories," Lett. Math. Phys. 111 (2021) 35, arxiv:2003.13713 [math-ph].

[472] M. Benini, A. Schenkel, and B. Vicedo, "Homotopical Analysis of 4d Chern-Simons Theory and Integrable Field Theories," Commun. Math. Phys. 389 no. 3, (2022) 1417-1443, arxiv : 2008.01829[[hep-th]

[473] V. Carmona, "Algebraic Quantum Field Theories: a homotopical view," 2021. arxiv: 2107.14176 [math-ph].

[474] S. Bruinsma, C. J. Fewster, and A. Schenkel, "Relative Cauchy evolution for linear homotopy AQFTs," arxiv:2108.10592 [math-ph].

[475] A. Anastopoulos and M. Benini, "Homotopy theory of net representations," arxiv:2201.06464 [math-ph].

[476] A. Grant-Stuart, "Spacetimes categories and disjointness for algebraic quantum field theory," arxiv:2201.09166 [math-ph]. [477] "Homotopical algebraic quantum field theory." https :

//ncatlab.org/nlab/show/homotopical algebraic quantum field theory.

[478] S. Doplicher, R. Haag, and J. E. Roberts, "Fields, observables and gauge transformations I," Commun. Math. Phys. 13 (1969) 1-23.

[479] S. Doplicher, R. Haag, and J. E. Roberts, "Fields, observables and gauge transformations. 2.," Commun. Math. Phys. 15 (1969) 173-200.

[480] D. Buchholz and K. Fredenhagen, "Locality and the Structure of Particle States," Commun. Math. Phys. 84(1982)1.

[481] J. Frohlich and T. Kerler, Quantum groups, quantum categories and quantum field theory. 6, 1993.

[482] H. Casini, M. Huerta, J. M. Magan, and D. Pontello, "Entropic order parameters for the phases of QFT," JHEP 04 (2021) 277, arxiv:2008.11748 [hep-th].

[483] H. Casini and J. M. Magan, "On completeness and generalized symmetries in quantum field theory," Mod. Phys. Lett. A 36 no. 36, (2021) 2130025, arxiv:2110.11358 [hep-th].

[484] V. Benedetti, H. Casini, and J. M. Magan, "Generalized symmetries of the graviton,"

JHEP 05 (2022) 045, arxiv:2111.12089 [hep-th].

[485] H. Araki, ”A Lattice of Von Neumann Algebras Associated with the Quantum Theory of a Free Bose Field,” Journal of Mathematical Physics 4 no. 11, (1963) 1343-1362.

[486] R. Brunetti, D. Guido, and R. Longo, ”Modular structure and duality in conformal quantum field theory,” Commun. Math. Phys. 156 (1993) 201-220, arxiv:funct-an/9302008.

[487] A. Beilinson and V. G. Drinfeld, Chiral algebras. American Mathematical Society, 2004.

[488] J. Lurie, ”On the Classification of Topological Field Theories,” Current Developments in Mathematics 2008 (2009) 129-280, arxiv:0905.0465 [math.CT].

[489] K. Costello, Renormalization and effective field theory, vol. 170 of Mathematical Surveys and Monographs. American Mathematical Society, 2011.

[490] K. Costello and O. Gwilliam, Factorization algebras in quantum field theory. Vol.1. New mathematical monographs: 31. Cambridge University Press, 2017.

[491] K. Costello and O. Gwilliam, Factorization Algebras in Quantum Field Theory. Vol.2. New Mathematical Monographs: 41. Cambridge University Press, 2021. [492] O. Gwilliam, Factorization algebras and free field theories. PhD thesis, Northwestern University, 2012.

[493] O. Gwilliam and K. Rejzner, ”Relating Nets and Factorization Algebras of Observables: Free Field Theories,” Communications in Mathematical Physics 373 (2017) 107-174. 1711.06674.

[494] O. Gwilliam and K. Rejzner, ”The observables of a perturbative algebraic quantum field theory form a factorization algebra,” arxiv:2212.08175 [math-ph].

[495] M. Benini, M. Perin, and A. Schenkel, ”Model-independent comparison between factorization algebras and algebraic quantum field theory on Lorentzian manifolds,” Commun. Math. Phys. 377 no. 2, (2019) 971-997, arxiv:1903.03396 [math-ph].

[496] B. Williams, ”The Virasoro vertex algebra and factorization algebras on Riemann surfaces,” Letters in Mathematical Physics 107 no. 12, (Aug, 2017) 2189-2237, 1603.02349.

[497] V. Gorbounov, O. Gwilliam, and B. R. Williams, ”Chiral differential operators via Batalin-Vilkovisky quantization,” Asterisque 419 (2020), 1610.09657.

[498] O. Gwilliam and B. Williams, ”The holomorphic bosonic string,” arxiv:1711.05823 [math-ph].

[499] C. Elliott, B. Williams, and P. Yoo, ”Asymptotic Freedom in the BV Formalism,” J. Geom. Phys. 123 (2018) 246-283, arxiv:1702.05973 [math-ph]

[500] B. R. Williams, ”Renormalization for Holomorphic Field Theories,” Communications in Mathematical Physics 374 (2018) 1693-1742, 1809.02661.

[501] O. Gwilliam and B. R. Williams, ”Higher Kac-Moody algebras and symmetries of holomorphic field theories,” Adv. Theor. Math. Phys. 25 no. 1, (2021) 129-239, arxiv:1810.06534

[math.QA]

[502] I. Saberi and B. R. Williams, ”Twisted characters and holomorphic symmetries,” arxiv: 1906.04221 [math-ph].

[503] I. Saberi and B. R. Williams, ”Superconformal algebras and holomorphic field theories,” arxiv:1910.04120 [math-ph].

[504] O. Gwilliam and B. R. Williams, ”A one-loop exact quantization of Chern-Simons theory,” 1910.05230.

[505] O. Gwilliam, E. Rabinovich, and B. R. Williams, ”Factorization algebras and abelian CS/WZW-type correspondences,” arxiv:2001.07888 [math.QA].

[506] C. Elliott and B. R. Williams, ”Holomorphic Poisson Field Theories,” arxiv: 2008.02302 [math-ph].

[507] I. Saberi and B. R. Williams, ”Constraints in the BV formalism: six-dimensional supersymmetry and its twists,” arxiv: 2009.07116 [math-ph].

[508] E. Rabinovich, ”Factorization Algebras for Classical Bulk-Boundary Systems,” arxiv: 2008.04953 [math.QA]

[509] D. Bruegmann, ”Vertex Algebras and Costello-Gwilliam Factorization Algebras,” arxiv: 2012.12214 [math.QA]

[510] E. Rabinovich, Factorization Algebras for Bulk-Boundary Systems. PhD thesis, University of California, Berkeley, 2021. 2111.01757.

[511] C. Elliott, O. Gwilliam, and B. R. Williams, ”Higher Deformation Quantization for Kapustin-Witten Theories,” arxiv: 2108.13392 [math-ph].

[512] G. Ginot, T. Tradler, and M. Zeinalian, ”Higher Hochschild Homology, Topological Chiral Homology and Factorization Algebras,” Communications in Mathematical Physics 326 (2010) 635-686,1011.6483

[513] G. Ginot, ”Notes on factorization algebras, factorization homology and applications,” Mathematical Aspects of Quantum Field Theories, Mathematical Physics Studies, pp. 429-552. 2013. 1307.5213

[514] J. Francis and D. Gaitsgory, ”Chiral Koszul duality,” Selecta Mathematica 18 (2011) 27-87, 1103.5803.

[515] E. Cliff, ”Universal factorization spaces and algebras,” Mathematical Research Letters 26 no. 4, (2019) 1059-1096, 1608.08122 .

[516] ”Factorization algebra.” https://ncatlab.org/nlab/show/factorization+algebra.

[517] S. Hollands, ”Quantum field theory in terms of consistency conditions. I. General framework, and perturbation theory via Hochschild cohomology,” SIGMA 5 (2009) 090, arxiv:0802.2198

[hep-th].

[518] M. Atiyah, ”Topological quantum field theories,” Publications Mathématiques de l’Institut des Hautes Études Scientifiques 68 no. 1, (Jan, 1988) 175-186. https://doi.org/10.1007/BF02698547.

[519] G. B. Segal, ”The Definition of Conformal Field Theory,” in Differential Geometrical Methods in Theoretical Physics, vol. 250, pp. 165-171. 1987.

[520] G. Segal, The definition of conformal field theory, p. 421-577. London Mathematical Society Lecture Note Series. Cambridge University Press, 2004.

[521] G. Segal, ”Three roles of quantum field theory.”, May, 2011.

[522] P. Teichner and S. Stolz, ”What is an elliptic object?,” Topology, geometry and quantum field theory, 247-343 (2004) 308 (06, 2004). https://math.berkeley.edu/ teichner/Papers/0xford.pdf.

[523] S. Stolz and P. Teichner, ”Supersymmetric field theories and generalized cohomology,” vol. 83 of Proceedings of Symposia in Pure Mathematics, pp. 279-340. 2011.

[524] ”Functorial field theory.” https://ncatlab.org/nlab/show/functorial+field+theory.

[525] M. Ludewig and A. Stoffel, ”A Framework for Geometric Field Theories and their Classification in Dimension One,” Symmetry, Integrability and Geometry: Methods and Applications (Jul, 2021), http://dx.doi.org/10.3842/SIGMA.2021.072.

[526] D. Grady and D. Pavlov, ”Extended field theories are local and have classifying spaces,” 2011.01208.

[527] D. Grady and D. Pavlov, ”The geometric cobordism hypothesis,” 2111.01095.

[528] M. Kontsevich and G. Segal, ”Wick Rotation and the Positivity of Energy in Quantum Field Theory,” Quart. J. Math. Oxford Ser. 72 no. 1-2, (2021) 673-699, arxiv:2105.10161 [hep-th].

[529] D. S. Freed and C. Teleman, ”Relative quantum field theory,” Commun. Math. Phys. 326 (2014) 459-476, arxiv:1212.1692 [hep-th].

[530] A. S. Cattaneo, P. Mnev, and N. Reshetikhin, ”Classical BV theories on manifolds with boundary,” Commun. Math. Phys. 332 (2014) 535-603, arxiv:1201.0290 [math-ph].

[531] A. S. Cattaneo, P. Mnev, and N. Reshetikhin, ”Perturbative quantum gauge theories on manifolds with boundary,” Commun. Math. Phys. 357 no. 2, (2018) 631-730, arxiv:1507.01221 [math-ph].

[532] A. S. Cattaneo, P. Mnev, and N. Reshetikhin, ”Perturbative BV theories with Segal-like gluing,” arxiv:1602.00741 [math-ph].

[533] A. S. Cattaneo, P. Mnev, and N. Reshetikhin, ”A cellular topological field theory,” Commun. Math. Phys. 374 no. 2, (2020) 1229-1320, arxiv:1701.05874 [math.AT].

[534] J. Dimock, ”Markov Quantum Fields on a Manifold,” Reviews in Mathematical Physics 16 no. 02, (Mar, 2004) 243-255, math-ph/0305017.

[535] U. Schreiber, ”AQFT from n-Functorial QFT,” Communications in Mathematical Physics 291 no. 2, (May, 2009) 357-401, 0806.1079.

[536] M. Dedushenko, ”Gluing. Part I. Integrals and symmetries,” JHEP 04 (2020) 175 arxiv:1807.04274 [hep-th].

[537] P. Kravchuk, J. Qiao, and S. Rychkov, ”Distributions in CFT. Part I. Cross-ratio space,” JHEP 05(2020)137, arxiv:2001.08778 [hep-th].

[538] P. Kravchuk, J. Qiao, and S. Rychkov, ”Distributions in CFT. Part II. Minkowski space,” JHEP 08 (2021) 094, arxiv:2104.02090 [hep-th].

[539] F. Gabbiani and J. Frohlich, ”Operator algebras and conformal field theory,” Commun. Math. Phys. 155 (1993) 569-640.

[540] R. Longo, ”Conformal subnets and intermediate subfactors,” Commun. Math. Phys. 237 (2003) 7–30, arxiv:math/0102196.

[541] Y. Kawahigashi and R. Longo, ”Classification of local conformal nets: Case c < 1,” Annals Math. 160 (2004) 493-522, arxiv:math-ph/0201015.

[542] Y. Kawahigashi and R. Longo, ”Classification of two-dimensional local conformal nets with c less than 1 and 2 cohomology vanishing for tensor categories,” Commun. Math. Phys. 244 (2004) 63-97, arxiv:math-ph/0304022.

[543] Y. Kawahigashi, ”Classification of operator algebraic conformal field theories in dimensions one and two,” in 14th International Congress on Mathematical Physics, pp. 476-485. 8, 2003. arxiv:math-ph/0308029.

[544] S. Carpi, R. Hillier, Y. Kawahigashi, R. Longo, and F. Xu, ”N =2 Superconformal Nets,” Commun. Math. Phys. 336 (2015) 1285-1328, arxiv:1207.2398 [math.0A].

[545] S. Carpi, R. Conti, R. Hillier, and M. Weiner, ”Representations of Conformal Nets, Universal C*-Algebras and K-Theory,” Commun. Math. Phys. 320 (2013) 275-300, arxiv:1202.2543 [math.OA]

[546] S. Carpi, Y. Kawahigashi, R. Longo, and M. Weiner, ”From vertex operator algebras to conformal nets and back,” arxiv:1503.01260 [math.OA]

[547] S. Carpi, ”Operator algebras and vertex operator algebras,” in The Fourteenth Marcel Grossmann Meeting. WORLD SCIENTIFIC, Nov, 2017. [548] J. E. Tener, ”Geometric realization of algebraic conformal field theories,” Advances in Mathematics 349 (2019) 488-563, arxiv: 1611.01176.

[549] J. E. Tener, ”Representation theory in chiral conformal field theory: from fields to observables,” arxiv:1810.08168 [math-ph].

[550] A. Bartels, C. L. Douglas, and A. G. Henriques, ”Conformal nets and local field theory,”

arxiv:0912.5307 [math.AT].

[551] A. Bartels, C. L. Douglas, and A. Henriques, ”Conformal Nets I: Coordinate-Free Nets,” International Mathematics Research Notices 2015 no. 13, (06, 2014) 4975-5052, arxiv: 1302. 2604

[552] A. Bartels, C. L. Douglas, and A. Henriques, ”Conformal nets II: conformal blocks,” Commun. Math. Phys. 354 (2017) 393-458.

[553] A. Bartels, C. L. Douglas, and A. Henriques, ”Conformal nets III: fusion of defects,” arxiv:1310.8263

[554] A. Bartels, C. L. Douglas, and A. Henriques, ”Conformal nets IV: The 3-category,” Algebraic & Geometric Topology 18 (2018) 897-956, arxiv : 1605.00662.

[555] A. Bartels, C. L. Douglas, and A. Henriques, ”Conformal Nets V: Dualizability,” Commun. Math. Phys. $391(2022)1-31$.

[556] A. Henriques, ”Three-tier CFTs from Frobenius algebras.”.

[557] ”Conformal net.” http://ncatlab.org/nlab/show/conformal net.

[558] A. A. Belavin, A. M. Polyakov, and A. B. Zamolodchikov, ”Infinite Conformal Symmetry in Two-Dimensional Quantum Field Theory,” Nucl. Phys. B 241 (1984) 333-380.

[559] T. Hartman, D. Mazac, D. Simmons-Duffin, and A. Zhiboedov, ”Snowmass White Paper: The Analytic Conformal Bootstrap,” in 2022 Snowmass Summer Study. 2, 2022. arxiv:2202.11012 [hep-th]

[560] S. M. Harrison, J. A. Harvey, and N. M. Paquette, ”Snowmass White Paper: Moonshine,” arxiv:2201.13321 [hep-th].

[561] E. Gerchkovitz, J. Gomis, and Z. Komargodski, ”Sphere Partition Functions and the Zamolodchikov Metric,” JHEP 11 (2014) 001, arxiv:1405.7271 [hep-th].

[562] D. Radicevic, ”The Ultraviolet Structure of Quantum Field Theories. Part 1: Quantum Mechanics,” arxiv:2105.11470 [hep-th]. [563] D. Radicevic, ”The Ultraviolet Structure of Quantum Field Theories. Part 2: What is Quantum Field Theory?,” arxiv:2105.12147 [hep-th].

[564] D. Radicevic, ”The Ultraviolet Structure of Quantum Field Theories. Part 3: Gauge Theories,” arxiv:2105.12751 [hep-th].

[565] J. Alexandre, ”Lifshitz-type Quantum Field Theories in Particle Physics,” Int. J. Mod. Phys. A 26 (2011) 4523-4541, arxiv:1109.5629 [hep-ph].

[566] P. Horava, ”Quantum Gravity at a Lifshitz Point,” Phys. Rev. D 79 (2009) 084008, arxiv:0901.3775 [hep-th].

[567] N. Seiberg, ”What is Quantum Field Theory?.” https://www.youtube.com/watch?v=GZvs-ae4YRA.

[568] N. Seiberg, ”Field Theories With a Vector Global Symmetry,” SciPost Phys. 8 no. 4,

(2020) 050, arxiv:1909.10544 [cond-mat.str-el].

[569] N. Seiberg and S.-H. Shao, ”Exotic $U(1)$ Symmetries, Duality, and Fractons in 3+1-Dimensional Quantum Field Theory,” SciPost Phys. 9 no. 4, (2020) 046, arxiv:2004.00015 [cond-mat.str-el].

[570] P. Gorantla, H. T. Lam, N. Seiberg, and S.-H. Shao, ”More Exotic Field Theories in 3+1 Dimensions,” SciPost Phys. 9 (2020) 073, arxiv:2007.04904 [cond-mat.str-el].

[571] P. Gorantla, H. T. Lam, N. Seiberg, and S.-H. Shao, ”Low-energy limit of some exotic lattice theories and UV/IR mixing,” Phys. Rev. B 104 no. 23, (2021) 235116, arxiv:2108.00020 [cond-mat.str-el].

[572] S. Hu and A. Losev, ”Feynman Geometries,”] in 60 Years of Yang-Mills Gauge Field Theories: CN Yang's Contributions to Physics, pp. 453-471.

[573] M. Kontsevich and Y. Soibelman, “Notes on A_∞-Algebras, A_∞-Categories and Non-Commutative Geometry,” Lect. Notes Phys. 757 (2009) 153-220. arxiv:math/0606241.