CSCI 420 Computer Graphics
Assignment 2 Roller Coasters
Extras: Derive the physically realistic equation

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Problem:
Derive the steps that lead to the physically realistic equation of updating the u.

$$u_{new} = u_{current} + (\Delta t) \frac{\sqrt{2g(h_{max} - h)}}{\left\| \frac{dp}{du} \right\|}$$

where Δt is the time step,

g is the gravity constant,

h_{max} is the maximum height of the track,

h is the current height of the roller coaster,

p is a function of u (i.e. $p(u)$) that computes the position (in 3D) of the roller coaster at $u = u_{current}$.

Note that $\frac{dp}{du}$ is the derivative of $p(u)$ with respect to u , and the derivative is evaulated at $u = u_{current}$. Also, $\left\| \frac{dp}{du} \right\|$ is the magitude (i.e. $mag = \sqrt{x^2 + y^2 + z^2}$) of the vector $\frac{dp}{du}$.

Proof:

In free fall, there are formulas for velocity and falling distance.

$$v = gt \qquad ①$$

$$\Delta h = \frac{1}{2}gt^2 \qquad ②$$

Suppose the highest point, that is, the height of the starting position is h_{max}, and the height of the current position is h.

$$\Delta h = h_{max} - h \qquad ③$$

From ① and ②, we get

$$v = \sqrt{2g\Delta h}$$

Then link ③, we get

$$v = \sqrt{2g(h_{max} - h)}$$

The displacement per unit time Δt can be derived from the above.

$$d = v\Delta t$$
$$= \Delta t\sqrt{2g(h_{max} - h)}$$

Suppose $T(u)$ is $\frac{dp}{du}$, the derivative of $p(u)$ with respect to u, then $\|T(u)\|$ is the displacement distance when u increases by 1.

And in the increase of u, the change in displacement is $\Delta t\sqrt{2g(h_{max}-h)}$.

According to the proportional relationship, the following equation is thus obtained.

$$1 : \|T(u)\| = \Delta u : \Delta t\sqrt{2g(h_{max}-h)}$$

$$\Rightarrow \quad \Delta u = \frac{\Delta t\sqrt{2g(h_{max}-h)}}{\|T(u)\|}$$

$$\Rightarrow \quad \Delta u = \frac{\Delta t\sqrt{2g(h_{max}-h)}}{\left\|\frac{dp}{du}\right\|}$$

Finally, the u_{new} is equal to the $u_{current}$ plus Δu, then we get.

$$u_{new} = u_{current} + (\Delta t) \frac{\sqrt{2g(h_{max} - h)}}{\left\| \frac{dp}{du} \right\|}$$