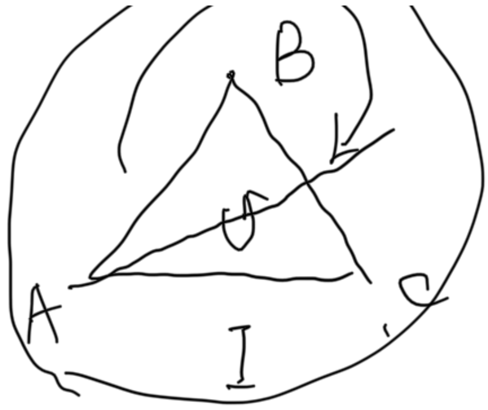
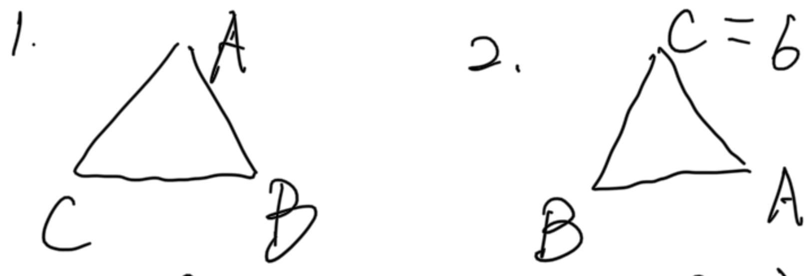


symmetry group: A B C C_3^2

I

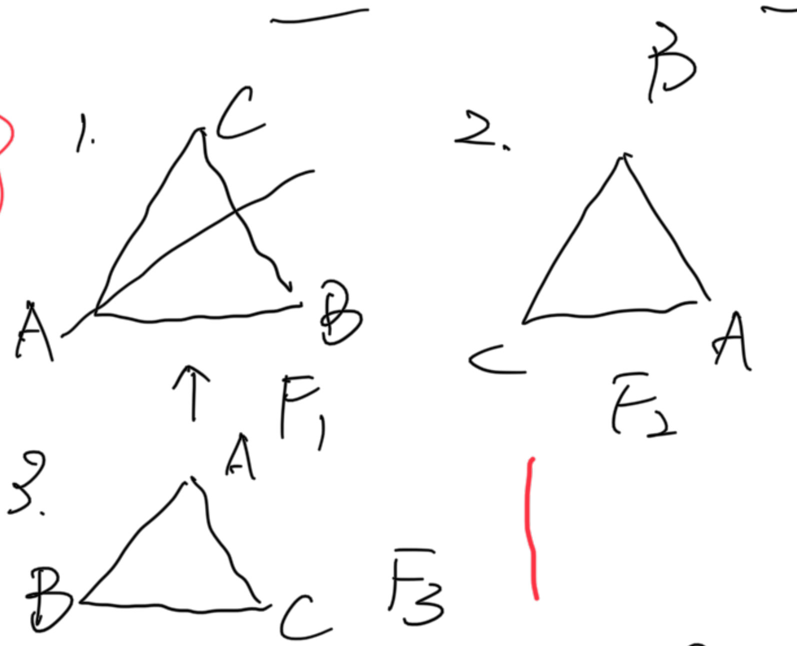


$120°$ R $240°$ R^2

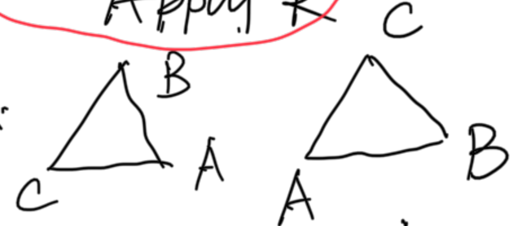
equilateral triangle

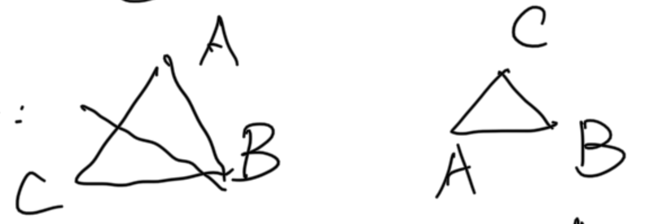
$\{I, R, R^2, F_1, | F_2, F_3\} = 6$

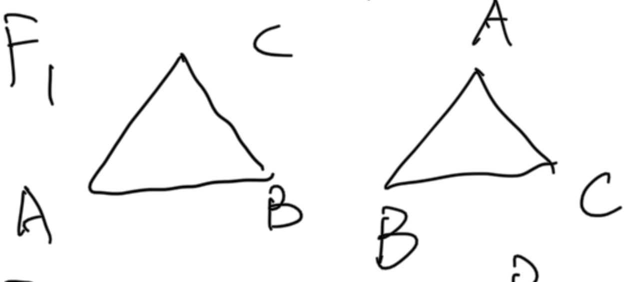
$R^3 = I$ $F_1^2 = I$

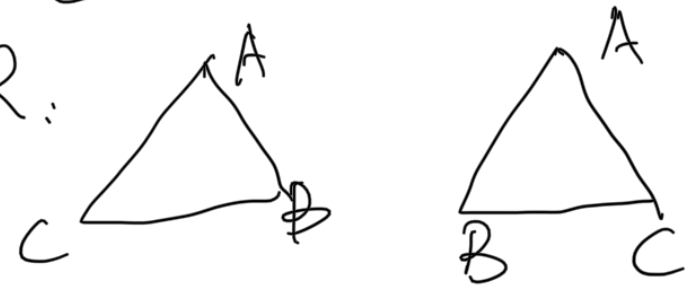


RF: ⇒ Apply F
Apply R

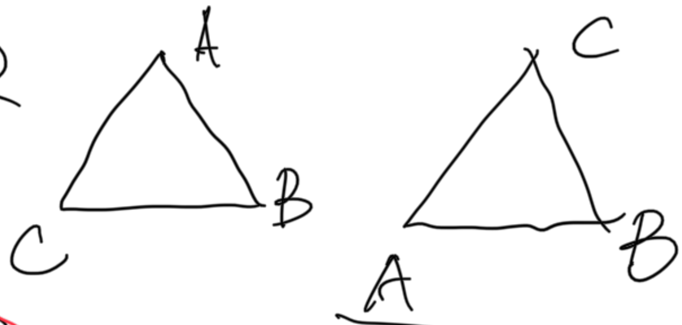
RF_2: 

F_2R: 

RF_1 

F_1R: 

RF_3. 

F_3R 

$RF \neq FR$

I. identity does nothing

operations: RF $R^2 \to R$

reverse: $I \to I$ $R \to R^2$ $F_1 \to F_1$

$F_2 \to F_2$ $F_3 \to F_3$

g∈G

I: Group: G is a set together with two operations.

$m: G \times G \to G$

$\Rightarrow \begin{cases} 1. \text{ multiplication. } & (\text{simple multi...}) \\ 2. \text{ inverse. } & i: G \to G \end{cases}$

operations obey the rules: 规则

$\begin{cases} 0 \times 0 = 0 \\ 0 = 0 \end{cases}$

1: Associativity For every $g, h, k \in G$.

$$m(m(g,h),k) = m(g, m(h,k))$$

$$\cancel{(ab)c = a(bc)}$$

2. Identity. There is an element $e \in G$. ~~s.t.~~ for every $g \in G$ s.t.

$$\boxed{m(g, e)} = g. \qquad m(e, g) = g$$

3. Inverse. For $g \in G$ s.t.

$$m(g, i(g)) = e = m(i(g), g)$$

□

II: $m(x, y) = xy$. $i(g) = g^{-1}$.

1. $(gh)k = g(hk)$
2. $ge = g = eg$.
3. $gg^{-1} = e g^{-1} g$.

III: 1. $(g * h) * k = g * (h * k)$

2. $g * e = g = e * g$

3. $g * g^{-1} = e = g^{-1} * g$. $\square$

Example: group? ~~$\{\} \in \{1\}$~~

$\cancel{\{\emptyset\}}$

$\{\}$ ←

$\cancel{\emptyset \in \{\}}$

1. multiplication. ← ✓
2. Associative. ✓
3. identity ×

$\boxed{\text{Not Group}}$

$\{a\}$

$\boxed{\text{Group}}$

1. multiplication. ← ✓ $\quad a*a \to a$
2. Associative. ✓

$g, h, k \in G$. $\quad g = h = k = a$.

$m(m(a,a), a) = a$

$m(a, m(a,a)) = a$

3. identity. ✓ $e = a$ $\quad g \in G \quad g = a$

$m(g,e) = m(a,a) = a = g$

$m(e,g) = m(a,a) = a = g$

4. inverse. ✓

$g \in G$, $g = a$

$m(g, g^{-1}) = m(a,a) = a = e$

$\{a\}$. multiplication. unique rule.
with this law of multiplication we get a group.

$\{a, b\}$ $\quad$ a: identity. ✓

$aa = a \quad ab = b$ ✓ $\quad a \to a$ $\quad$ inverse ✓

$ba = b \quad bb = a$

inverse

$b \to b$

$\boxed{\text{Group}}$

$(aa)b = a(ab)$

$ab = ab$ ✓

Lemma: $f: A \to B$

$g: B \to C$

$h: C \to D$

$\cancel{h \circ (g \circ f) = (h \circ g) \circ f}$

$\boxed{h \circ (g \circ f) = (h \circ g) \circ f}$

proof: 1. $A \to D \iff A \to D$

2. same value. $a \in A \Rightarrow d \in D$

$(h \circ (g \circ f))(a) = h((g \circ f)(a))$

$= h(g(f(a)))$

$((h \circ g) \circ f)(a) = (h \circ g)(f(a))$

$= h(g(f(a)))$.

$\{I, R, R^2, F_1, F_2, F_3\}$ is a group.

multiplication. composition of symmetries.

symmetry $\mathbb{R}^2 \Rightarrow \mathbb{R}^2$

function

Associative ✓

identity

inverse

abelian: for every g and $h \in G$.

$gh = hg$.

G: abelien.

$\cancel{\{a\}}$ $\cancel{\{\}}$ $\{a\}$ $\{a, b\}$ abelien

$W = \{1, 2, \cdots\}$ addition $(a+b)+c = a+(b+c)$

Lemma addition ~~multiplication~~ is associative under complex number

identity ✗ Not group

$W = \{0, 1, 2, \cdots\}$ identity: 0. inverse ✗

$1 + ? \geq 1 \Rightarrow$ ✗

$\mathbb{Z} = \{\cdots -2, -1, 0, 1, 2 \cdots\}$ identity ✓ inverse ✓

$\mathbb{Q}, \mathbb{R}, \mathbb{C}$ ✓ $\boxed{\text{Group}}$

multiplication. $\mathbb{Z}$: 1. associative
2. identity. 1
3. inverse. ~~$2 \times \frac{1}{2} = 1$~~ ✗

$\mathbb{Q}$: 1. associative. ✓
2. identity: 1
3. inverse. $2 \times \frac{1}{2}$ ✓ 0 ✗

$\mathbb{Q}^* = \mathbb{Q} / \{0\}$ ✓ $\boxed{\text{Group}}$

$\mathbb{R}^*$

$\mathbb{C}^*$

Abelian

$gh = hg$

矩阵 matrix.