CSC373 Fall'19
Tutorial 4
Mon. Oct. 7, 2019

Q1 Ford Fulkerson

Consider the following network:

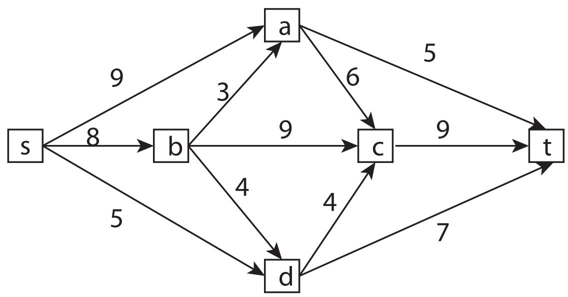

Figure 1:

(a) Compute a maximum flow in this network, using the Ford-Fulkerson algorithm: find augmenting paths and use them to augment the flow, one path at a time. For each augmenting path, take the time to write down the residual capacity and the resulting augmentation in the flow.

(b) Consider the cut $X_0 = (\{s, b, c, d\}, \{a, t\})$. Identify all forward and all backward edges across X_0, then compute the capacity and the flow across X_0 (for your maximum flow from part (a)).

(c) Find a cut in the network above whose capacity is equal to the value of your maximum flow (this provides a guarantee that your flow really is maximum). Use the algorithm outlined in the proof of the Ford-Fulkerson theorem.

Q2 Teaching Assignment

Consider the following problem:

Input: Set of profs $p_1, ..., p_n$ with teaching loads $L_1, ..., L_n$, and set of courses $c_1, ..., c_m$ with number of sections $S_1, ..., S_m$, along with subsets of courses that each prof is available to teach – each prof p_i has its own subset $A_i \subseteq \{c_1, ..., c_m\}$.

Output: Assignment of profs to courses such that:

- each prof p_i assigned exactly L_i courses,
- each course c_j assigned exactly S_j profs,

- no prof assigned a course outside their available set,
- no prof teaches multiple sections of the same course.

Show how to represent this problem as a network flow, and how to solve it using network flow algorithms. Justify carefully that your solution is correct and can be obtained in polynomial time.

Q3 Mobile Computing

Consider a set of mobile computing clients in a certain town who each need to be connected to one of several possible "base stations". We'll suppose there are n clients, with the position of each client specified by its (x, y) coordinates in the plane. There are also m base stations, each of whose positions is specified by (x, y) coordinates as well.

We wish to connect each client to exactly one base station. Our choice of connections is constrained in the following ways. There is a "range parameter" r – a client can only be connected to a base station that is within distance r. There is also a "load parameter" L – no more than L clients can be connected to any single base station.

Show how to represent this problem as a network flow, and how to solve it using network flow algorithms. Justify carefully that your solution is correct and can be obtained in polynomial time.