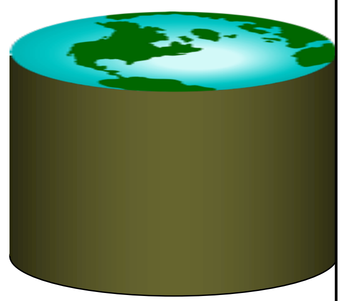
Tree-Structured Indexes

Lecture 3
Chapter 10

"If I had eight hours to chop down a tree, I'd spend six sharpening my ax."

Abraham Lincoln



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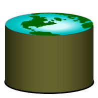


Introduction

- ***Recall: 2 alternatives for data entries*** k*:
  - <**k**, rid of data record with search key value **k**>
  - <**k**, list of rids of data records with search key **k**>
- **Choice is orthogonal to the *indexing technique* used to locate data entries** k***.**
- **Tree-structured indexing techniques support both *range searches* and *equality searches*.**
- ***ISAM*: static structure; *B+ tree*: dynamic, adjusts gracefully under inserts and deletes.**
- **ISAM = ???**

  Indexed Sequential Access Method

2

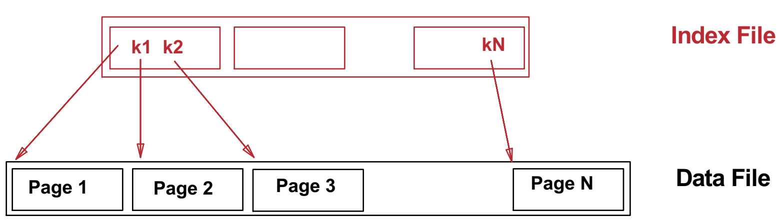
A Note of Caution

- **ISAM is an old-fashioned idea**
  - B+-trees are usually better, as we'll see
    - Though not *always*
- **But, it's a good place to start**
  - Simpler than B+-tree, but many of the same ideas
- **Upshot**
  - **Don't** brag about being an ISAM expert on your resume
  - **Do** understand how they work, and tradeoffs with B+-trees

3

Range Searches

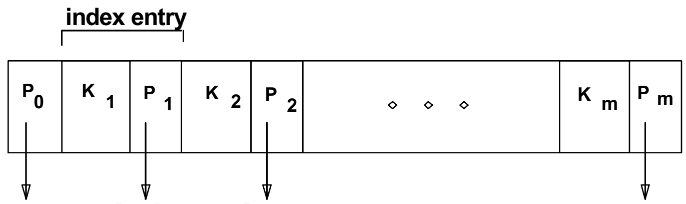
- ***``Find all students with gpa > 3.0'***
  - If data is in sorted file, do binary search to find first such student, then scan to find others.
  - Cost of binary search can be quite high.
- **Simple idea: Create an `index' file.**
  - Level of indirection again!



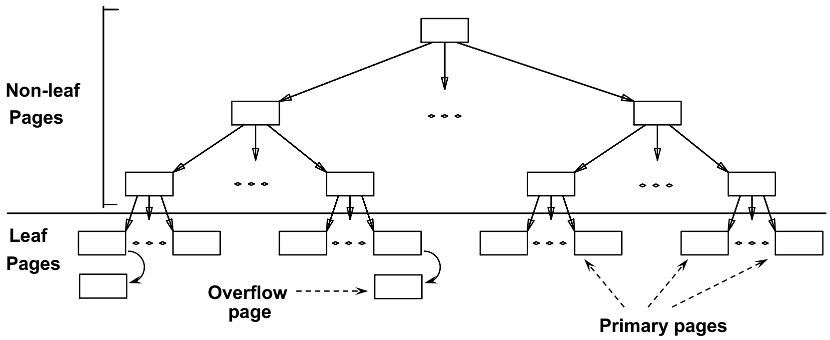
☛ *Can do binary search on (smaller) index file!*

4

ISAM



- **Index file may still be quite large. But we can apply the idea repeatedly!**

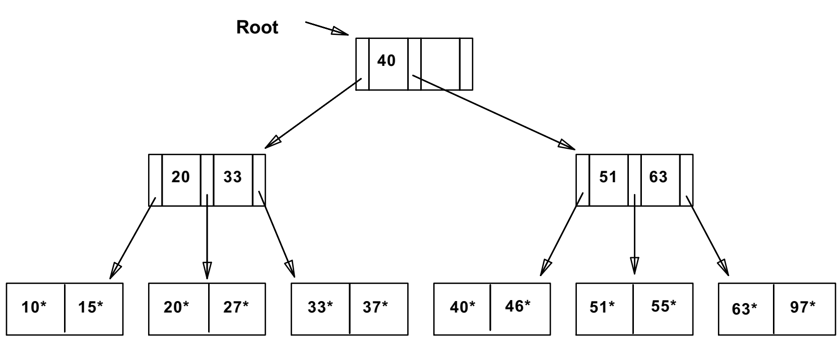


☛ *Leaf pages contain data entries.*

5

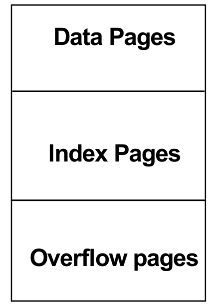
Example ISAM Tree

- **Each node can hold 2 entries**



6

Comments on ISAM



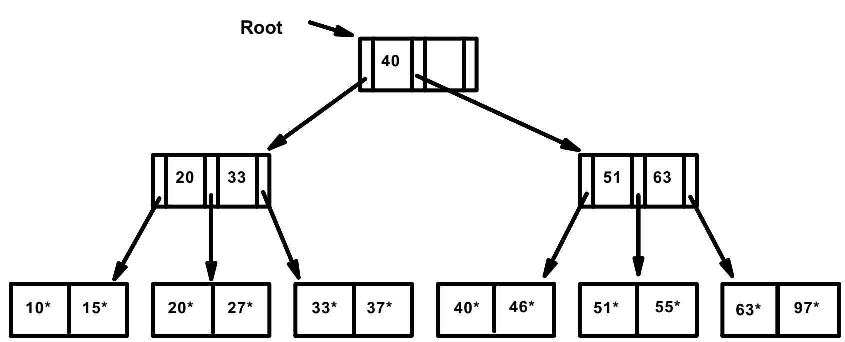
- ***File creation***: Leaf (data) pages allocated sequentially, sorted by search key.
  Then index pages allocated.
  Then space for overflow pages.
- ***Index entries***: <search key value, page id>; they `direct' search for ***data entries***, which are in leaf pages.
- ***Search***: Start at root; use key comparisons to go to leaf.
  Cost $\propto \log_F N$; F = # entries/index pg, N = # leaf pgs
- ***Insert***: Find leaf where data entry belongs, put it there. (Could be on an overflow page).
- ***Delete***: Find and remove from leaf; if empty overflow page, de-allocate.

☛ **Static tree structure**: *inserts/deletes affect only leaf pages.*

7

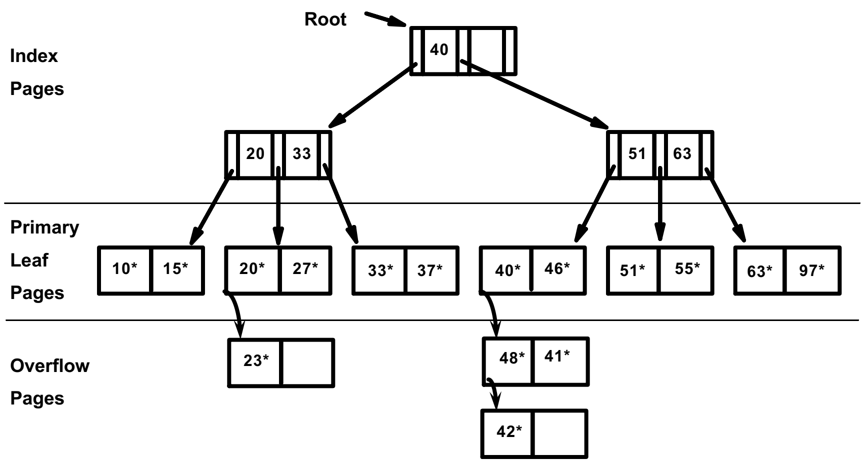
Example ISAM Tree

- **Each node can hold 2 entries; no need for `next-leaf-page' pointers. (Why?)**



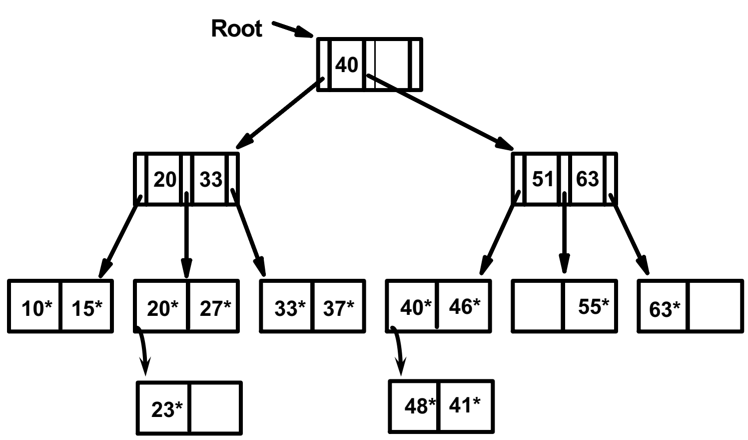
8

After Inserting 23*, 48*, 41*, 42* ...



9

... then Deleting 42*, 51*, 97*



☛ *Note that 51 appears in index levels, but 51* not in leaf!*

10

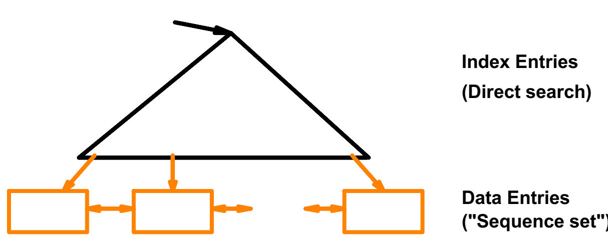
ISAM ---- Issues?

- **Pros**
  - ????
- **Cons**
  - ????

11

B+ Tree: The Most Widely Used Index

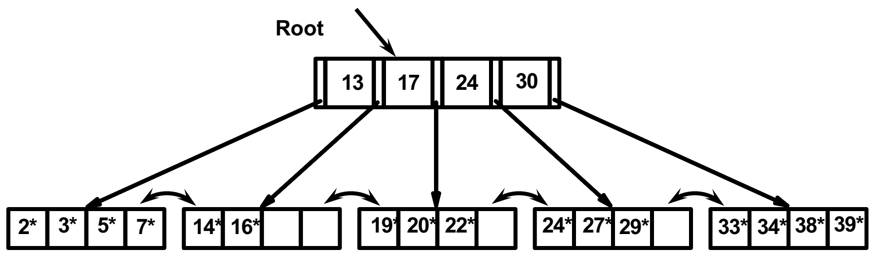
- **Insert/delete at log $_F$ N cost; keep tree *height-balanced*.   (F = fanout, N = # leaf pages)**
- **Minimum 50% occupancy (except for root). Each node contains d <= *m* <= 2d entries. The parameter d is called the *order* of the tree.**
- **Supports equality and range-searches efficiently.**



12

Example B+ Tree

- **Search begins at root, and key comparisons direct it to a leaf (as in ISAM).**
- **Search for 5*, 15*, all data entries >= 24* ...**



☛ *Based on the search for 15*, we <u>know</u> it is not in the tree!*

13

B+ Trees in Practice

- **Typical order: 100. Typical fill-factor: 67%.**
  - average fanout = 133
- **Typical capacities:**
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- **Can often hold top levels in buffer pool:**
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes

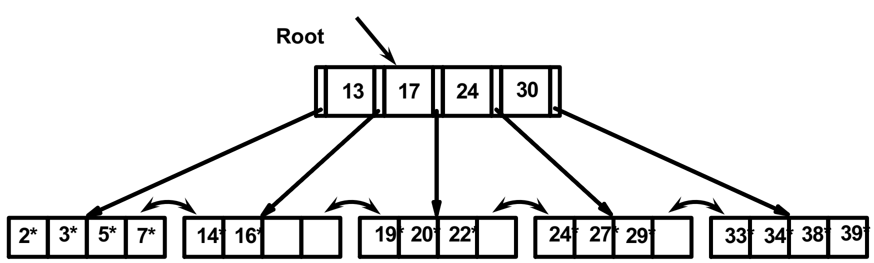
14

Inserting a Data Entry into a B+ Tree

- **Find correct leaf *L.***
- **Put data entry onto *L*.**
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.
- **This can happen recursively**
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- **Splits "grow" tree; root split increases height.**
  - Tree growth: gets *wider* or *one level taller at top.*

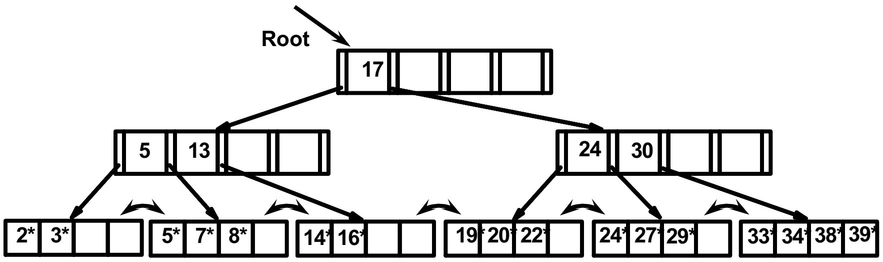
15

Example B+ Tree - Inserting 8*



16

Example B+ Tree - Inserting 8*

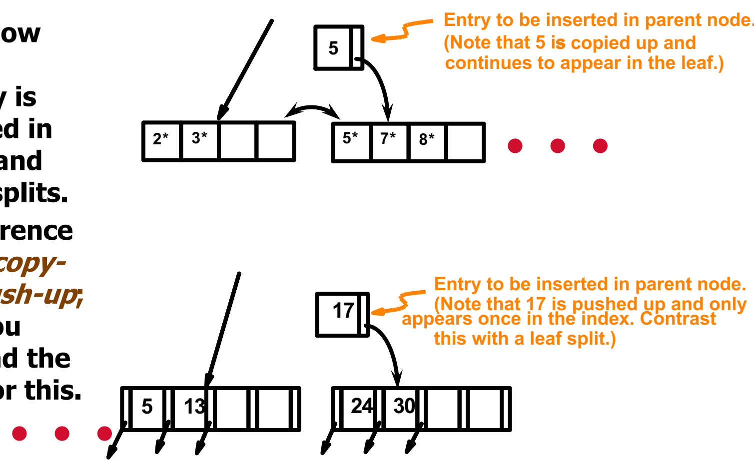


❖ Notice that root was split, leading to increase in height.

17

Inserting 8* into Example B+ Tree

- **Observe how minimum occupancy is guaranteed in both leaf and index pg splits.**
- **Note difference between *copy-up* and *push-up*; be sure you understand the reasons for this.**



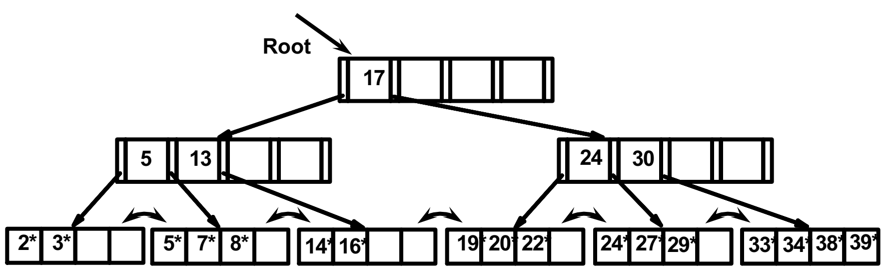
18

Deleting a Data Entry from a B+ Tree

- **Start at root, find leaf *L* where entry belongs.**
- **Remove the entry.**
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from *sibling (adjacent node with same parent as L)*.
    - If re-distribution fails, *merge* *L* and sibling.
- **If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.**
- **Merge could propagate to root, decreasing height.**

19

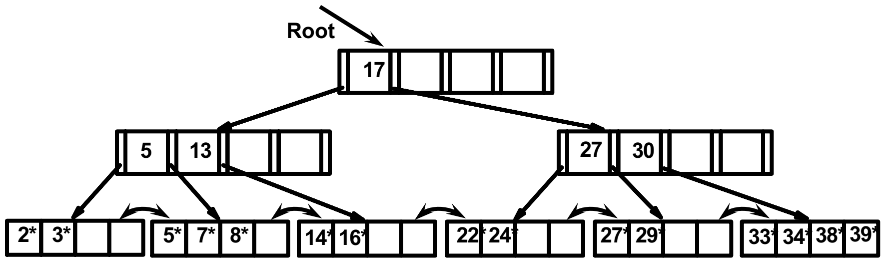
Example Tree (including 8*) Delete 19* and 20* ...



- **Deleting 19* is easy.**

20

Example Tree (including 8*) Delete 19* and 20* ...

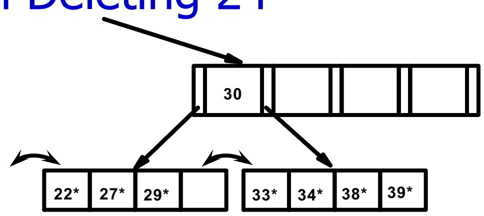


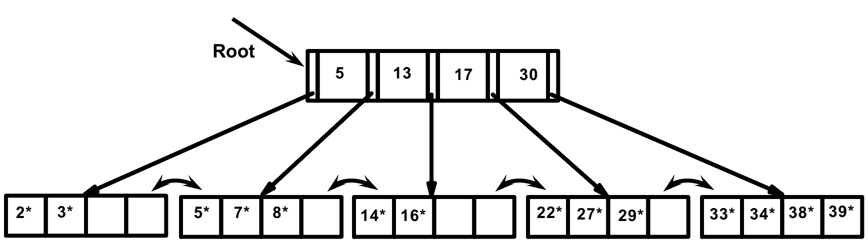
- **Deleting 19* is easy.**
- **Deleting 20* is done with re-distribution. Notice how middle key is *copied up*.**

21

... And Then Deleting 24*

- **Must merge.**
- **Observe `*toss*' of index entry (on right), and `*pull down*' of index entry (below).**

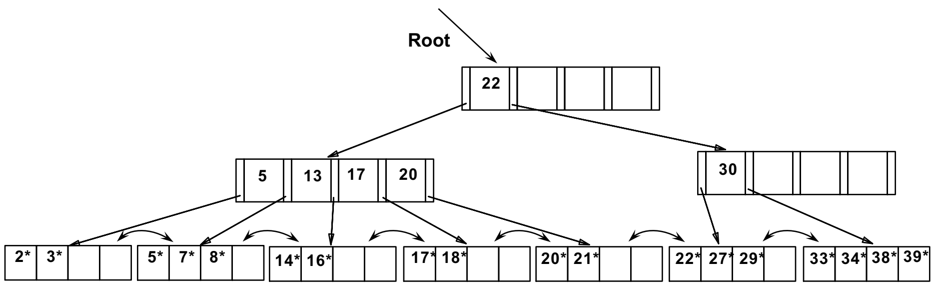




22

Example of Non-leaf Re-distribution

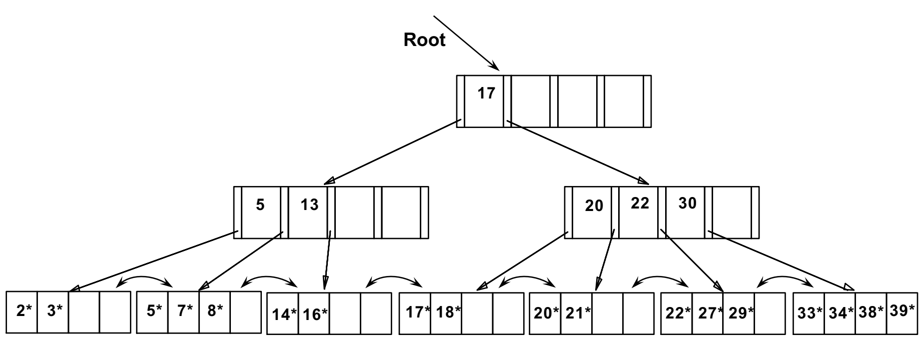
- **Tree is shown below *during deletion* of 24*. (What could be a possible initial tree?)**
- **In contrast to previous example, can re-distribute entry from left child of root to right child.**



23

After Re-distribution

- **Intuitively, entries are re-distributed by `*pushing through*' the splitting entry in the parent node.**
- **It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well for illustration.**



24

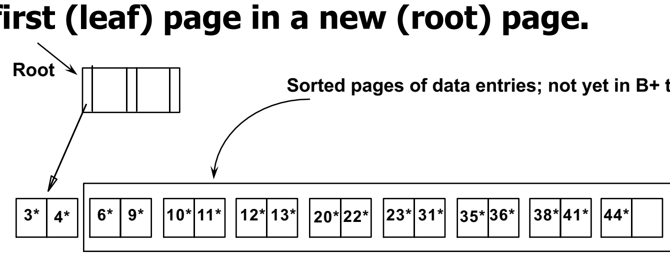
Prefix Key Compression

- **Important to increase fan-out. (Why?)**
- **Key values in index entries only `direct traffic'; can often compress them.**
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt*, *David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- **Insert/delete must be suitably modified.**

25

Bulk Loading of a B+ Tree

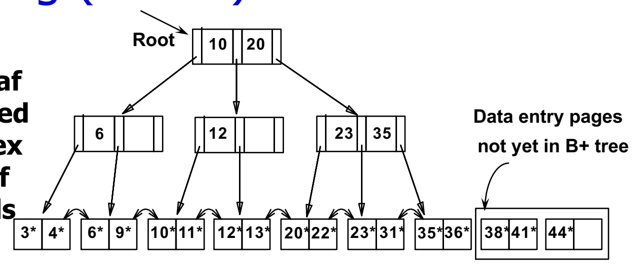
- **If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.**
  - Also leads to minimal leaf utilization --- why?
- ***<u>Bulk Loading</u>*** **can be done much more efficiently.**
- ***Initialization*****: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.**

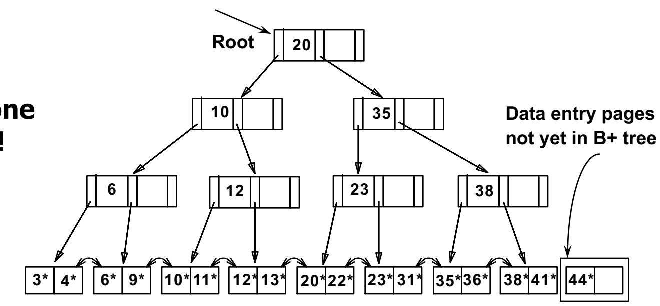


26

Bulk Loading (Contd.)

- **Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)**
- **Much faster than repeated inserts, especially when one considers locking!**





27

Summary of Bulk Loading

- **Option 1: multiple inserts.**
  - Slow.
  - Does not give sequential storage of leaves.
- **Option 2: *Bulk Loading***
  - Has advantages for concurrency control.
  - Fewer I/Os during build.
  - Leaves will be stored sequentially (and linked, of course).
  - Can control "fill factor" on pages.

28

A Note on `Order'

- ***Order*** **(d) concept replaced by physical space criterion in practice (`*at least half-full*').**
  - Index pages can typically hold many more entries than leaf pages.
  - Variable sized records and search keys mean different nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (2)).
- **Many real systems are even sloppier than this --- only reclaim space when a page is *completely* empty.**

29

Summary

- **Tree-structured indexes are ideal for range-searches, also good for equality searches.**
- **ISAM is a static structure.**
  - Only leaf pages modified; overflow pages needed.
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant.
- **B+ tree is a dynamic structure.**
  - Inserts/deletes leave tree height-balanced; $\log_F N$ cost.
  - High fanout (**F**) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.

30

Summary (Contd.)

- Typically, 67% occupancy on average.
- Usually preferable to ISAM, adjusts to growth gracefully.
- If data entries are data records, splits can change rids!

- **Key compression increases fanout, reduces height.**
- **Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.**
- **Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.**