



组 号 1

实验题目 数学实验测试文档

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1 模板说明

1.1 引入模板

使用如下 \LaTeX 代码:

```
\documentclass[options]{mathexpthesis}
```

拥有几个可选项:

- **bwprint/colorprint** 控制打印时的颜色与超链接的边框.
- **withoutpreface** 控制扉页的输出.
- **withouttitlepage** 控制标题页的输出, 最好将摘要放在此页.
- **openbookmarknumber** 开启 PDF 书签的编号, 注意此方法会导致编译警告.

1.2 模板参数

- `\title` 实验文档标题, 在 document 中需要使用 `\maketitle`, 该参数不必须填写.
- `\groupno` 小组编号, 该参数必须填写.
- `\membera,b,c,d` 成员姓名 (仅支持恰好四人的小组), 该参数必须填写.

1.3 编译

目前仅支持 $\text{Xe}\text{\LaTeX}$ 编译.

2 简单的使用说明

2.1 摘要与关键字

使用方式:

```
\begin{abstract}
  Abstract here.
\keywords{\LaTeX, typesetting}
\end{abstract}
```

2.2 节标题

模板中的指令经过了重定义, 但使用方法不变:

```
\section{A section}
\subsection{A subsection}
\subsubsection{A subsubsection}
```

具体上, 修改了一定的间距, 重写了一个计数器指令.

2.3 公式与定理环境

使用了 `amsthm`, `amsmath`, `amssymb`, `amsfonts` 宏包. 使用方式不变.

行内公式使用 `$$` 即可. 行间公式使用 `equation`, `equarray`, `align` 等方式.

请注意英文的拼写:

- `lemma` 引理
- `theorem` 定理
- `proof` 证明
- `assumption` 假设
- `definition` 定义
- `example` 例

注意, 此处的计数器是**章节全局性**的计数器.

还有一些其它的定义, 不过用处不算太大, 此处不介绍.

这里使用一个我曾经写过的例子.

定理 2.1 (Riesz 定理) 若 $f_n \xrightarrow{m} f$, 则 $\exists \{f_{n_k}\} \subset \{f_n\}, s.t. \quad f_{n_k} \rightarrow f \quad a.e.$

证明: $f_n \xrightarrow{m} f$, 则 $\forall \varepsilon > 0, \delta > 0, \exists n \geq 1, s.t. \quad n \geq N, m(E(|f_n - f| \geq \varepsilon)) < \delta$. 于是 $\forall k \in \mathbb{N}^*$, 令 $\varepsilon = \frac{1}{k}, \delta = \frac{1}{2^k}$, 可以依次选取自然数 $n_1, n_2, \dots, n_k, \dots, s.t.$

$$m(E(|f_{n_k} - f| \geq \frac{1}{k})) < \frac{1}{2^k}.$$

下面证明 $f_{n_k} \rightarrow f \quad a.e.$

令

$$E_0 = \bigcap_{N=1}^{\infty} \bigcup_{k=N}^{\infty} E(|f_k - f| \geq \frac{1}{k}).$$

对每个 $N = 1, 2, \dots$, 有

$$m(E_0) \leq m(\bigcup_{k=N}^{\infty} E(|f_{n_k} - f| \leq \frac{1}{k})) \leq \sum_{k=N}^{\infty} m(E(|f_{n_k} - f| \geq \frac{1}{k})) < \sum_{k=N}^{\infty} \frac{1}{2^k} = \frac{1}{2^{N-1}}.$$

令 $N \rightarrow \infty$, 可知 $m(E_0) = 0$. 由 De Morgan 公式得,

$$E - E_0 = \bigcup_{N=1}^{\infty} \bigcap_{k=N}^{\infty} E(|f_k - f| \geq \frac{1}{k}).$$

所以 $\forall x \in E - E_0, \exists N \geq 1, s.t. \quad k \geq N$,

$$|f_{n_k}(x) - f(x)| < \frac{1}{k}.$$

故 $f_{n_k}(x) \rightarrow f(x)$, 则在 E 上 $f_{n_k} \rightarrow f \quad a.e.$

□

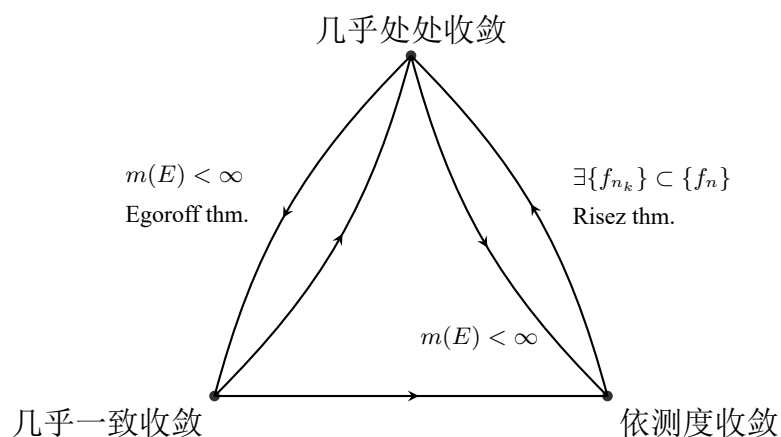


图 1 Example of TikZ

2.4 图片插入

对于未指定后缀的图片，将以`.pdf`，`.eps`，`.jpg`，`.png`的顺序查找。

图片支持的目录：

```
{./}, {figures/}, {figure/}, {pictures/}, {picture/}, {pic/},
↪ {pics/}, {image/}, {images/}
```

插入图片的方式：

```
\begin{figure}
  \centering
  \caption{The caption} % caption above
  \includegraphics{figures/...}
  \label{fig-x}
\end{figure}
```

在 `figure` 环境中也可以使用 TikZ 作图。以下是一个例子。

2.5 表格插入

支持简单的表格，推荐使用 Excel2LaTeX 工具一键转换各种表格。一般表格的创建是用 `tabular`，`table` 这样的命令来完成的。

建议采用三线表。以下是一个三线表的示例。

LaTeX 代码由插件生成。

```
\begin{table}[htbp]
  \centering
  \begin{tabular}{lcc}
```

| Type | Biomass($g \cdot m^{-2}$) | Area Required(m^2) |
|-----------------|-----------------------------|------------------------|
| rain forest | 45000 | 6666.67 |
| seasonal forest | 35000 | 8571.43 |
| grassland | 4000 | 75000 |

表 1 Minimum area needed in different environment

```

\toprule
\multicolumn{1}{c}{Type} & Biomass( $g \cdot m^{-2}$ ) & Area
\rightarrow Required( $m^2$ ) \\
\midrule
rain forest & 45000 & 6666.67 \\
seasonal forest & 35000 & 8571.43 \\
grassland & 4000 & 75000 \\
\bottomrule
\end{tabular}

\caption{Minimum area needed in different environment}
\label{tab:addlabel}
\end{table}

```

2.6 参考文献

使用以下代码：

```

\begin{thebibliography}{99}
  \bibitem{bib:one} ....
  \bibitem{bib:two} ....
\end{thebibliography}

```

使用 `\cite` 命令可以实现一个“比较大”的引用 [1]. 使用新写的 `\upcite` 命令则是上标引用^[2].

2.7 附录

使用：

```
\begin{appendices}
  \section{...}
  \subsection{...}
\end{appendices}
```

即可创建附录.

附录中可以放置一些实验时产生的很占地方的图片, 也可以放置代码. 推荐 `minted` 宏包, 需要 Python 安装一些额外文件并将其放入系统的环境变量中:

```
pip install Pygments
```

从代码生成的角度上, `minted` 的效果优于 `lstlistings`.

注意: 目前附录仅支持最多两层, 否则目录与编号会出错.

3 其它注意事项

3.1 符号的使用

句号请写成“.”, 不要使用“。”.

不需要滥用符号, 公式标号与定理环境.

所有符号应使用相应的说明, 也需要一个整体的表格概述符号的意义.

3.2 图片与表格的位置

不要忘加 `htb`.

部分情况不要忘加 `\centering`.

3.3 使用许可与鸣谢

模板协议为 LPPL (直接用就行了).

此文档中除去示例代码的内容, 协议为 LPPL (即不要未经许可使用我的 `matlab` 代码和 `tikz` 代码).

由于模板集成了 `latexstudio/GMCMthesis`^[3] 与 `latexstudio/CUMCMthesis`^[4] 的文档模板, 该部分的版权为版权所属人所有, 此模板属于 `Derived Work` (这是鸣谢).

3.4 已知问题

`Package hyperref Warning: Token not allowed in a PDF string (Unicode)`. 这个问题不会影响编译, 原因是 `hyperref` 处理编号时使用了 `LATEX` 语法, 这些语法不能被 PDF 接受. 最后结果中的书签仍然会正常生成. 目前这个问题通过添加一个额外选项的方法解决.

3.5 其它

以下是示例的参考文献内容和示例的附录. 如需使用网址, `url` 宏包可能会更好看一点. 有时 `url` 可能会导致强制换行的问题, 此时可以人工换行.

参考文献

[1] `upcite` reference example

[2] `cite` reference example

[3] <https://github.com/latexstudio/GMCMthesis>

[4] <https://github.com/latexstudio/CUMCMthesis>

附录 A 更新记录

2019/2/25 初次更新

2019/2/26 修复了 `hyperref` 链接错误的问题

2019/2/26 调整了一部分代码的顺序, 提高可读性

附录 B 一些 matlab 测试代码

B.1 单纯形法-Core

```
function [xval, fval] = SimplexMethod(c, A, b, epsilon)
    % Simplex method for linear programming.
    % This program is to solve functions like:
    % min f = cx such that Ax = b, x >= 0, b >= 0
    % c, A, b: equation above
    % epsilon: error value when calculating a valid point

    if (nargin == 3)
        epsilon = 1e-6;
    end

    if (size(c, 2) ~= size(A, 2)) || (size(c, 1) ~= 1) || ...
        (size(A, 1) ~= size(b, 1)) || (size(b, 2) ~= 1)
        error("Invalid matrix.");
    end
```



```

end

% Find a solution first.
extA = [A, eye(size(A, 1))];
extB = b;
extC = [zeros(size(c)), -ones(size(b')), 0];
inX = size(c, 2) + 1: size(c, 2) + size(A, 1);
extTable = [extA extB; extC]; % Generate a table
% Manipulate the table
for i = 1:size(A, 1)
    extTable(end, :) = extTable(end, :) + extTable(i, :);
end

% Pivot, step by step
[val, cind] = max(extTable(end, 1: end - 1));
while (val > 0)
    minval = inf;
    for i = 1:size(extTable, 1) - 1
        if (extTable(i, cind) * inf >= 0)
            val = extTable(i, end) / extTable(i, cind);
            if (val < minval)
                minval = val;
                rind = i; % find row index
            end
        end
    end
    if (isinf(minval))
        error("Unable to find a valid initial point.");
    end
    point = extTable(rind, cind); % find the point
    inX(rind) = cind; % calculate inner columns at the same time
    extTable(rind, :) = extTable(rind, :) ./ point;
    % Elimination
    for i = 1:size(extTable, 1)
        if (i ~= rind)

```

```

        extTable(i, :) = extTable(i, :) - extTable(i, cind) .*
        ↪ extTable(rind, :);
    end
end
[val, cind] = max(extTable(end, 1: end - 1));
end
if (abs(extTable(end, end)) >= epsilon)
    error("Unable to find a valid initial point.");
else
    tempX = zeros(size(extTable, 2), 1);
    tempX(inX) = extTable(1: end - 1, end);
    initX = tempX(1: size(c, 2)); % get initial point of original LP
end
% change extTable to let useful vectors in
for i = 1: size(A, 1)
    if (inX(i) > size(c, 2)) % move condition
        ckey = find(extTable(i, 1: size(c, 2)) ~= 0, 1);
        if (~isempty(ckey)) % get the key, apply the change
            point = extTable(i, ckey);
            inX(i) = ckey;
            extTable(i, :) = extTable(i, :) ./ point;
            % Elimination without the bottom
            for j = 1: size(extTable, 1) - 1
                if (j ~= i)
                    extTable(j, :) = extTable(j, :) - extTable(j,
                    ↪ ckey) .* extTable(i, :);
                end
            end
        else % don't get the key, remove the line
            warning("Surplus condition found. Removing it.")
            inX(i) = -1; % Tag the key
        end
    end
end
table = extTable(inX > 0, [1: size(A, 2), end]); % construct table

```

```

% re-calculate conditional number
table = [table; zeros(1, size(table, 2))]; % make room first
table(end, end) = c * initX;
inX = inX(inX > 0);
w = c(inX) / table(1: end - 1, inX); % get factor w
for j = 1: size(table, 2) - 1
    table(end, j) = w * table(1: end - 1, j) - c(j);
end
% Pivot, step by step
[val, cind] = max(table(end, 1: end - 1));
while (val > 0)
    minval = inf;
    for i = 1: size(table, 1) - 1
        if (table(i, cind) * inf >= 0)
            val = table(i, end) / table(i, cind);
            if (val < minval)
                minval = val;
                rind = i; % find row index
            end
        end
    end
    if (isinf(minval))
        error("This problem is unbounded.")
    end
    point = table(rind, cind); % find the point
    inX(rind) = cind; % calculate inner columns at the same time
    table(rind, :) = table(rind, :) ./ point;
    % Elimination
    for i = 1: size(table, 1)
        if (i ~= rind)
            table(i, :) = table(i, :) - table(i, cind) .*
                ↪ table(rind, :);
        end
    end
    [val, cind] = max(table(end, 1: end - 1));

```

```

end
tempX = zeros(size(table, 2), 1);
tempX(inX) = table(1: end - 1, end);
xval = tempX(1: size(c, 2)); % get initial point of original LP
fval = table(end, end);
end

```

B.2 单纯形法-Full

```

function [xval, fval] = SimplexSolver(c, A, b, Aeq, beq, lb, ub, epsilon)
% Uniform simplex solver for simplex method.
% Solves problem like:
% min f = cx, such that A * x <= b, Aeq * x = beq, lb <= x <= ub
% epsilon: error value when calculating a valid point
% This program works in larger range than the core simplex method.

% Autofill
if (nargin == 3)
    Aeq = []; beq = []; lb = []; ub = []; epsilon = 1e-6;
elseif (nargin == 5)
    lb = []; ub = []; epsilon = 1e-6;
elseif (nargin == 6)
    ub = []; epsilon = 1e-6;
elseif (nargin == 7)
    epsilon = 1e-6;
end
origC = c;

% Validate
if (~isempty(A) && ~isempty(b) && ...
    ((size(c, 2) ~= size(A, 2)) || (size(c, 1) ~= 1) || ...
    (size(A, 1) ~= size(b, 1)) || (size(b, 2) ~= 1))) || ...
    (~isempty(Aeq) && ~isempty(beq) && ((size(c, 2) ~= size(Aeq, 2))
    ⇨ || ...
    (size(Aeq, 1) ~= size(beq, 1)) || (size(beq, 2) ~= 1))) || ...

```

```

    (~isempty(lb) && (size(lb, 2) ~= 1 || size(lb, 1) ~= size(c, 2)))
    ↪ || ...
    (~isempty(ub) && (size(ub, 2) ~= 1 || size(ub, 1) ~= size(c, 2)))
    error("Invalid matrix.");
end

% Integrate lowerbounds and upperbounds into constraints
if (isempty(lb))
    lb = -inf(size(c))';
end
if (isempty(ub))
    ub = +inf(size(c))';
end

% create movement table for recovering the data later
% line 1 demonstrates movement type
% value 0 for x - u type + v - x type
% value 1 for x - u type
% value 2 for v - x type
% value 3 for x1 - x2 type
% line 2 demonstrates movement place
% value n for place n
% value -1 for not taking a place

movement_table = zeros(2, size(c, 2));
place = size(c, 2) + 1;
for i = 1: size(c, 2)
    if (ub(i) - lb(i) < 0)
        error("lowerbound is greater than upperbound.");
    end
    if (lb(i) ~= -inf) && (ub(i) ~= +inf)
        movement_table(1, i) = 0;
        movement_table(2, i) = -1;
        % Add constraint for full bounded condition
        if (~isempty(A))

```

```

        b(:) = b(:) - A(:, i) * lb(i);
    end
    if (~isempty(Aeq))
        beq(:) = beq(:) - Aeq(:, i) * lb(i);
    end
    temprow = zeros(size(c));
    temprow(i) = 1;
    A = [A; temprow];
    b = [b; ub(i) - lb(i)];
elseif (lb(i) ~= -inf) && (ub(i) == +inf)
    movement_table(1, i) = 1;
    movement_table(2, i) = -1;
    if (~isempty(A))
        b(:) = b(:) - A(:, i) * lb(i);
    end
    if (~isempty(Aeq))
        beq(:) = beq(:) - Aeq(:, i) * lb(i);
    end
elseif (lb(i) == -inf) && (ub(i) ~= +inf)
    movement_table(1, i) = 2;
    movement_table(2, i) = -1;
    c(i) = -c(i);
    if (~isempty(A))
        b(:) = b(:) - A(:, i) * ub(i);
        A(:, i) = -A(:, i);
    end
    if (~isempty(Aeq))
        beq(:) = beq(:) - Aeq(:, i) * ub(i);
        Aeq(:, i) = -Aeq(:, i);
    end
elseif (lb(i) == -inf) && (ub(i) == +inf)
    movement_table(1, i) = 3;
    movement_table(2, i) = place;
    place = place + 1;
    % Add variable for unbounded condition

```

```

        c = [c, -c(i)];
        if (~isempty(A))
            A = [A, -A(:, i)];
        end
        if (~isempty(Aeq))
            Aeq = [Aeq, -Aeq(:, i)];
        end
    end
end

% Add surplus variables for inequalities and make it stardardized
for i = 1: size(A, 1)
    tempcol = zeros(size(A, 1), 1);
    if b(i) < 0
        A(i, :) = -A(i, :);
        b(i) = -b(i);
        tempcol(i) = -1;
        A = [A, tempcol];
        c = [c, 0];
    else
        tempcol(i) = 1;
        A = [A, tempcol];
        c = [c, 0];
    end
    if (~isempty(Aeq))
        Aeq = [Aeq, zeros(size(Aeq, 1), 1)];
    end
end

% Make equality constraints startardized
for i = 1: size(Aeq, 1)
    if (beq(i) < 0)
        Aeq(i, :) = -Aeq(i, :);
        beq(i) = -beq(i);
    end
end

```

```

end

% Solve the LP using the core simplex method module
[tempxval, ~] = SimplexMethod(c, [A; Aeq], [b; beq], epsilon);

xval = nan(size(origC, 2), 1);
% From movement_table to recover final value
for i = 1: size(movement_table, 2)
    switch (movement_table(1, i))
        case {0, 1}
            xval(i) = tempxval(i) + lb(i);
        case 2
            xval(i) = ub(i) - tempxval(i);
        case 3
            xval(i) = tempxval(i) - tempxval(movement_table(2, i));
    end
end
end
fval = origC * xval;

end

```