The analytical NES value of aREA

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Here the analytical NES value of aREA is derived.

The expression signature S^{exp} is transformed into S after rank transformation and quantile transformation. The ES and NES are calculated based on the signature S.

The ES value is calculated based on the following formula:

$$ES = \frac{\sum w_i(s_i^{t1q}(1-|M_i|) + s_i^{t2q}M_i)}{\sum w_i}$$

The ES value have two parts, ES_1 and ES_2:

$$ES = ES_1 + ES_2$$

$$ES_1 = \frac{\sum w_i s_i^{t1q}(1-|M_i|)}{\sum w_i}$$

$$ES_2 = \frac{\sum w_i s_i^{t2q} M_i}{\sum w_i}$$

Both ES_1 and ES_2 follows normal distribution, which is proved bellow: ES_1 is the sum of some weighted s_i^{t1q}, $s_i^{t1q} \sim N(0,1)$ and $\frac{w_i s_i^{t1q}(1-|M_i|)}{\sum w_i} \sim N(0, \frac{(w_i(1-|M_i|))^2}{(\sum w_i)^2})$. It is known that the sum of variables from normal distribution also follows the normal distribution with means equal to the sum of the variables' means and variance equal to the sum of the variables' variance. Thus

$$ES_1 \sim N(0\,, \frac{\sum(w_i(1-|M_i|))^2}{(\sum w_i)^2})$$

Similarly,

$$ES_2 \sim N(0\,, \frac{\sum(w_i M_i)^2}{(\sum w_i)^2})$$

Thus, the ES is the sum of two variables comes from two normal distribution, it is easy to know

$$ES \sim N\left(0, \frac{\sum\left(w_i(1-|M_i|)\right)^2}{(\sum w_i)^2} + \frac{\sum(w_i M_i)^2}{(\sum w_i)^2}\right)$$

$$= N\left(0, \frac{\sum\left(w_i(1-|M_i|)\right)^2 + \sum(w_i M_i)^2}{(\sum w_i)^2}\right)$$

Finally

$$NES = \frac{ES}{\sqrt{Var(ES)}}$$
$$= \frac{ES * \sum w_i}{\sqrt{\sum\left(w_i(1-|M_i|)\right)^2 + \sum(w_i M_i)^2}}$$
$$= \frac{\frac{\sum w_i\left(s_i^{t1q}(1-|M_i|) + s_i^{t2q} M_i\right)}{\sum w_i}}{\sqrt{\frac{\sum\left(w_i(1-|M_i|)\right)^2 + \sum(w_i M_i)^2}{(\sum w_i)^2}}}$$
$$= \sum w_i\left(s_i^{t1q}(1-|M_i|) + s_i^{t2q} M_i\right) / \sqrt{\sum\left(w_i(1-|M_i|)\right)^2 + \sum(w_i M_i)^2}$$

Thus $NES = \frac{ES*\sum w_i}{\sqrt{\sum\left(w_i(1-|M_i|)\right)^2+\sum(w_i M_i)^2}}$

or in the full expression

$NES = \sum w_i\left(s_i^{t1q}(1-|M_i|) + s_i^{t2q} M_i\right) / \sqrt{\sum\left(w_i(1-|M_i|)\right)^2 + \sum(w_i M_i)^2}$