Image Restoration and Reconstruction

Preview

- The principal goal of restoration techniques is to i mprove an image in some predefined sense.
- Restoration attempts to reconstruct an image that has been degraded by using a priori knowledge of t he degradation process.

- Image restoration: objective process
    - Involves formulating a criterion of goodness.
- Image enhancement: subjective process
    - Heuristic procedures.

A Model of Image Degradation

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

h: Degradation function

η : Additive noise

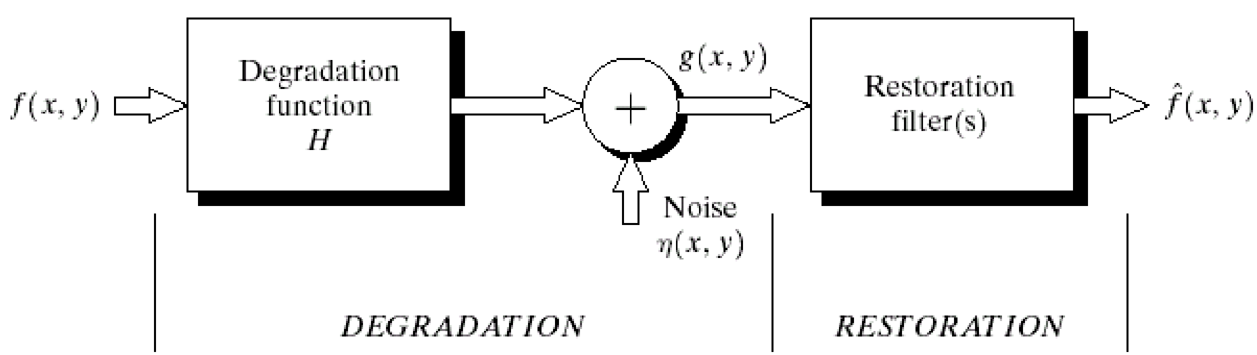


FIGURE 5.1 A model of the image degradation/restoration process.

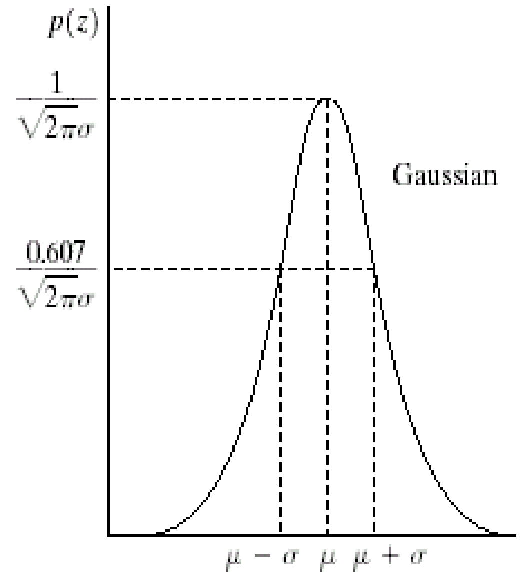
Restoration

- Given **g(x,y)**, some **knowledge about H**, and some knowledge about the noise term, obtain an estimate of the original image.

$$\underline{g(x,y)} = \underline{h(x,y)} * f(x,y) + \underline{\eta(x,y)}$$

Noise Models – Gaussian Noise

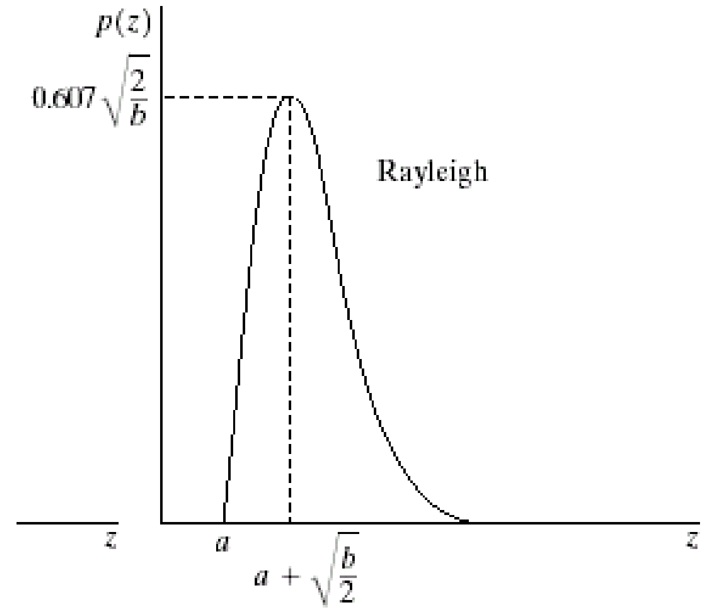
$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$



- Mean, Variance?

Noise Models – Rayleigh Noise

$$p(z) = \begin{cases} \dfrac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

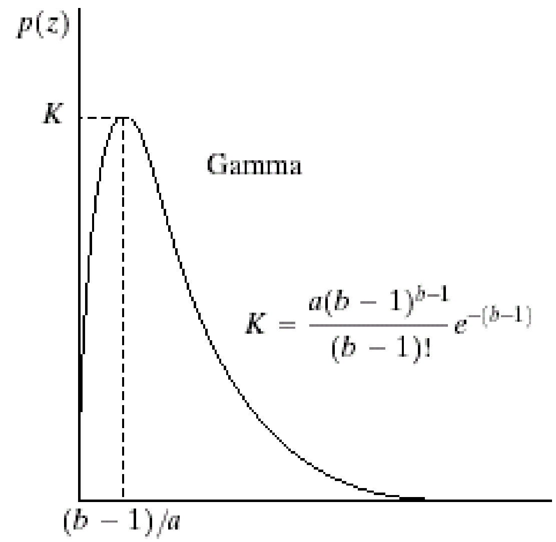


- Mean, Variance

$$\mu = a + \sqrt{\pi b/4} \qquad \sigma^2 = \frac{b(4-\pi)}{4}$$

Noise Models – Erlang (Gamma) Noise

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

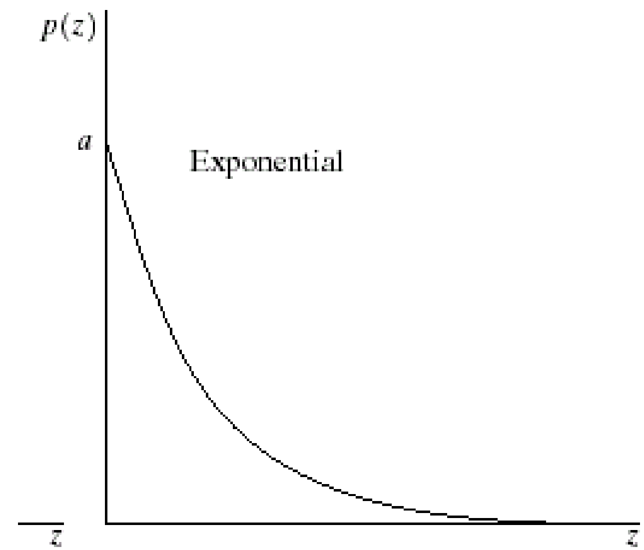


- Mean, Variance

$$\mu = \frac{b}{a} \qquad \sigma^2 = \frac{b}{a^2}$$

Noise Models – Exponential Noise

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

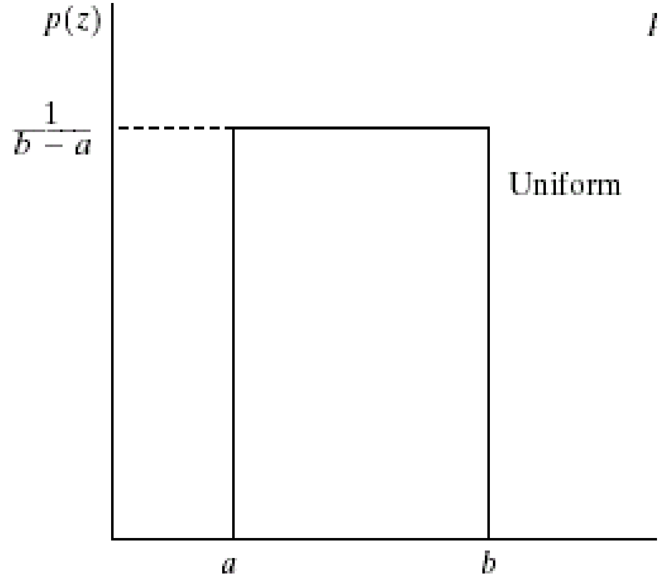


- Mean, Variance

$$\mu = \frac{1}{a} \qquad \sigma^2 = \frac{1}{a^2}$$

Noise Models – Uniform Noise

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

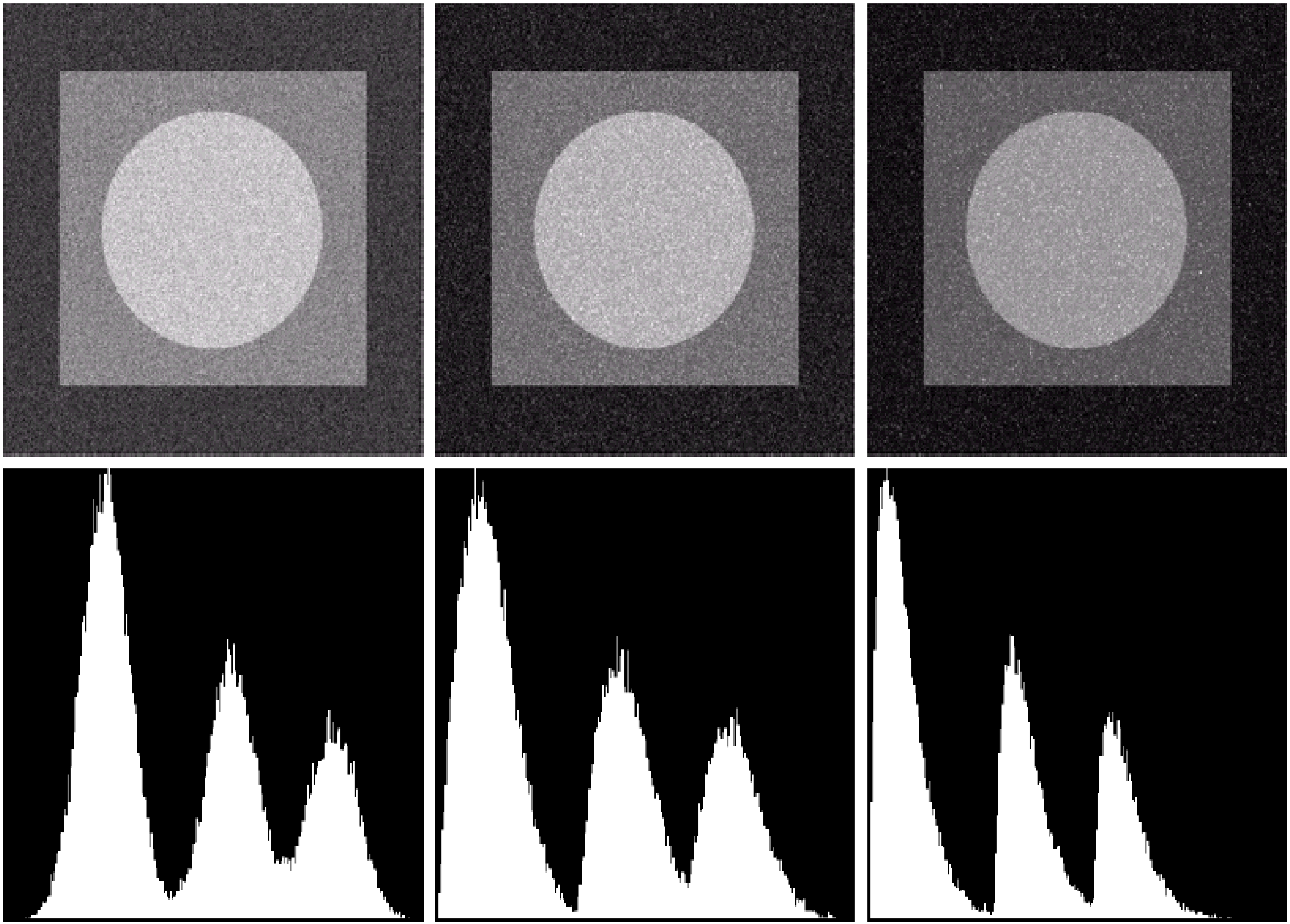


- Mean, Variance

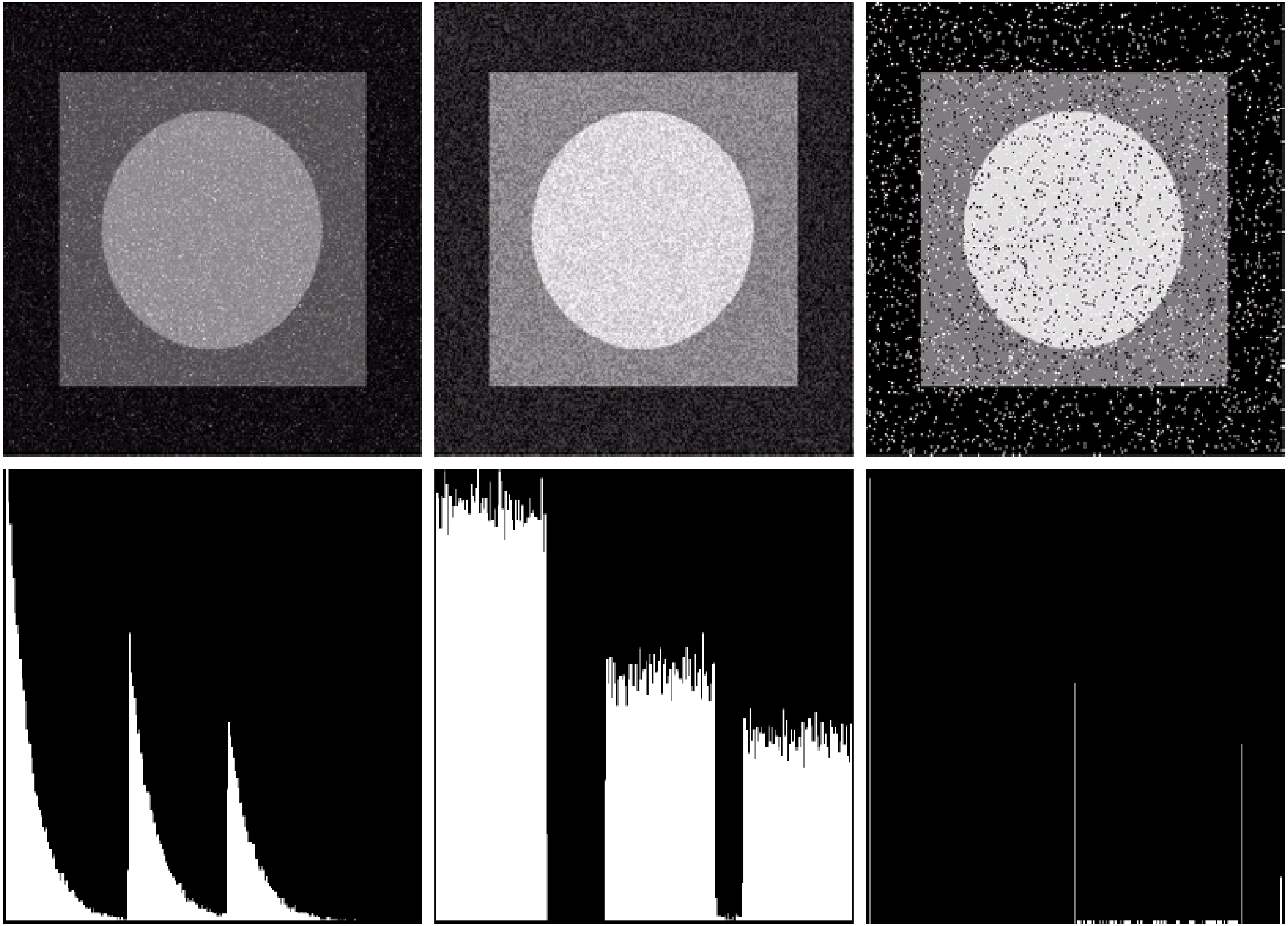
$$\mu = \frac{a+b}{2} \qquad \sigma^2 = \frac{(b-a)^2}{12}$$

Noise Models – Impulse (Salt-and-Pepper) Noise

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



Gaussian Rayleigh Gamma



Exponential Uniform Salt & Pepper

Estimation of Noise Parameters

- Estimate the parameters of the PDF from small patches of reasonably constant background intensity.

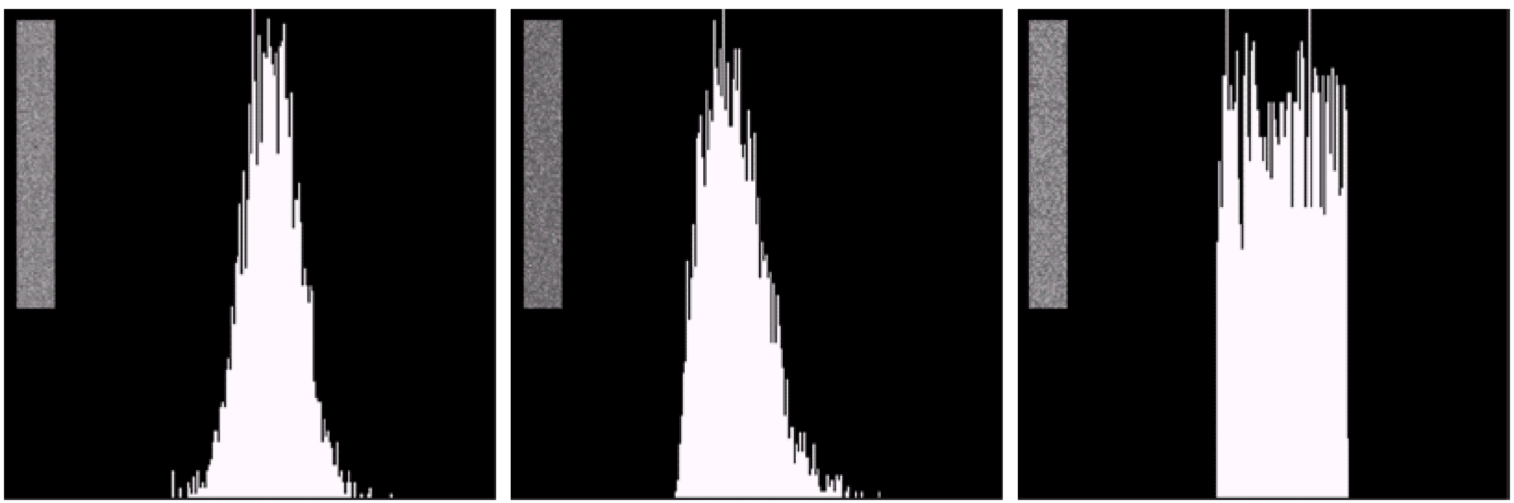


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration in the Presence of Noise Only

- Spatial filtering is the method of choice in situations when only additive random noise is present.

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

Mean Filters

- Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- Geometric mean filter

$$\hat{f}(x,y) = \left(\prod_{(s,t)\in S_{xy}} g(s,t) \right)^{\frac{1}{mn}}$$

Mean Filters

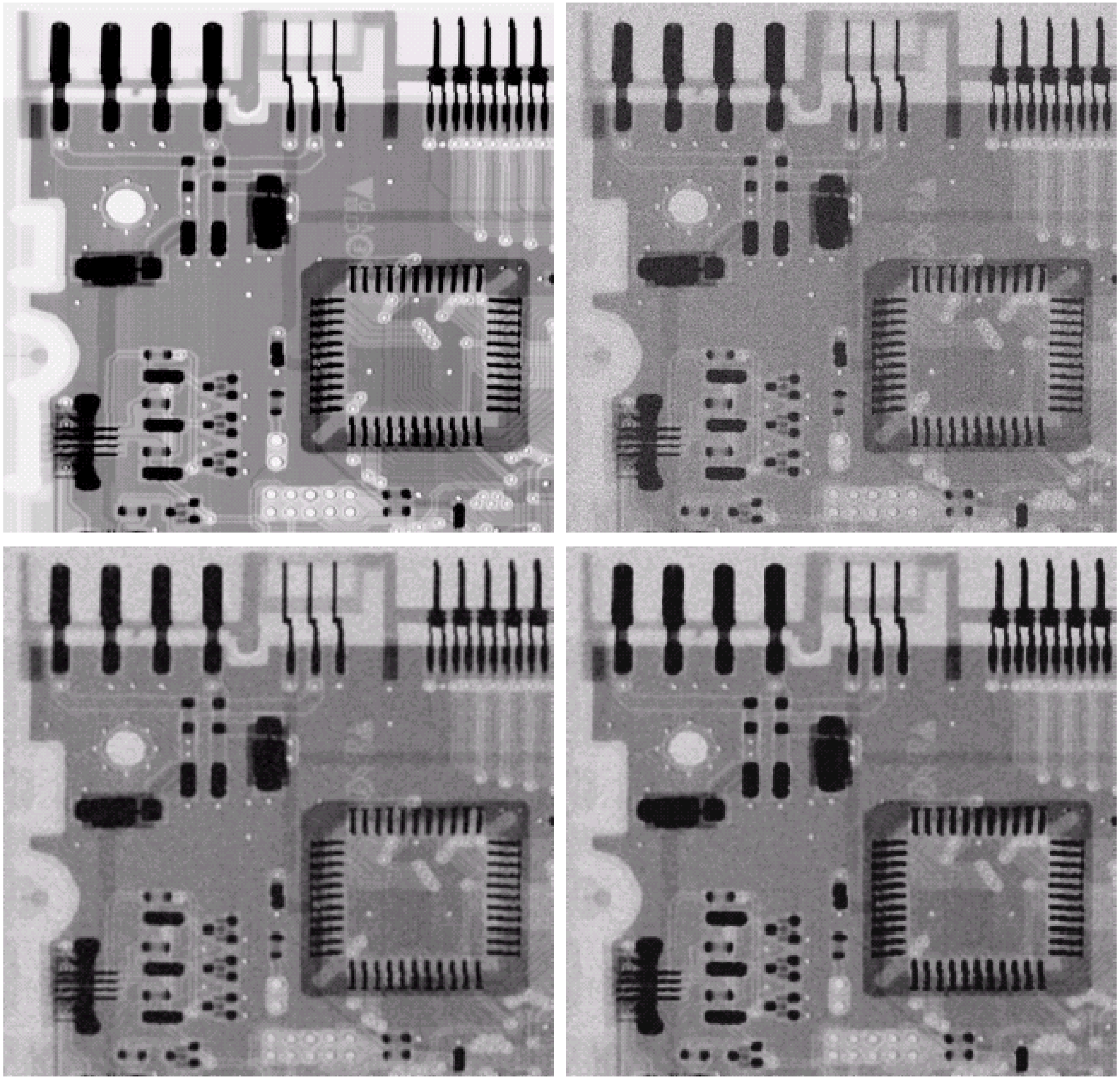
- Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- Contraharmonic mean filter

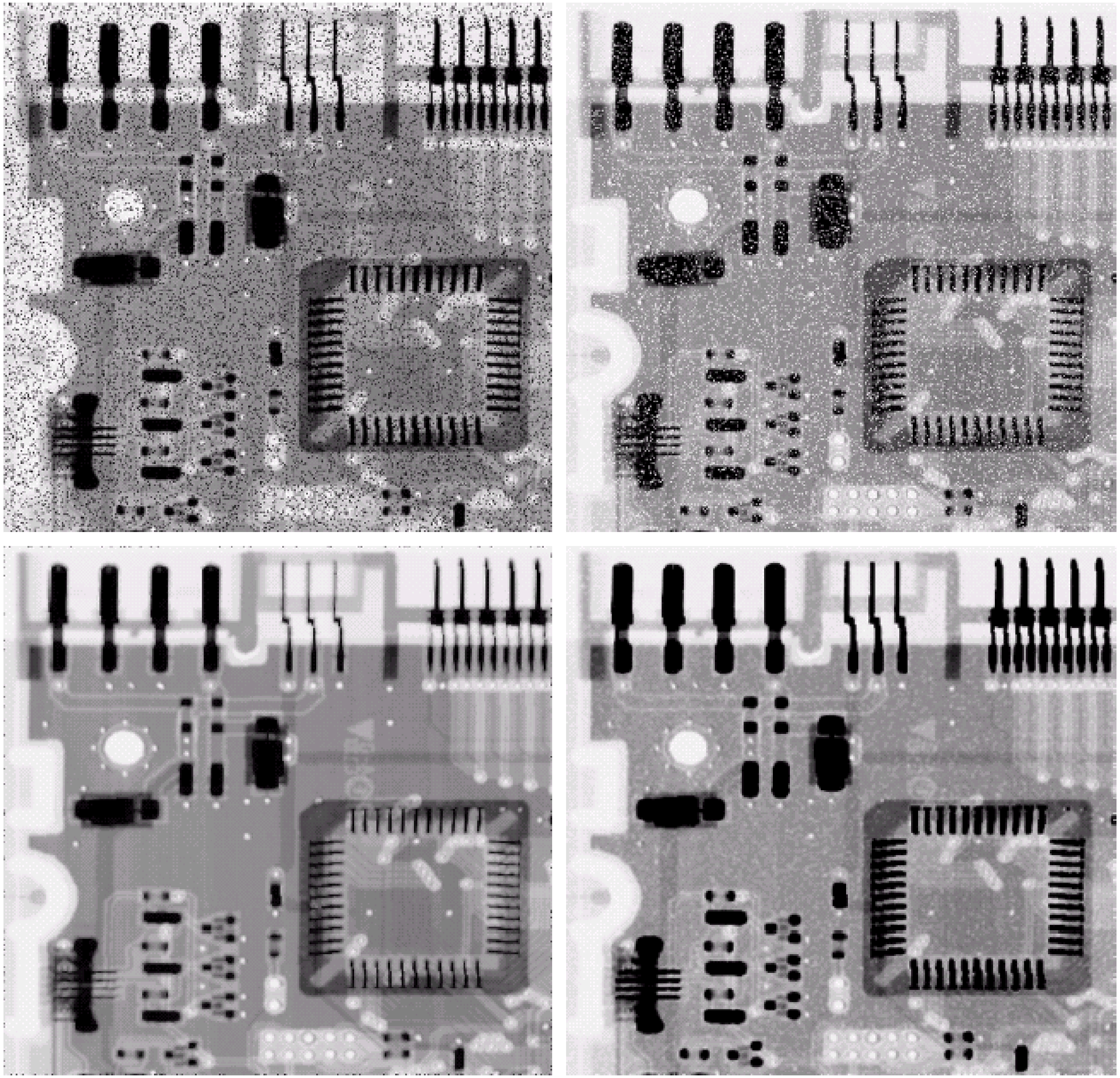
$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Positive Q is suitable for eliminating pepper noise. Negative Q is suitable for eliminating salt noise.



a	b
c	d

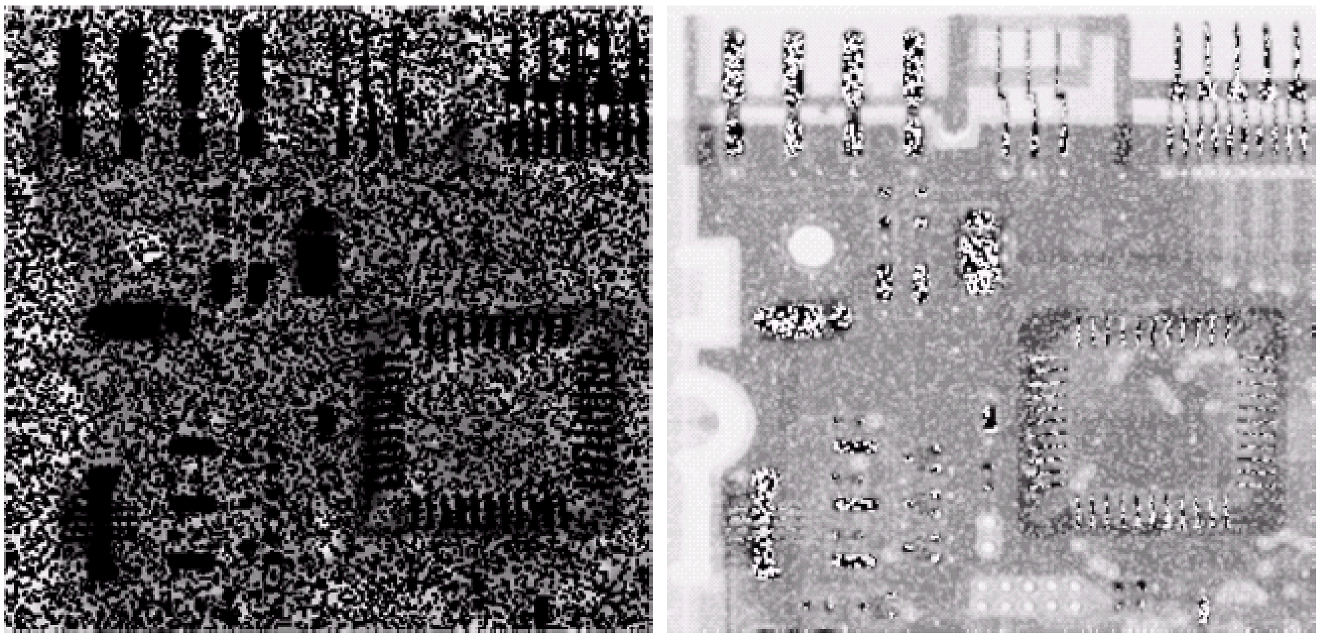
FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Wrong Sign in Contraharmonic Filtering



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Order-Statistic Filters

- Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\left\{g(s,t)\right\}$$

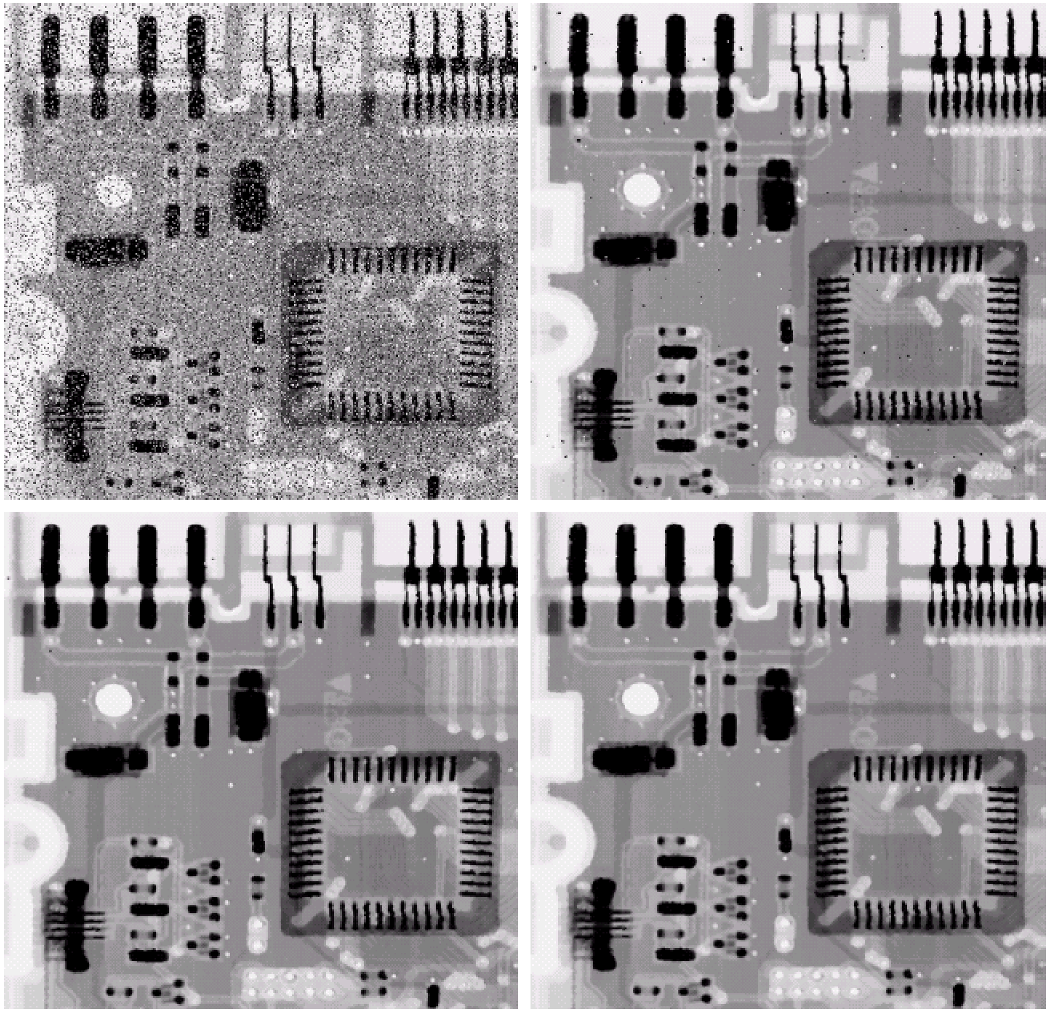
- Max and min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\left\{g(s,t)\right\} \qquad \hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\left\{g(s,t)\right\}$$

Order-Statistic Filters

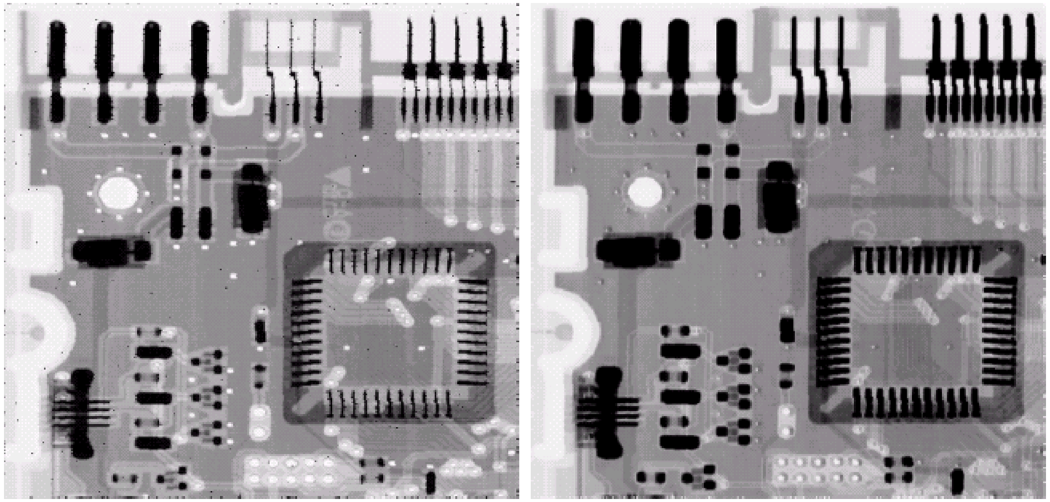
- Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left(\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\} \right)$$



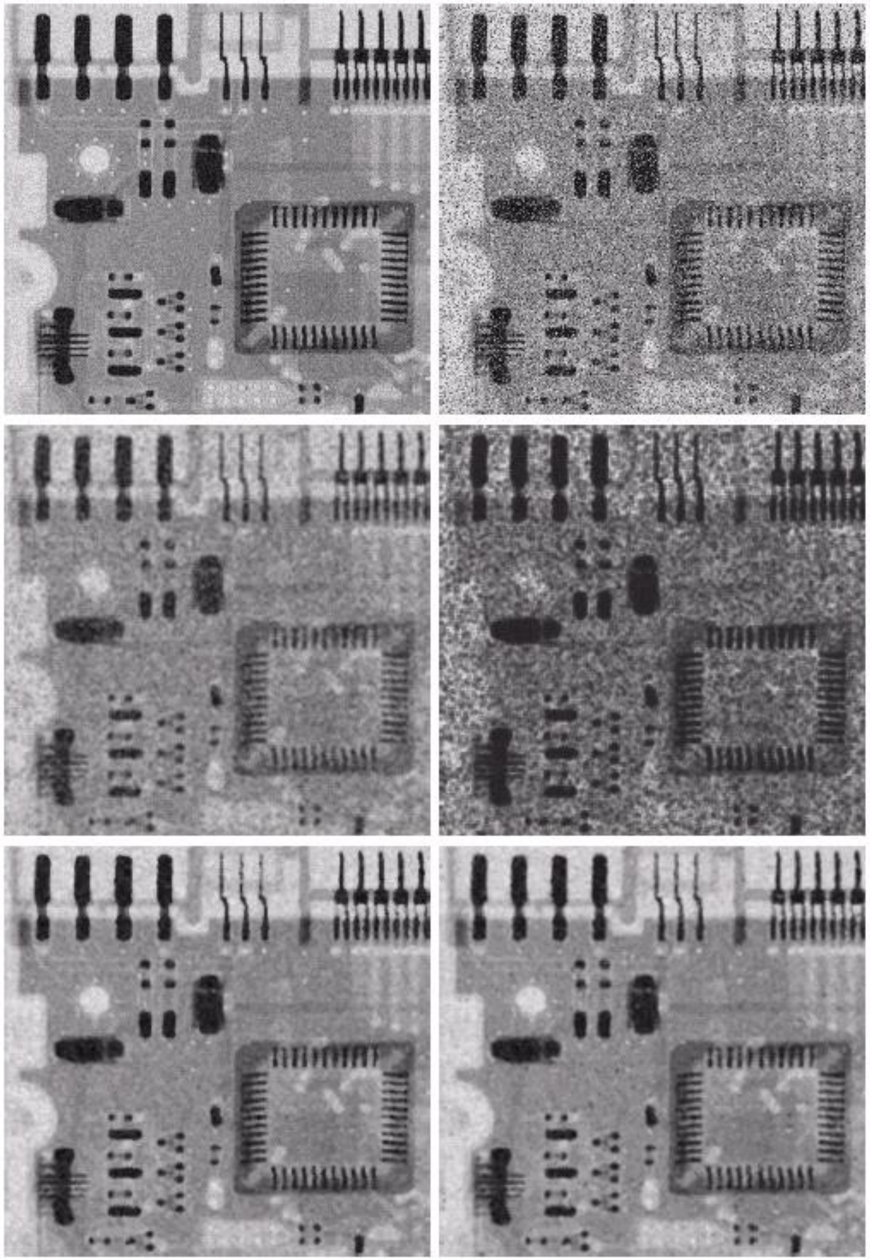
a b
c d

FIGURE 5.10
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.



a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5×5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

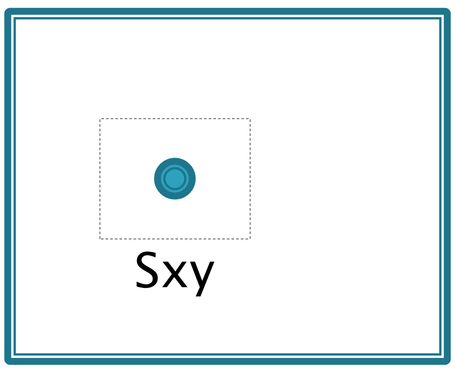
Adaptive Filters

- Previous Filters: Once selected, the filters are applied to an image **without regard for how image characteristics vary** from one point to another.

- Adaptive Filters whose behavior **changes based on statistical characteristics** of the image inside the filter region.

Adaptive, Local Noise Reduction Filter

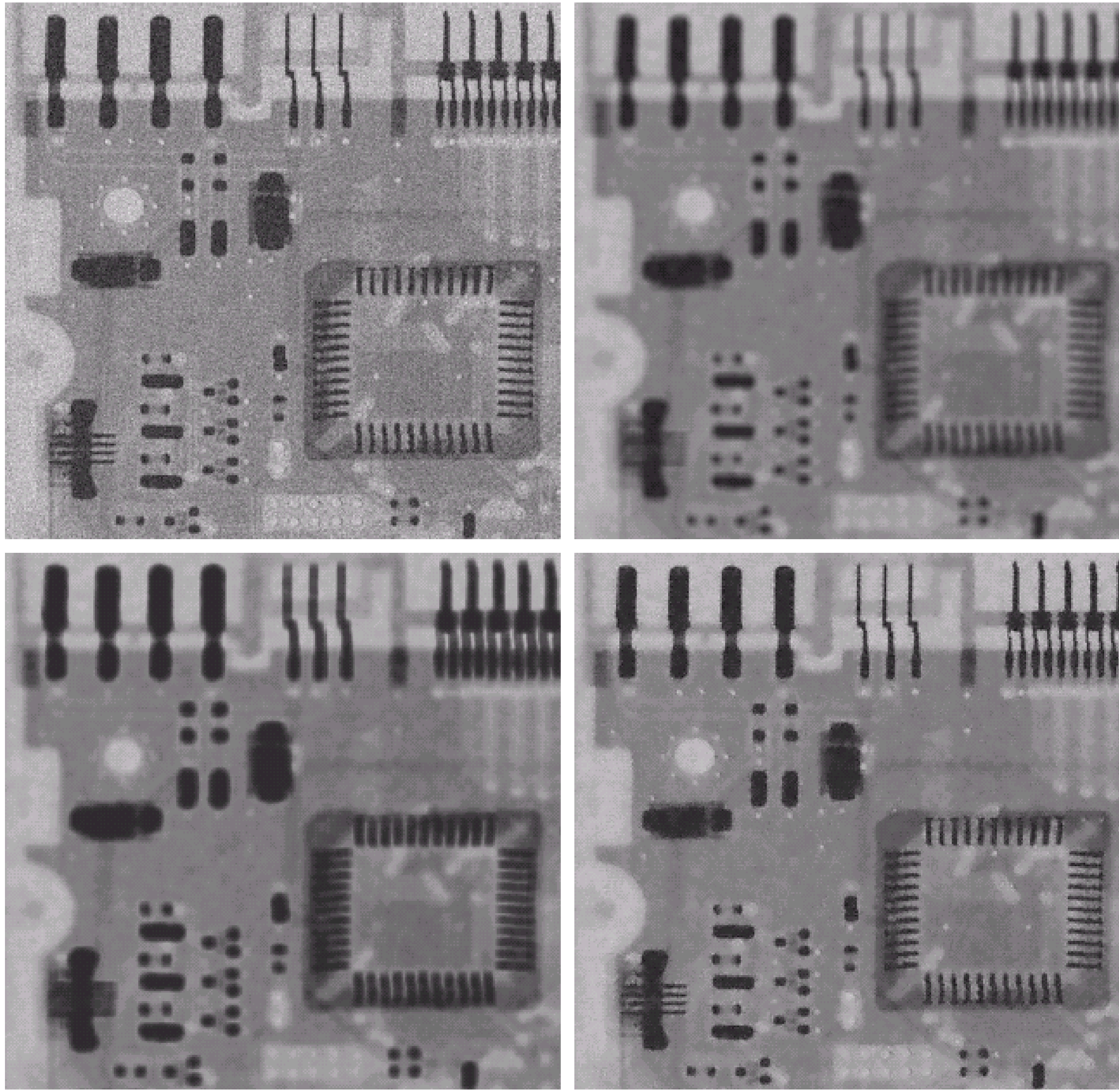
- Four quantities
  - g(x,y): the value of the noisy image at (x,y)
  - σ_η^2 : the variance of the noise corrupting f(x,y)
  - m_L: the local mean of the pixels in Sxy (Local region)
  - σ_L^2 : the local variance of the pixels in Sxy



Adaptive, Local Noise Reduction Filter

- Concept
  - If σ_η^2 is zero, → zero-noise case (No noise)
  - If σ_L^2 is high relative to σ_η^2 → edges
    - The filter should return a value close to $g(x,y)$
  - If $\sigma_L^2 = \sigma_\eta^2$, → inside objects
    - We want the filter to return the arithmetic mean value of the pixels in Sxy

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left(g(x,y) - m_L\right)$$



a b
c d

FIGURE 5.13
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.
(b) Result of arithmetic mean filtering.
(c) Result of geometric mean filtering.
(d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Adaptive Median Filter

Level *A*: $A1 = z_{median} - z_{min}$

$A2 = z_{median} - z_{max}$

If $A1>0$ and $A2<0$, goto level B ➔ $z_{min}<z_{median}<z_{max}$

Else increase window size

If window size $<= S_{max}$ repeat level A

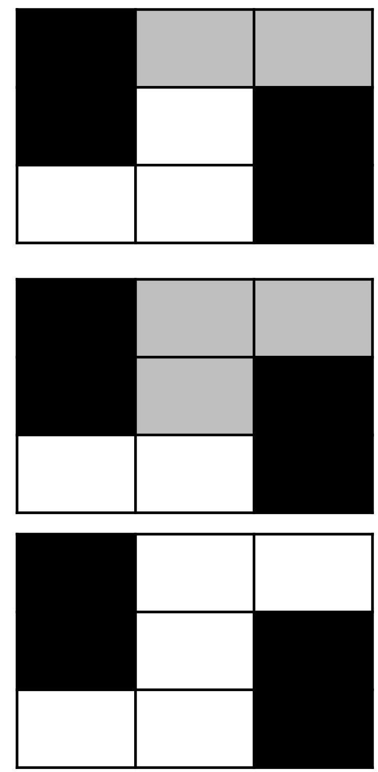
Else return z_{xy}

Level *B*: $B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

If $B1 > 0$ and $B2 < 0$, return z_{xy}

Else return z_{median}



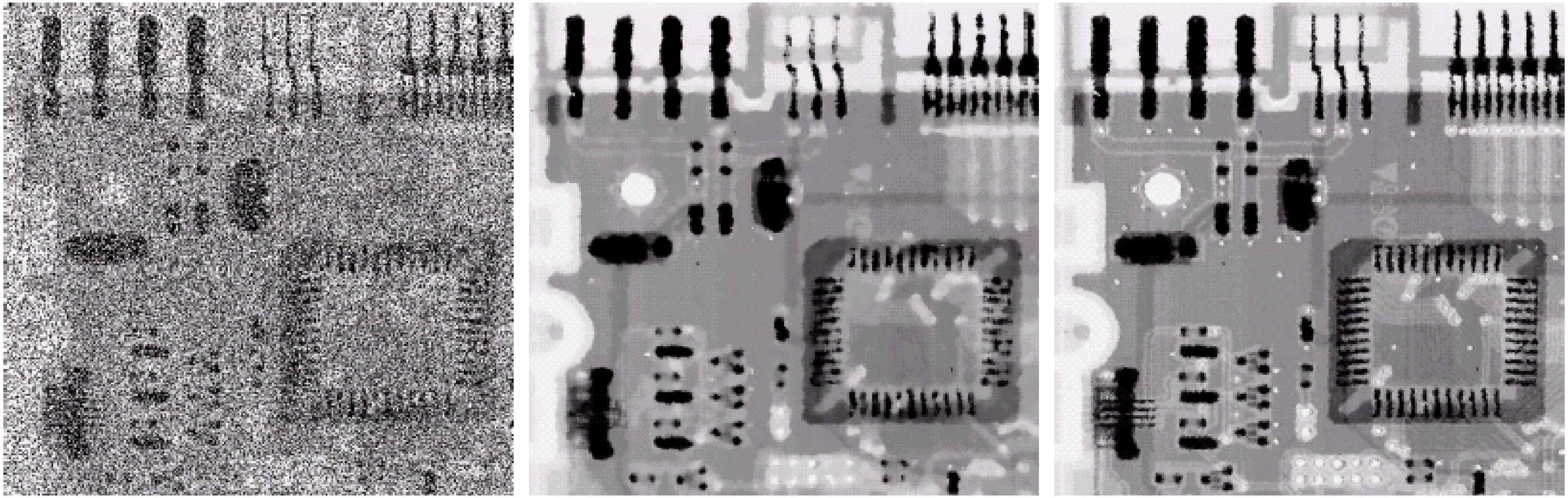
z_{min} = minimum gray level value in S_{xy}

z_{max} = maximum gray level value in S_{xy}

z_{median} = median of gray levels in S_{xy}

z_{xy} = gray level value at pixel (x,y)

S_{max} = maximum allowed size of S_{xy}



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

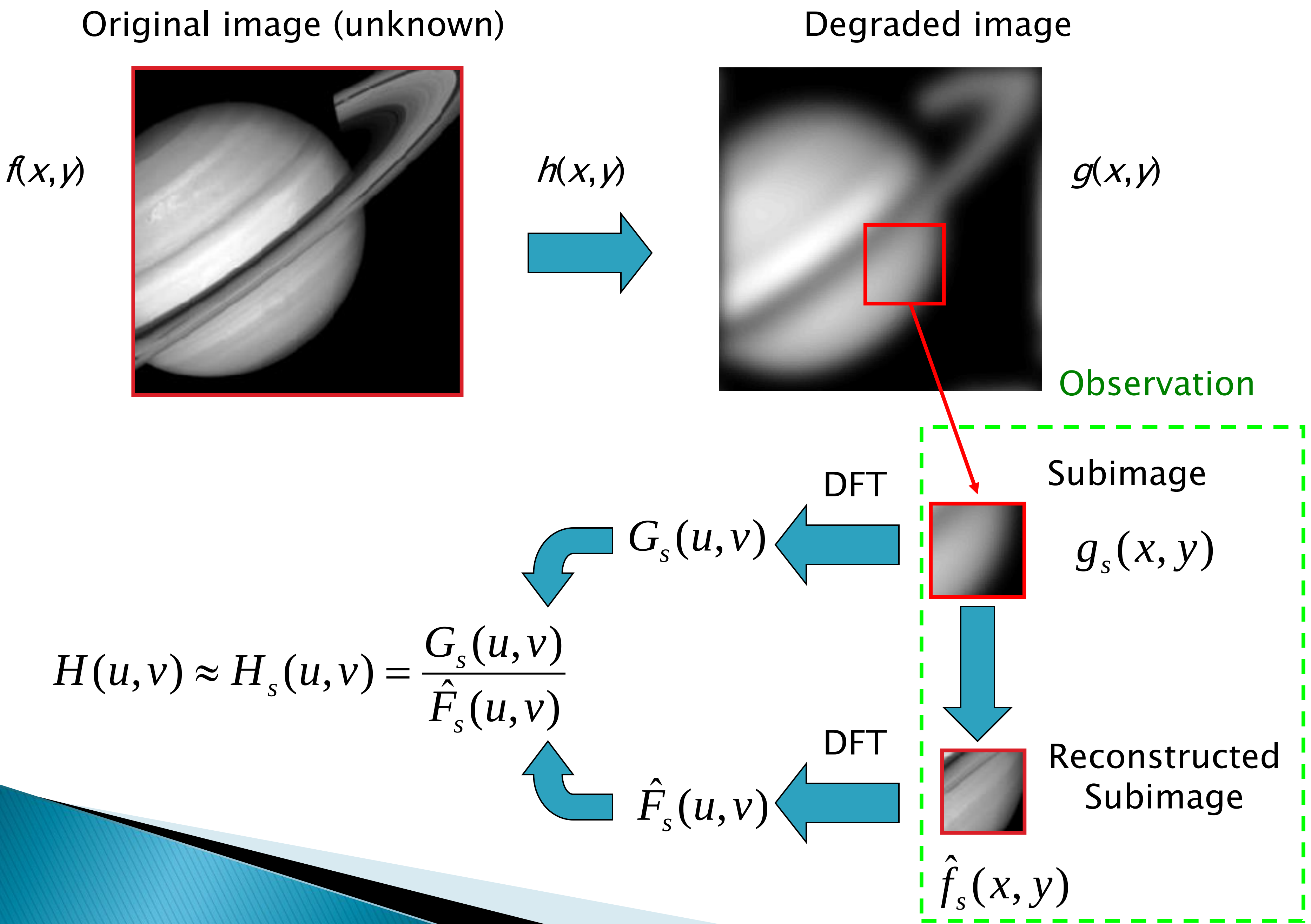
Estimating the Degradation Function

- There are three principal ways to estimate the degradation function for use in image restoration
  - Observation
  - Experimentation
  - Mathematical modeling

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

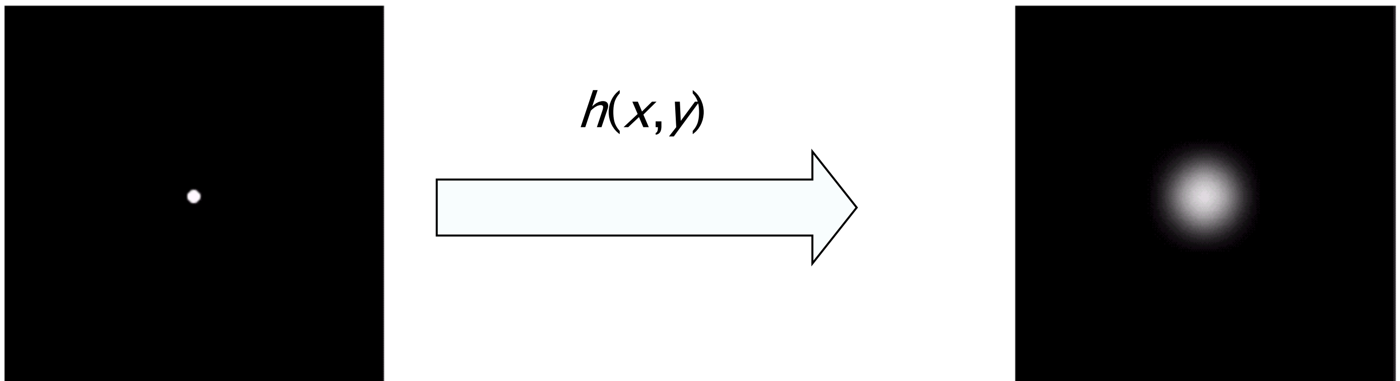
If we know exactly $h(x,y)$, regardless of noise, we can do deconvolution to get $f(x,y)$ back from $g(x,y)$.



Original image (unknown)
Degraded image
$f(x,y)$
$h(x,y)$
$g(x,y)$
Observation
Subimage
$g_s(x,y)$
DFT
$G_s(u,v)$
Reconstructed Subimage
$\hat{f}_s(x,y)$
DFT
$\hat{F}_s(u,v)$
$$H(u,v) \approx H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

Estimating the Degradation Function

If equipment similar to the equipment used to acquire the degraded image is available, it is possible in principle to obtain an accurate estimate of the degradation.



An impluse of light

$\delta(x,y)$

Degraded impluse

$g(x,y)$

$$H(u,v) = \frac{F(g(x,y))}{F(\delta(x,y))} = F(g(x,y))$$

Estimation by Modeling

Atmospheric Turbulence model

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

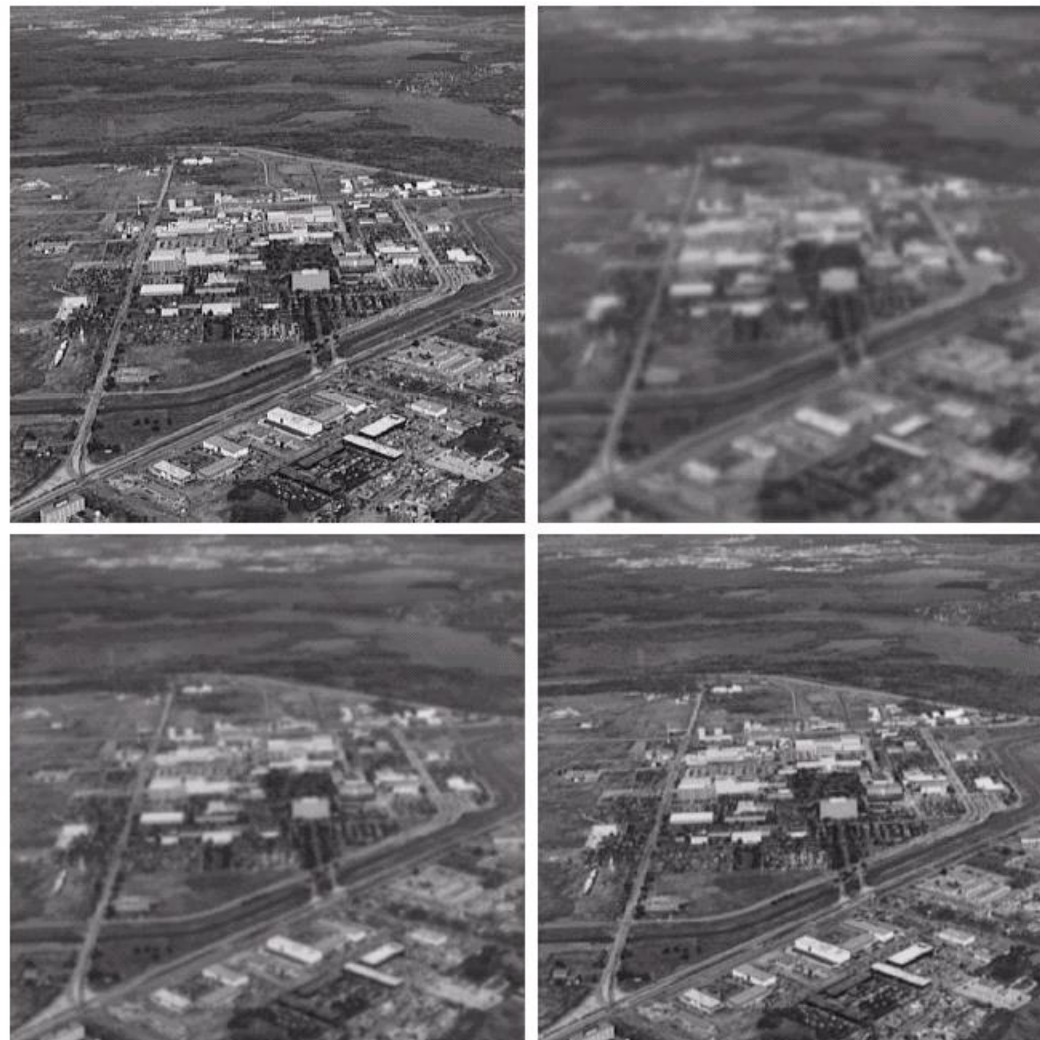


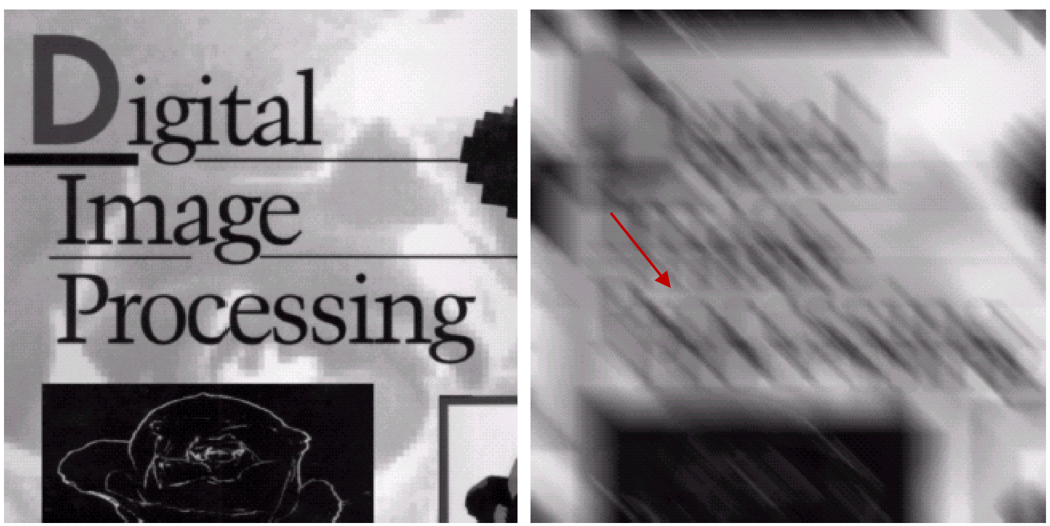
FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Estimation by Modeling

- Image blurring

$(x_0(t), y_0(t))$: Time varying components of motion

$$g(x,y) = \int_0^T f(x + x_0(t), y + y_0(t))dt$$



$$H(u,v) = \frac{T}{\pi(ua+vb)}\sin(\pi(ua+vb))e^{-j\pi(ua+vb)}$$

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Inverse Filter

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

Inverse filter

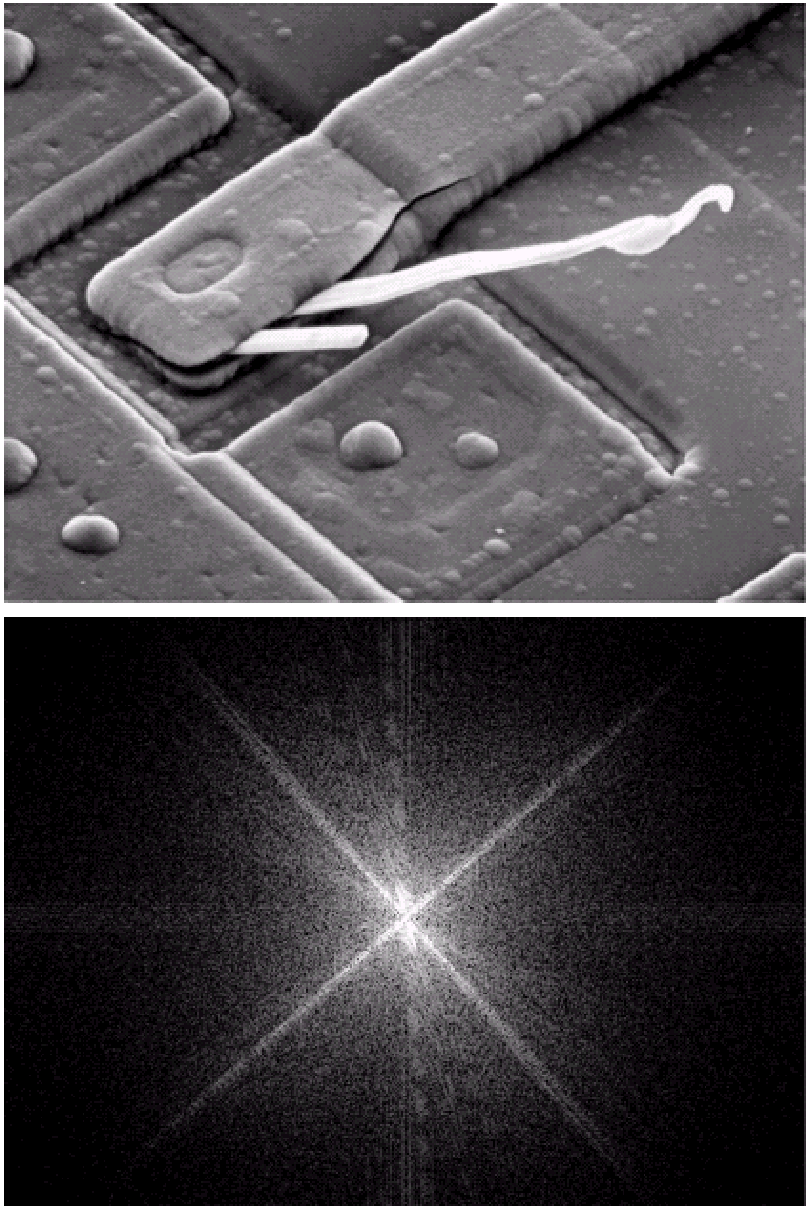
$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Example

Atmospheric Turbulence model

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

What if H(u,v) is small?



a
b

FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.