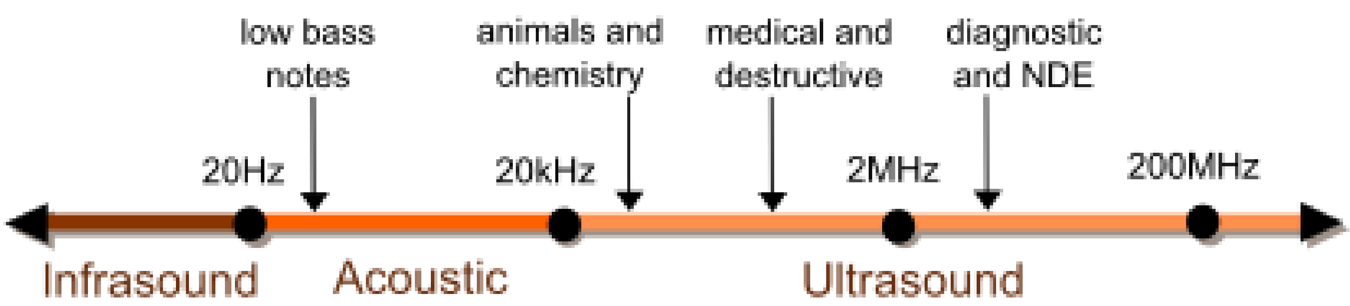
Filtering in the Frequency Domain

Sound

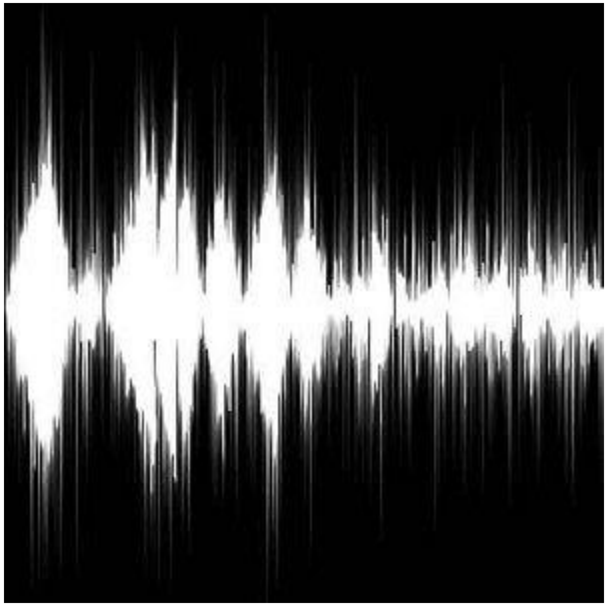
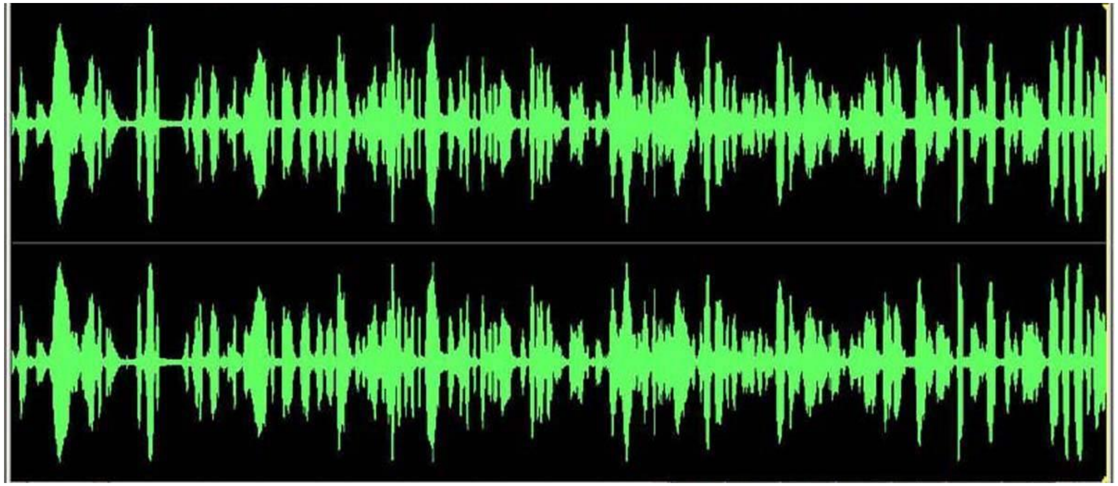
- Human: ~20kHz
- Bat: ~100kHz, up to 200kHz



http://blog.naver.com/PostView.nhn?blogId=sorisemna&logNo=40120837059

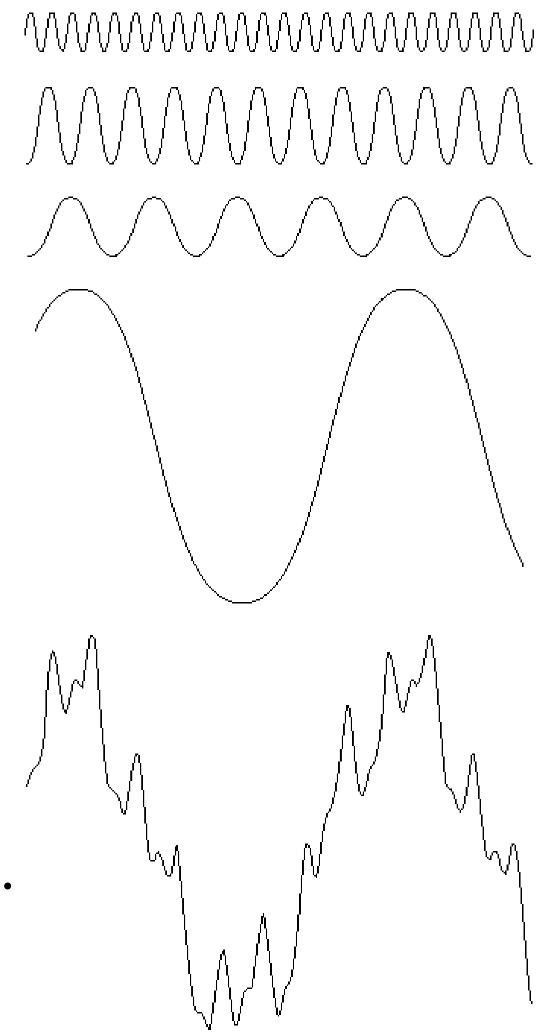
Soundwave

- A **voice frequency** is one of the frequencies, within part of the audio range.

Fourier's Idea

- Any function can be expressed as the sum of sines of different freqs.



$$f(x) = a_0 \cos b_0 x + a_1 \cos b_1 x + a_2 \cos b_2 x + ...$$

FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

Fourier Transform

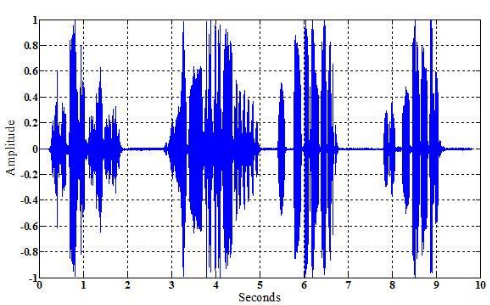
Time, spatial Domain Signals

Fourier Transform →

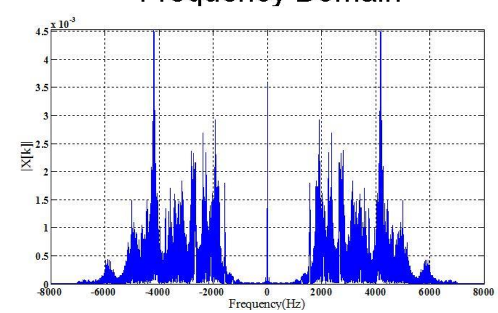
← Inverse Fourier Transform

Frequency Domain Signals

Time Domain

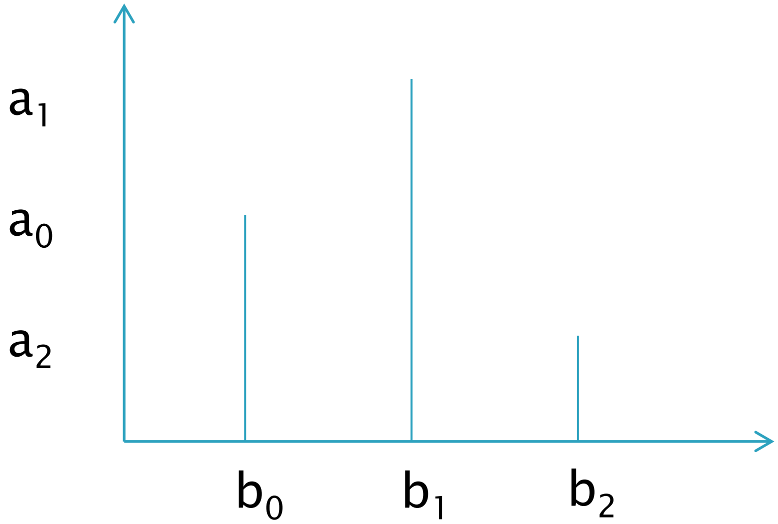


Frequency Domain



Simple Conceptual Example

$$f(x) = a_0 \cos b_0 x + a_1 \cos b_1 x + a_2 \cos b_2 x$$



Fourier Transform

- Continuous Signal

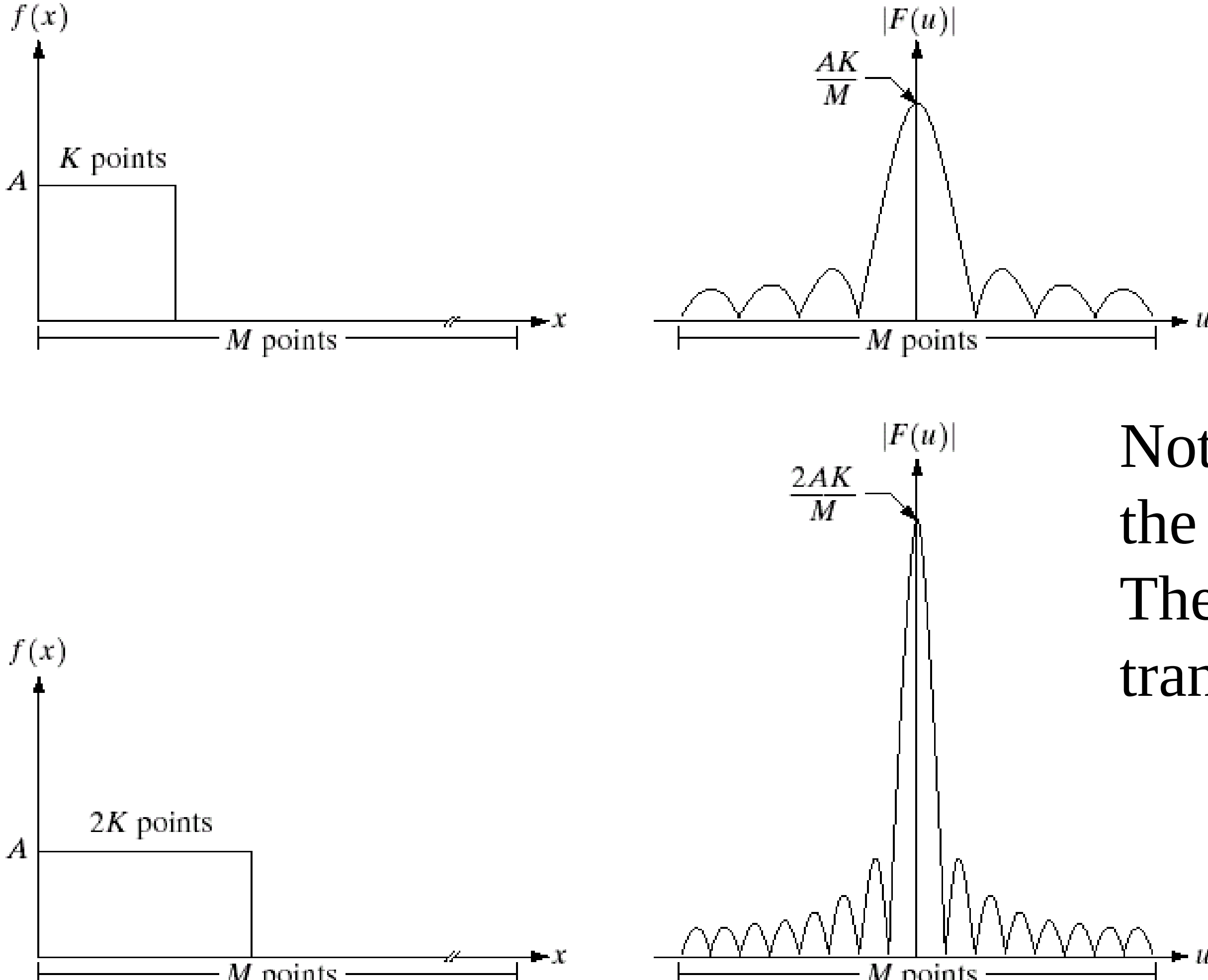
Fourier Transform
$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

Inverse Fourier Transform
$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

- Discrete Signal

Fourier Transform
$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \qquad u = 0,\ldots,M-1$$

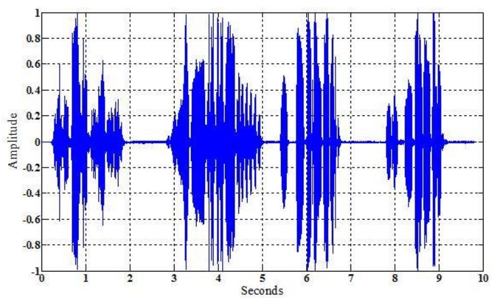
Inverse Fourier Transform
$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \qquad x = 0,\ldots,M-1$$



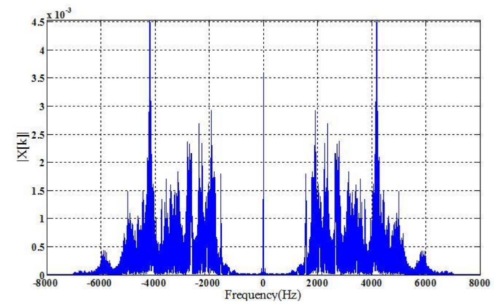
Notice that the longer the time domain signal, The shorter its Fourier transform

Meaning of Frequency Domain

Time Domain



Frequency Domain



Two-Dimensional DFT

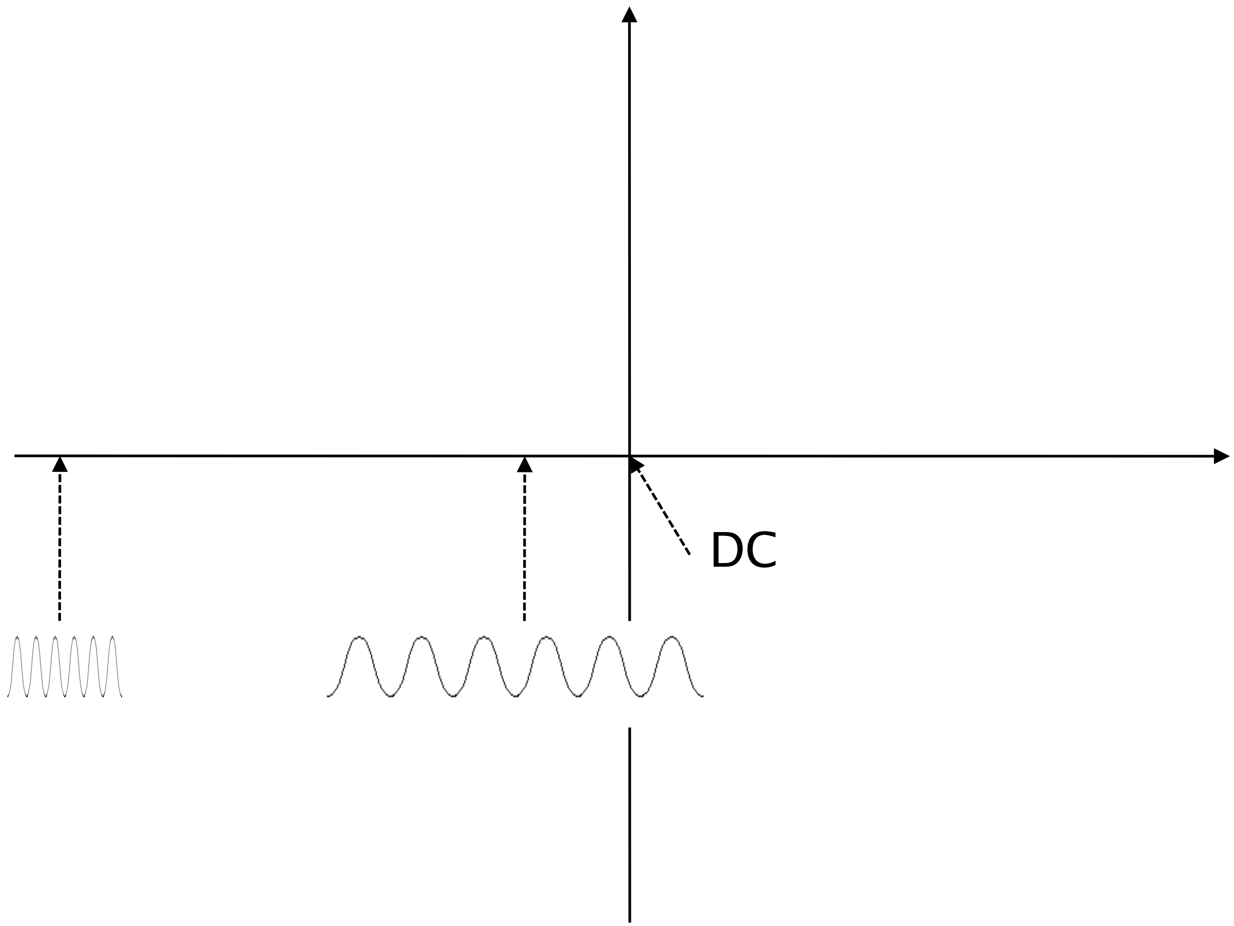
2-D DFT

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

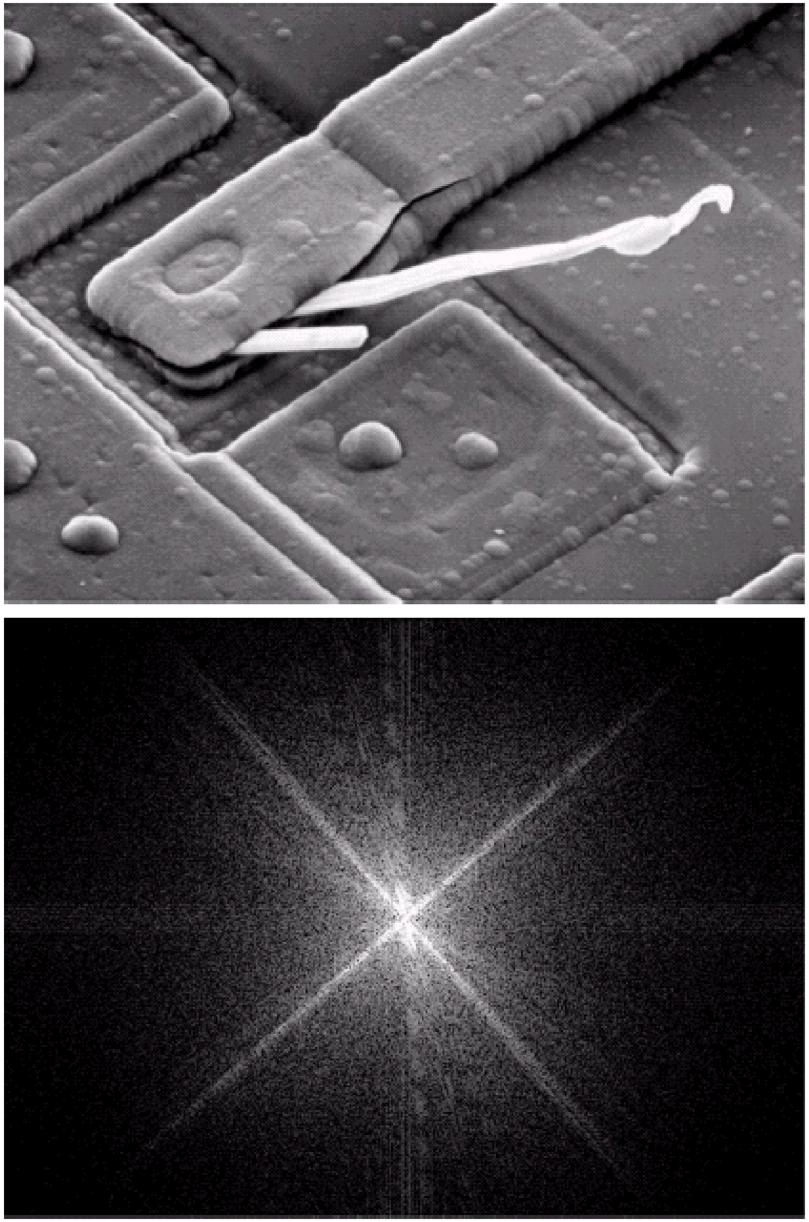
2-D IDFT

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

Property



Frequency Domain

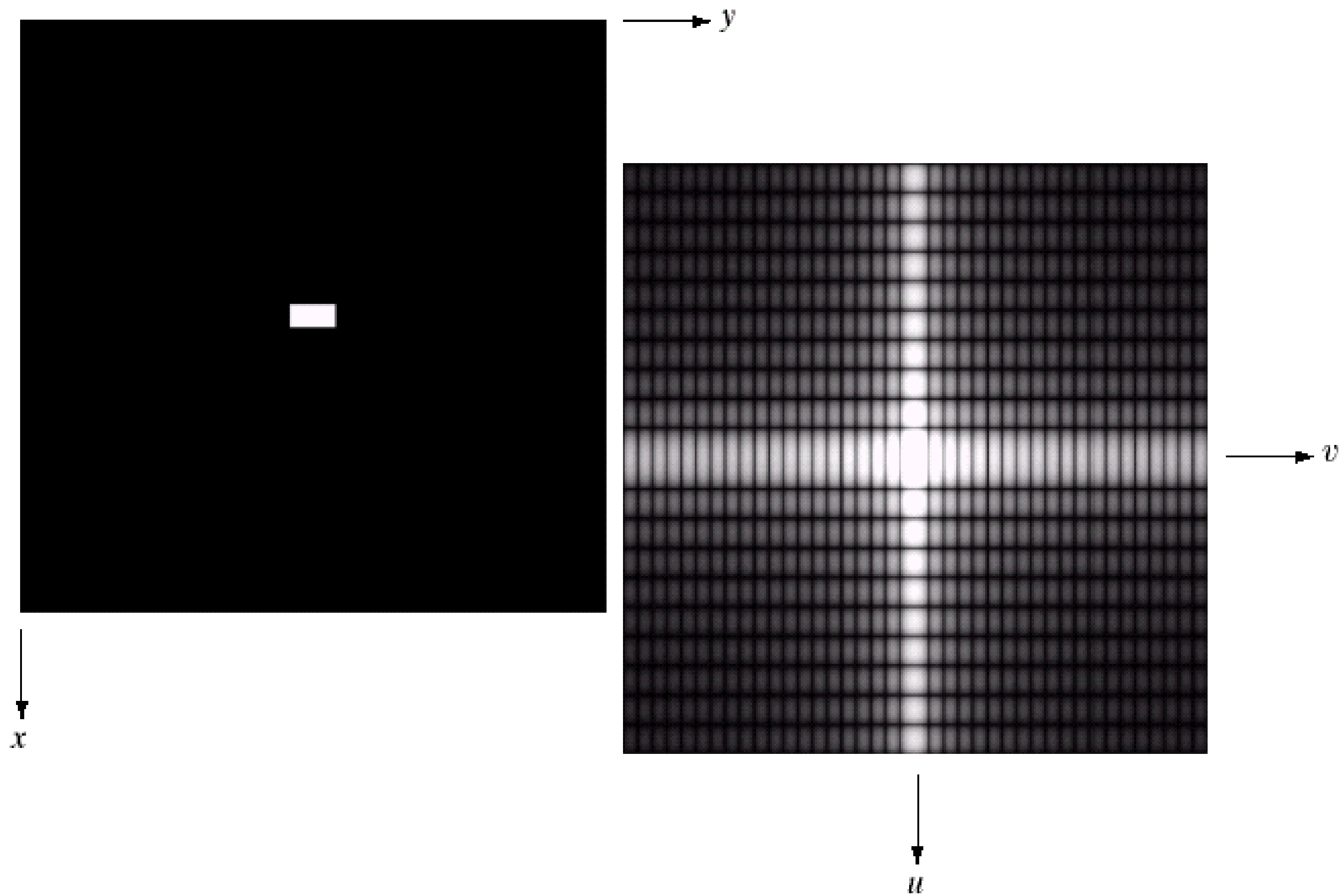


a
b

FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

a b

FIGURE 4.3
(a) Image of a 20×40 white rectangle on a black background of size 512×512 pixels.
(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.



Basics of Filtering in the Frequency Domain

$$g(x,y)=\sum_{i=-a}^{a}\sum_{j=-b}^{b}h(i,j)\,f(x-i,y-j)$$

Frequency domain filtering operation

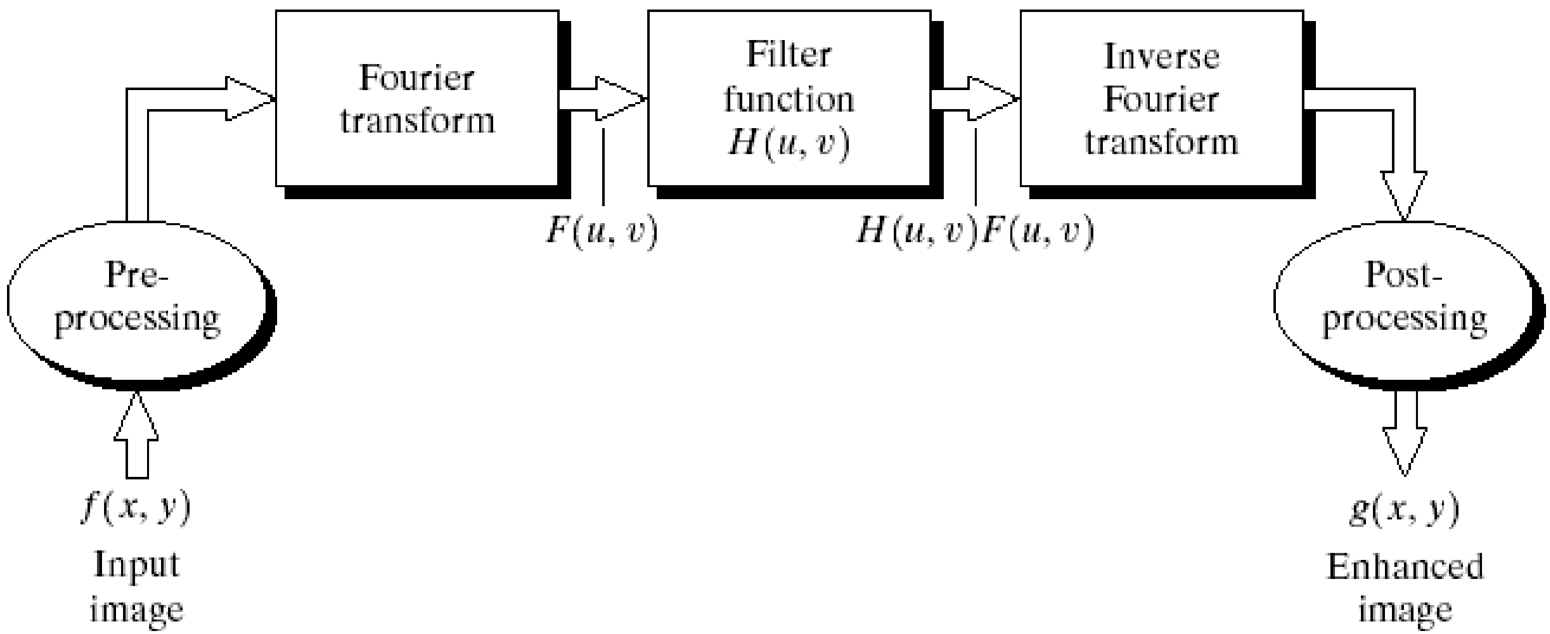


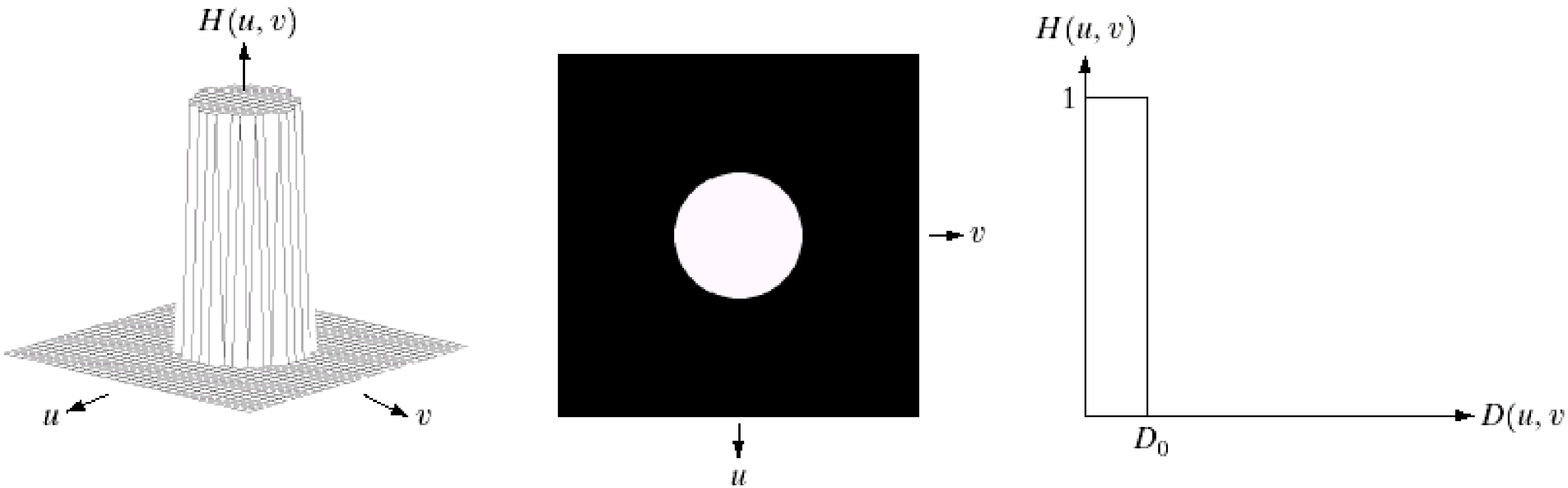
Image Smoothing Using Frequency Domain Filters

- Smoothing (Blurring) is achieved in the freque ncy domain by dropping out the high frequen cy components.

$$G(u,v) = H(u,v)\, F(u,v)$$

$F(u,v)$: Fourier transform of the image
$H(u,v)$: Filter transform function

Ideal Lowpass Filters



a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

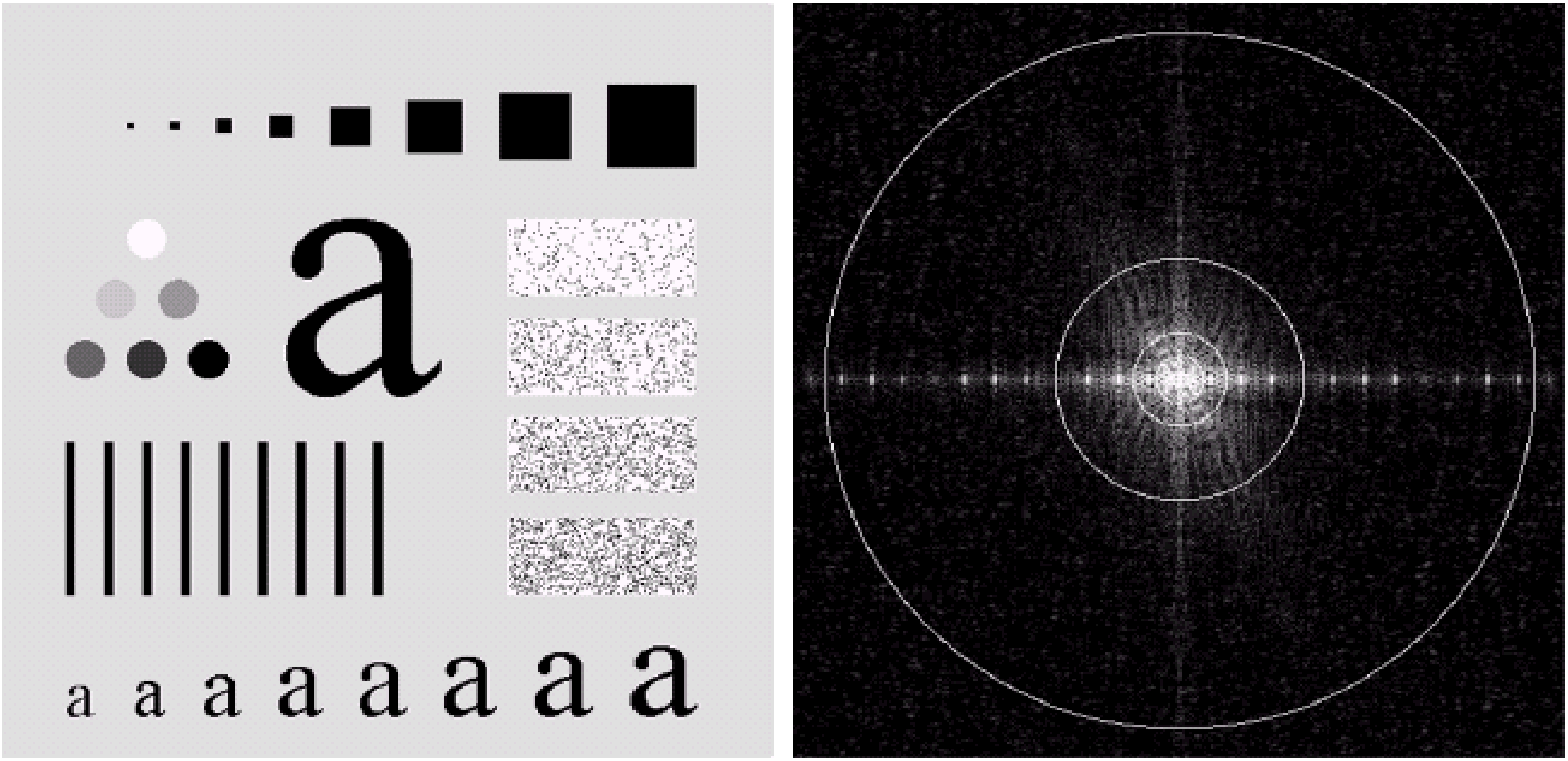
Ideal Lowpass Filters

- Transfer function for the ideal low pass filter

$$H(u,v)=\begin{cases}1 \text{ if } D(u,v)\le D_0\\ 0 \text{ if } D(u,v)>D_0\end{cases}$$

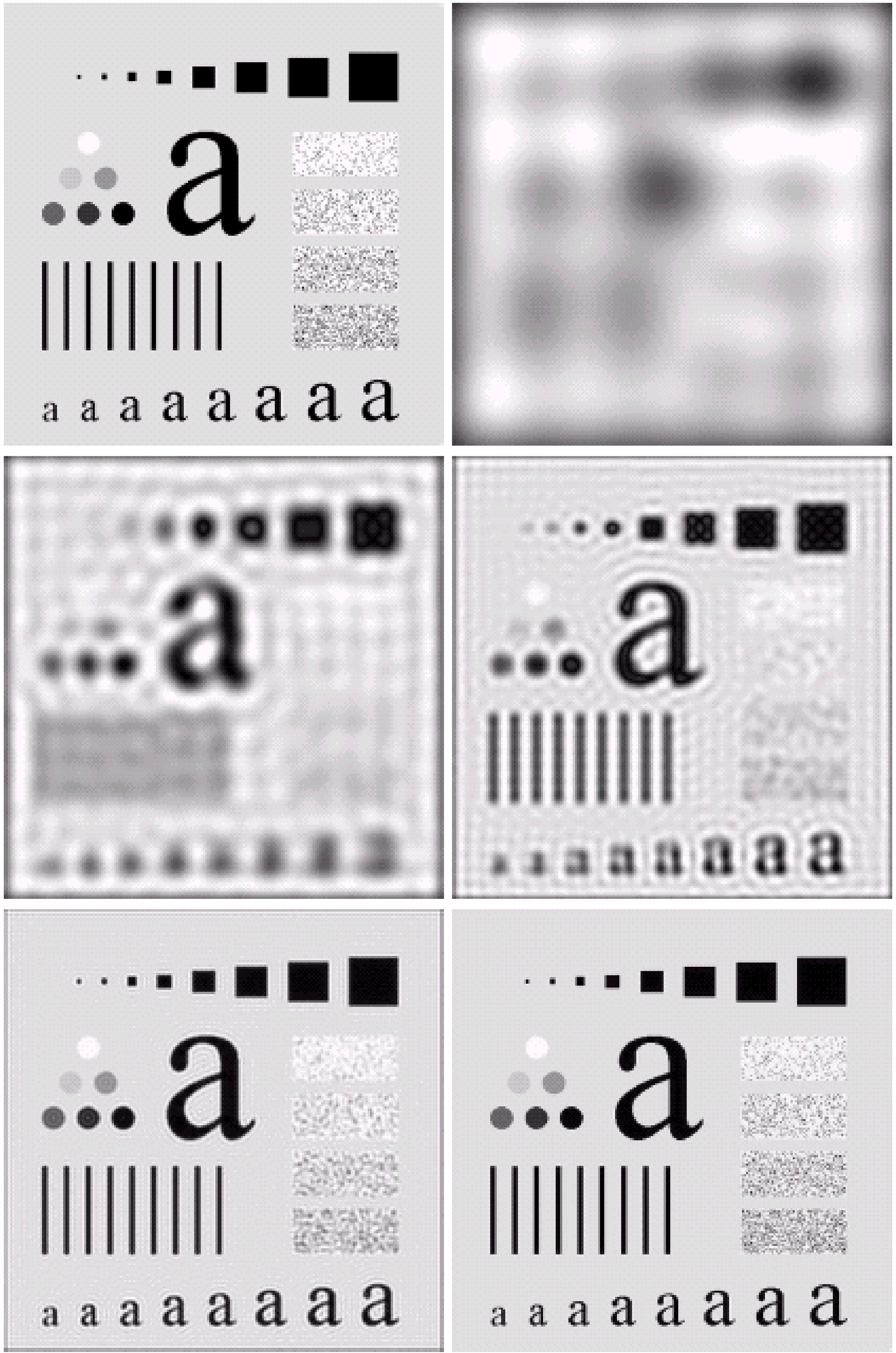
D(u,v) is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle.

$$D(u,v)=[(u-P/2)^2+(v-Q/2)^2]^{1/2}$$



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.



Blurring and Ringing

a b
c d
e f

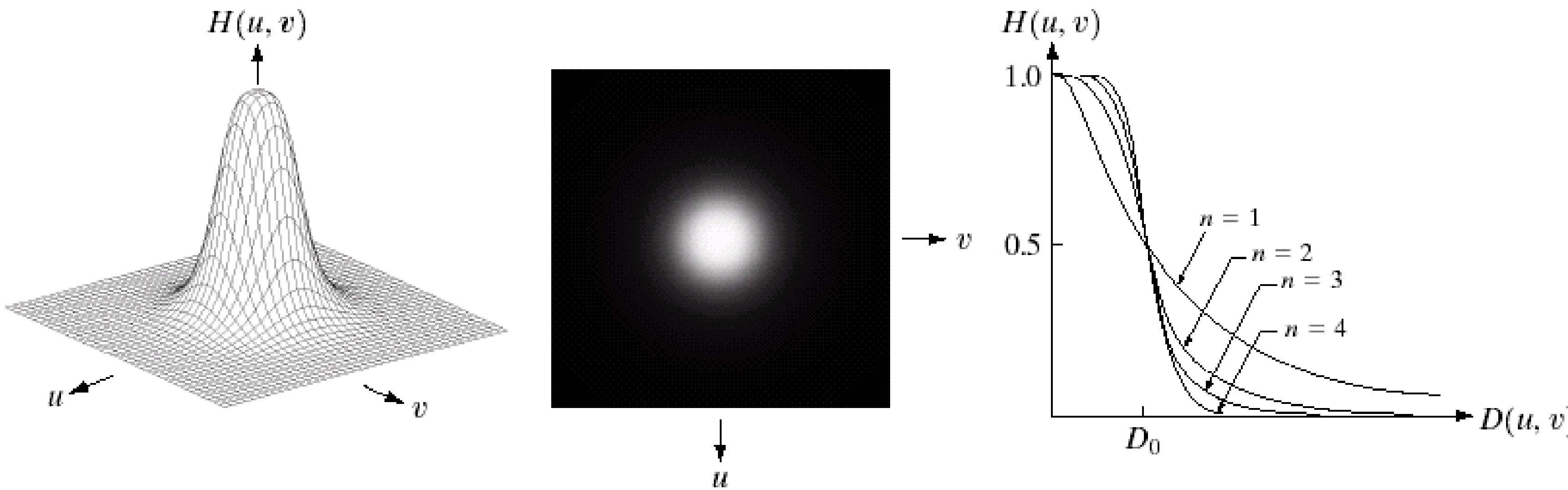
FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

Butterworth Lowpass Filters

- The transfer function of a Butterworth lowpass filter (BLPF) of order n, and with cutoff frequency at distance D_0 from the origin, is defined as

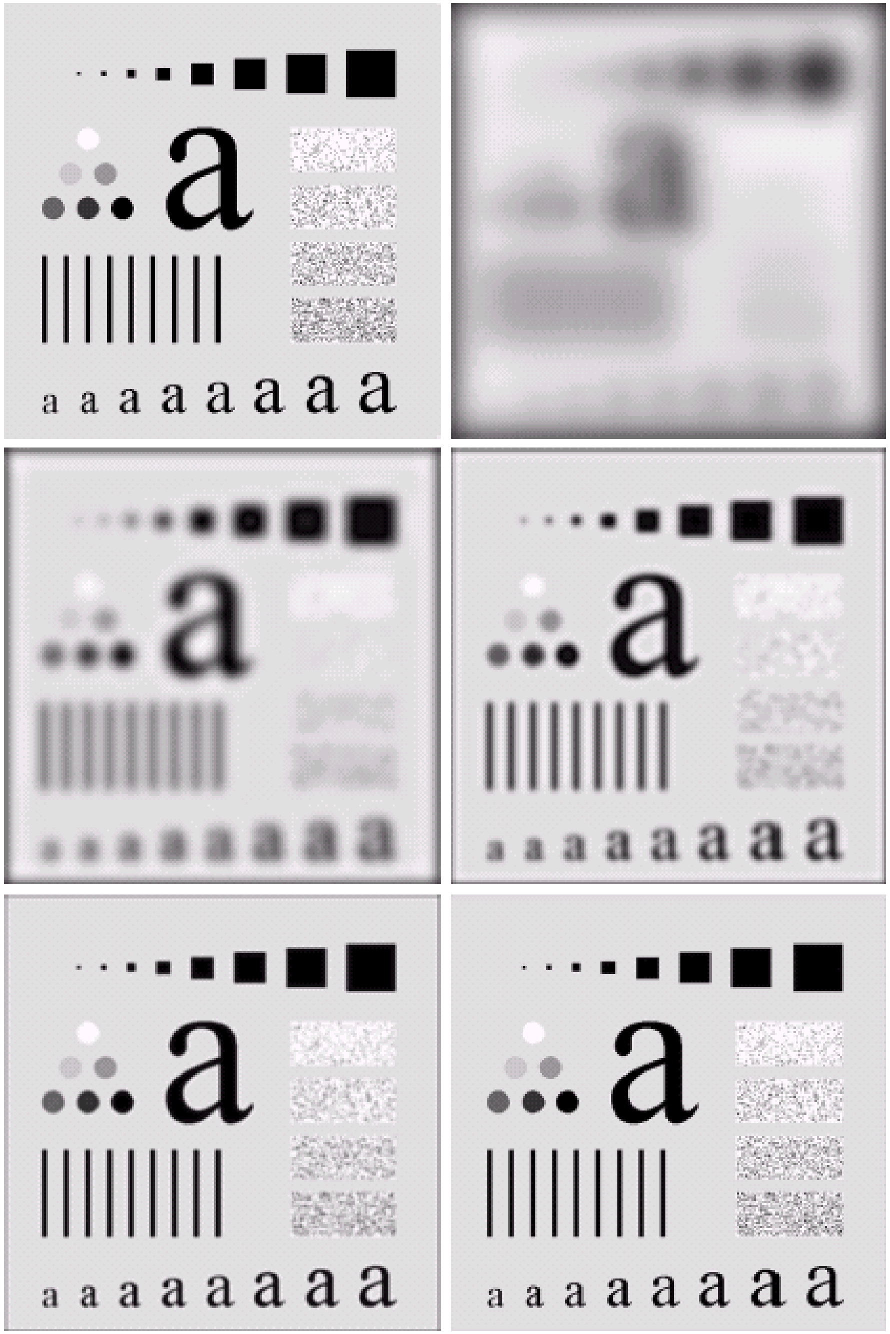
$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$

Butterworth Lowpass Filters



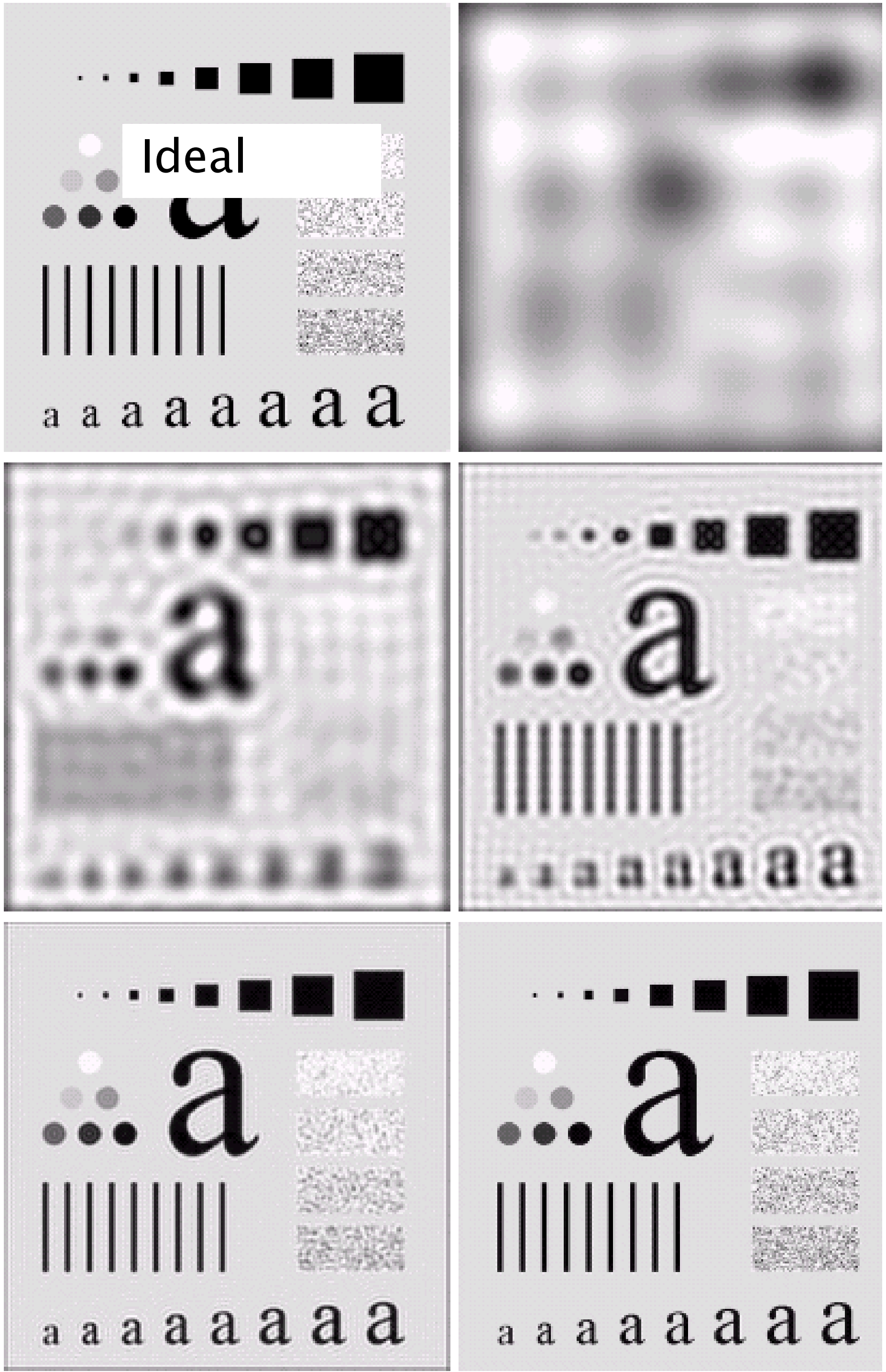
a b c

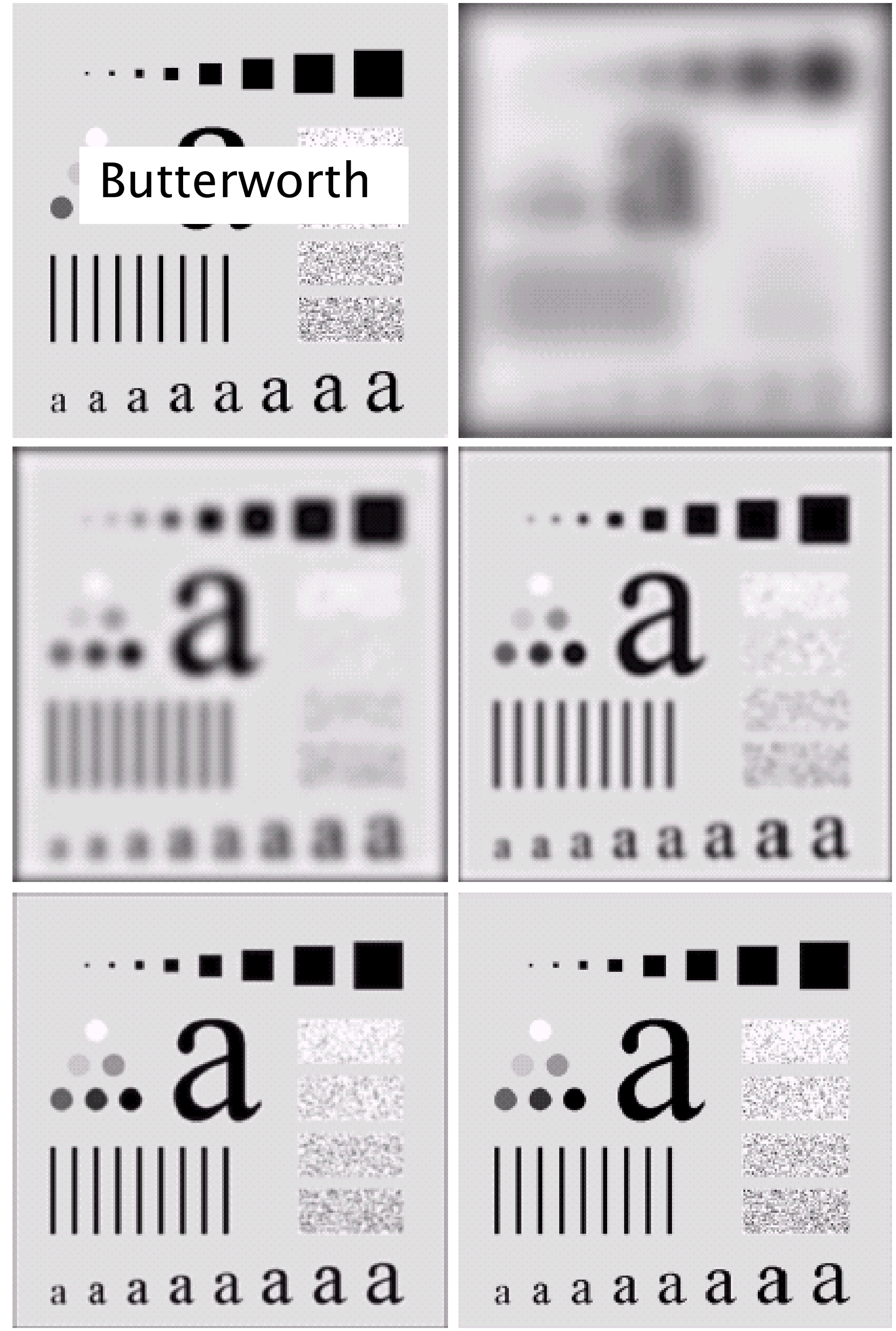
FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.





a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Gaussian Lowpass Filters

$$H(u,v) = e^{-D^2(u,v)/2D_0^{\,2}}$$

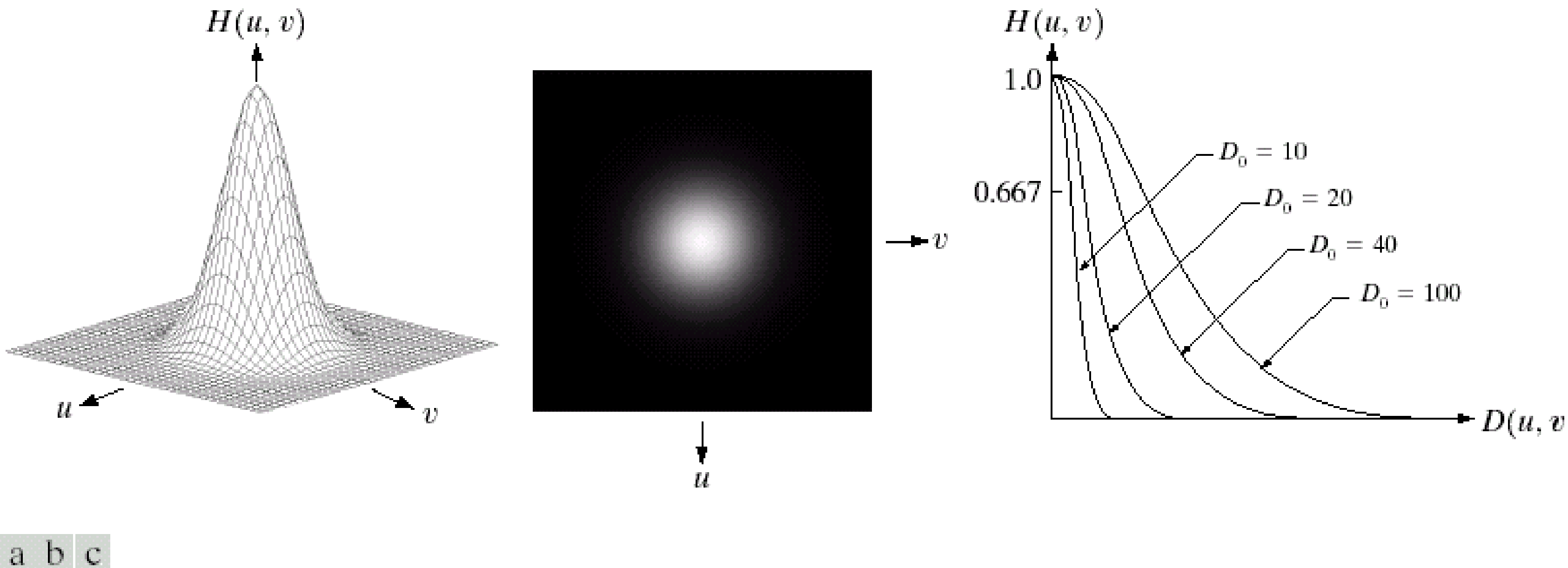


FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

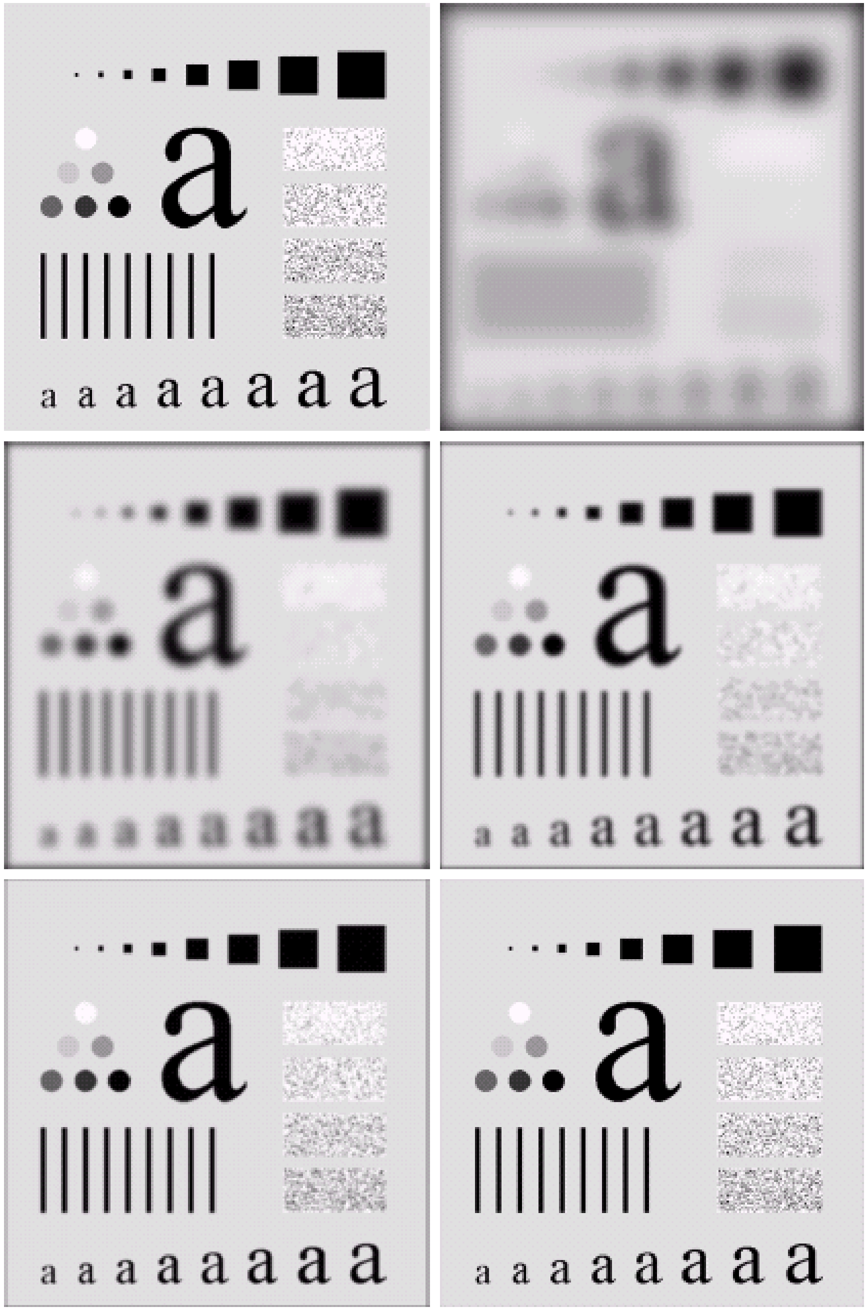


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a	b
c	d
e	f

Practical Applications of Lowpass Filtering

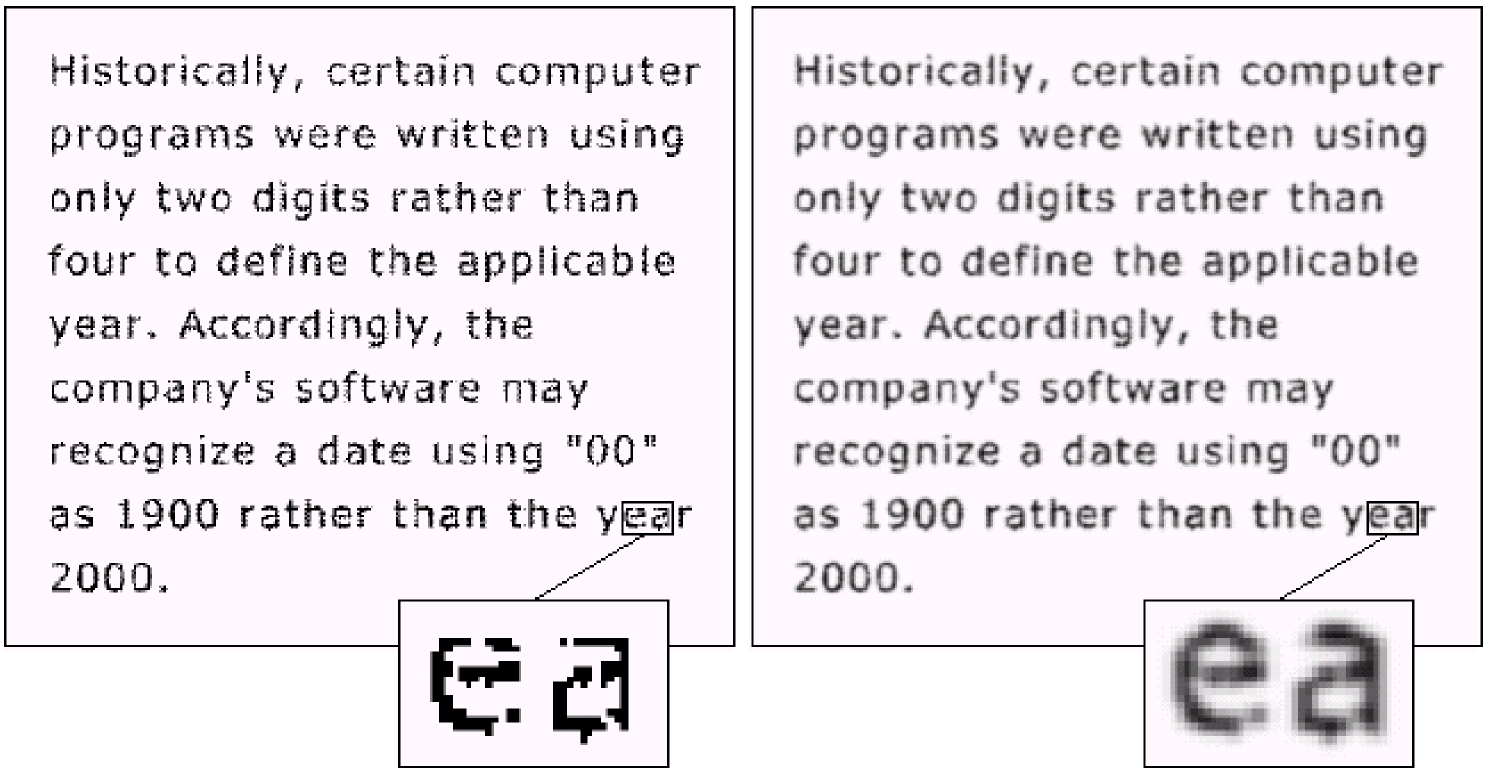
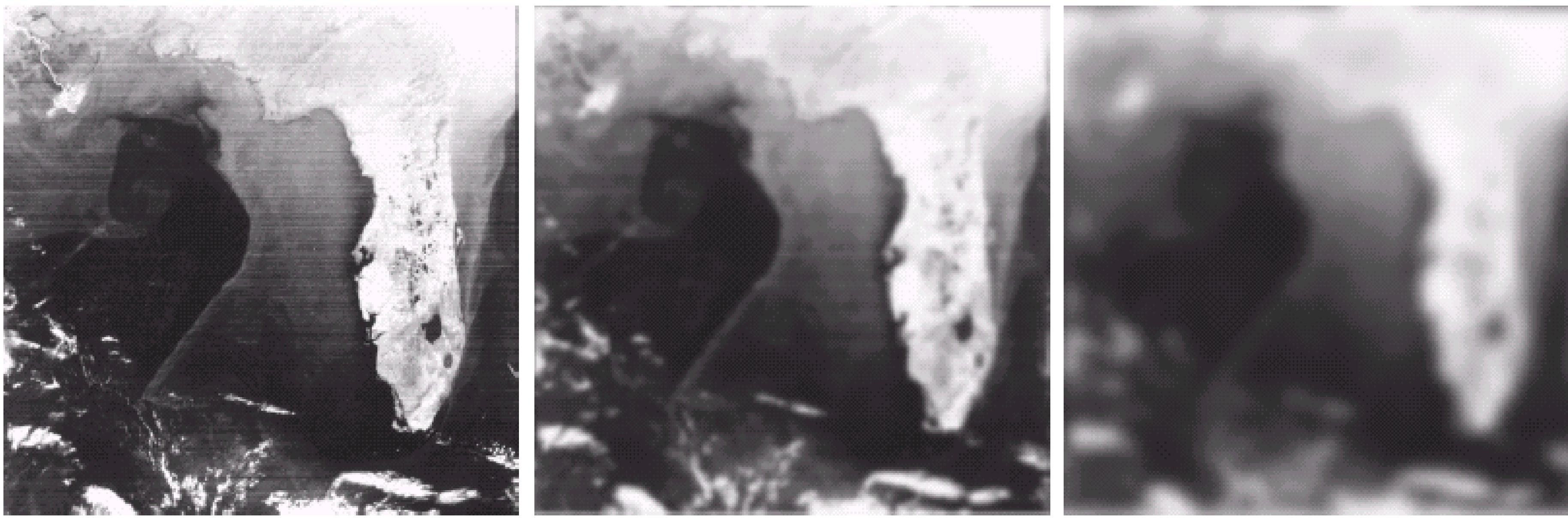


FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Practical Applications of Lowpass Filtering

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

Practical Applications of Lowpass Filtering



a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

Image Sharpening Using Frequency Domain Filters

- High pass filters
  - Only pass the high frequencies, drop the low ones

- A highpass filter is obtained from a given lowpass filter

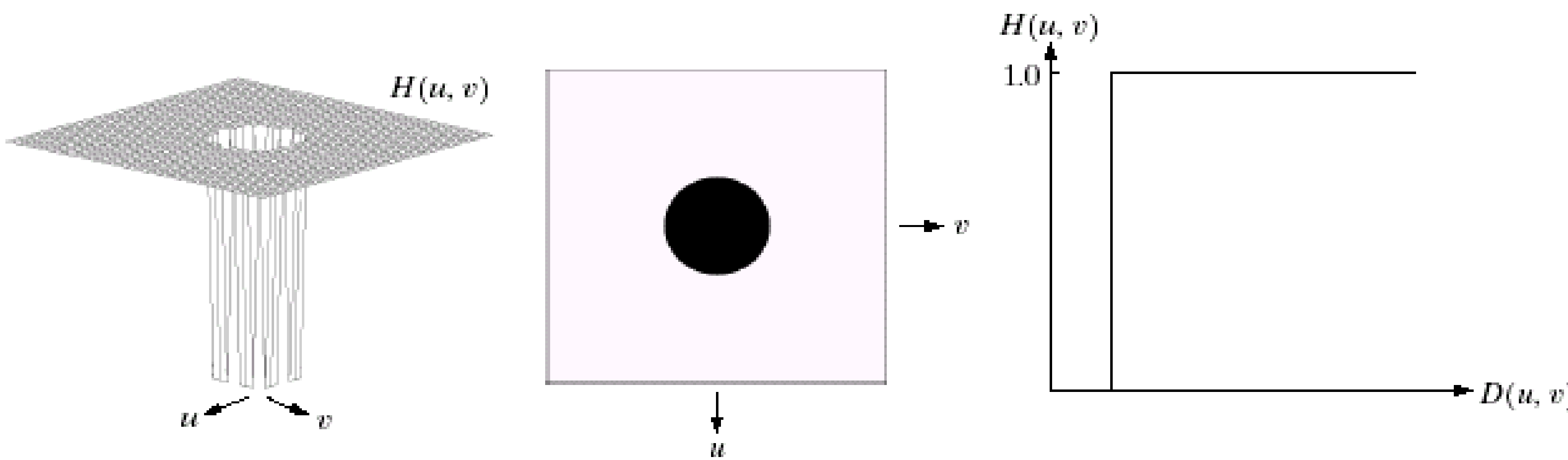
  - $H_{hp}(u, v) = 1 - H_{lp}(u, v)$

Ideam Highpass Filter

- A 2-D ideal highpass filter is defined as

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

where $D(u,v)$ = Distance from (u,v) to the ce nter of the mask.

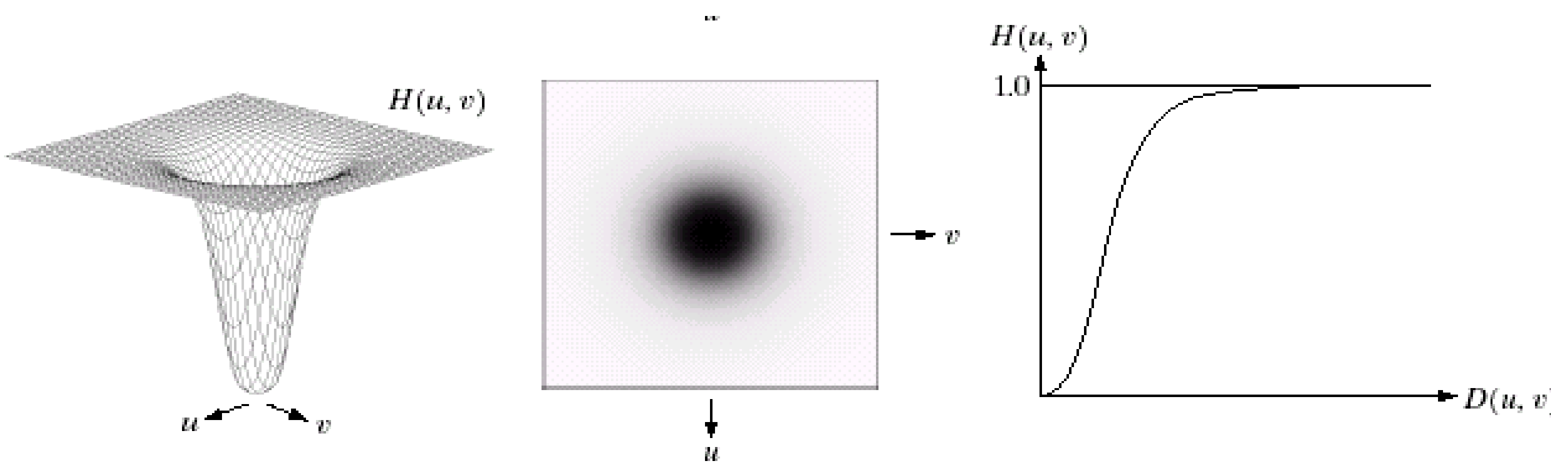


Butterworth High Pass Filters

The Butterworth high pass filter is given as:

$$H(u,v) = \frac{1}{1+[D_0 / D(u,v)]^{2n}}$$

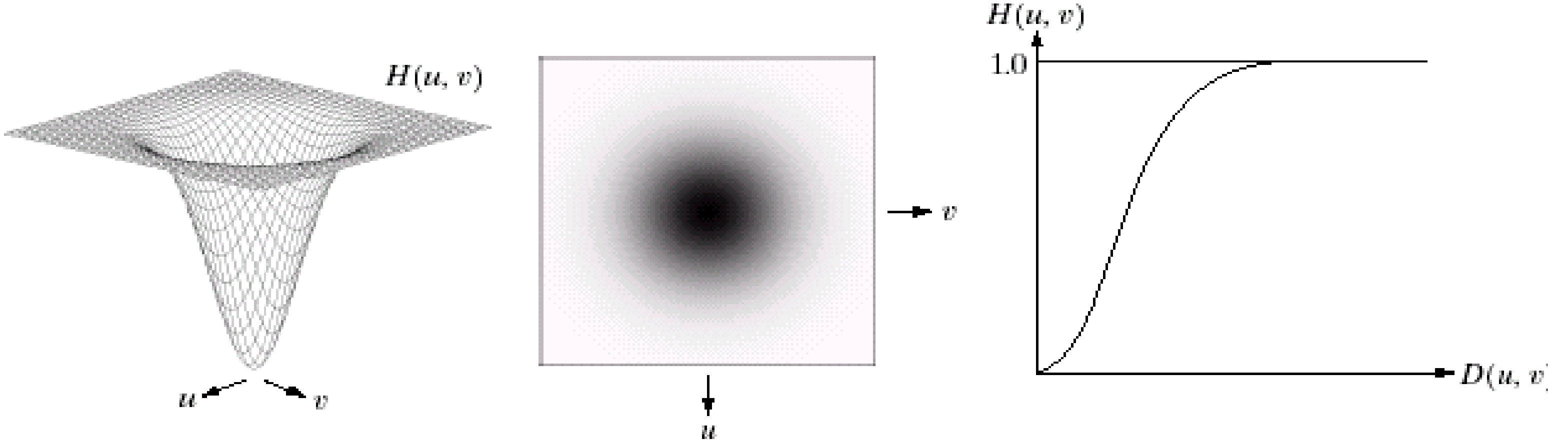
where n is the order

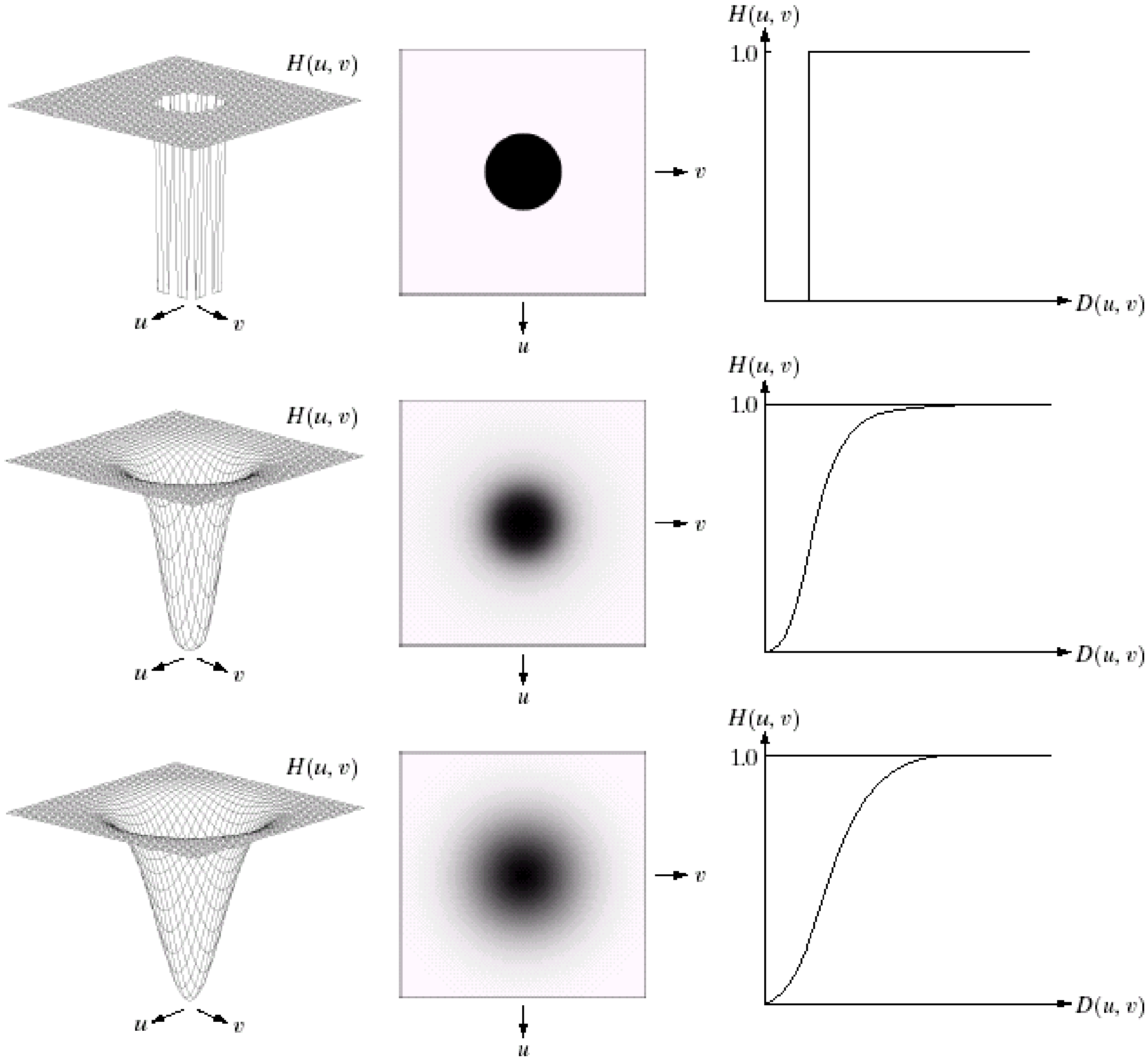


Gaussian High Pass Filters

- The Gaussian high pass filter is given as

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

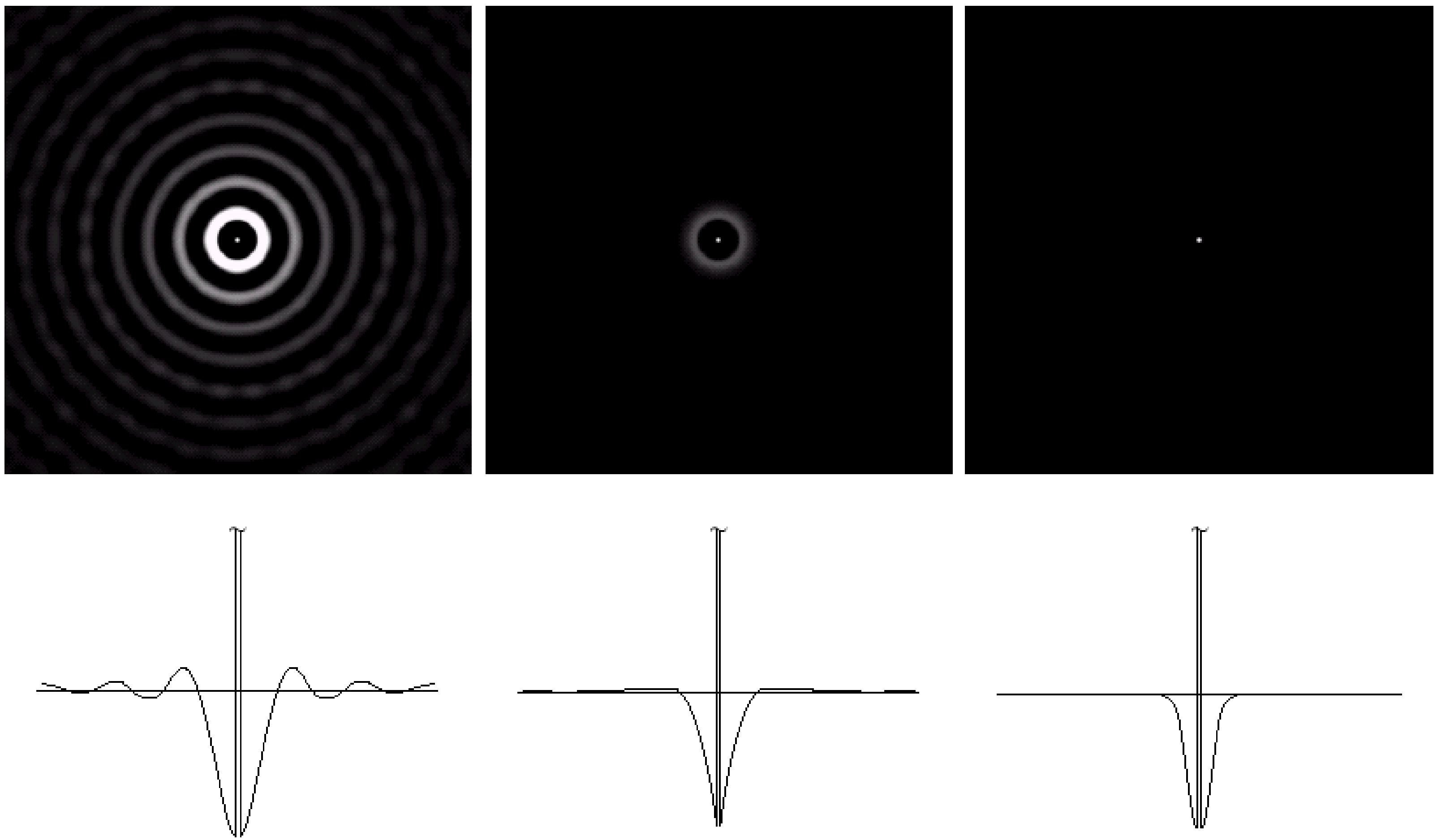




a	b	c
d	e	f
g	h	i

FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

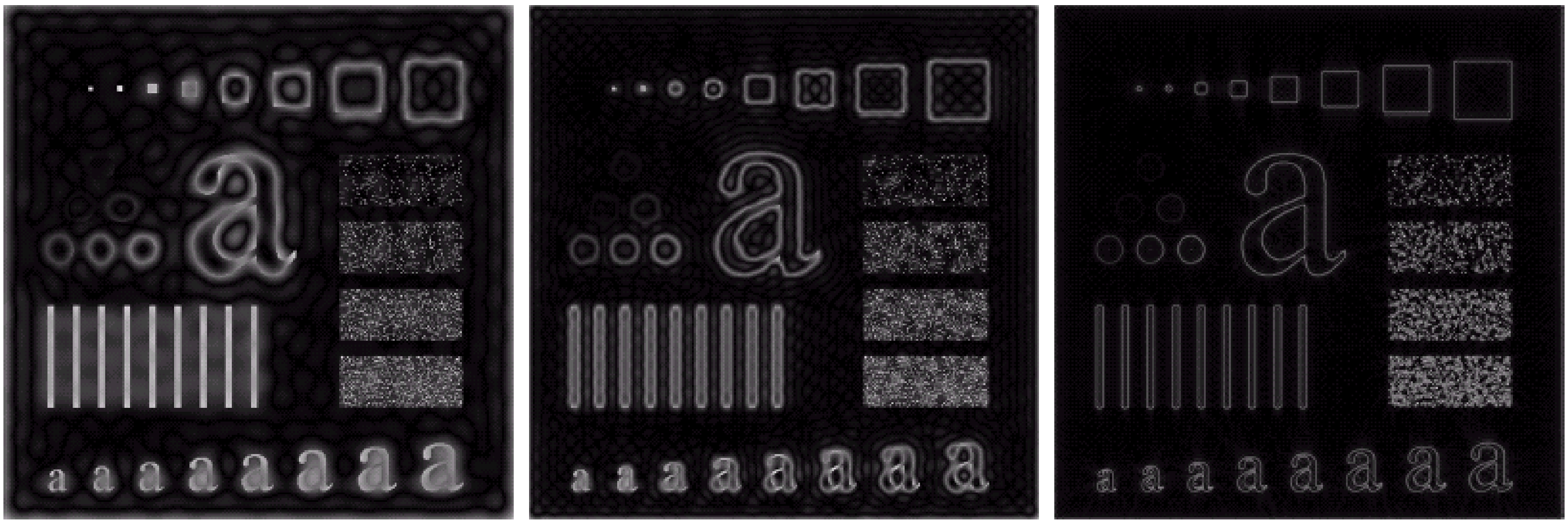
Spatial Representation



a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

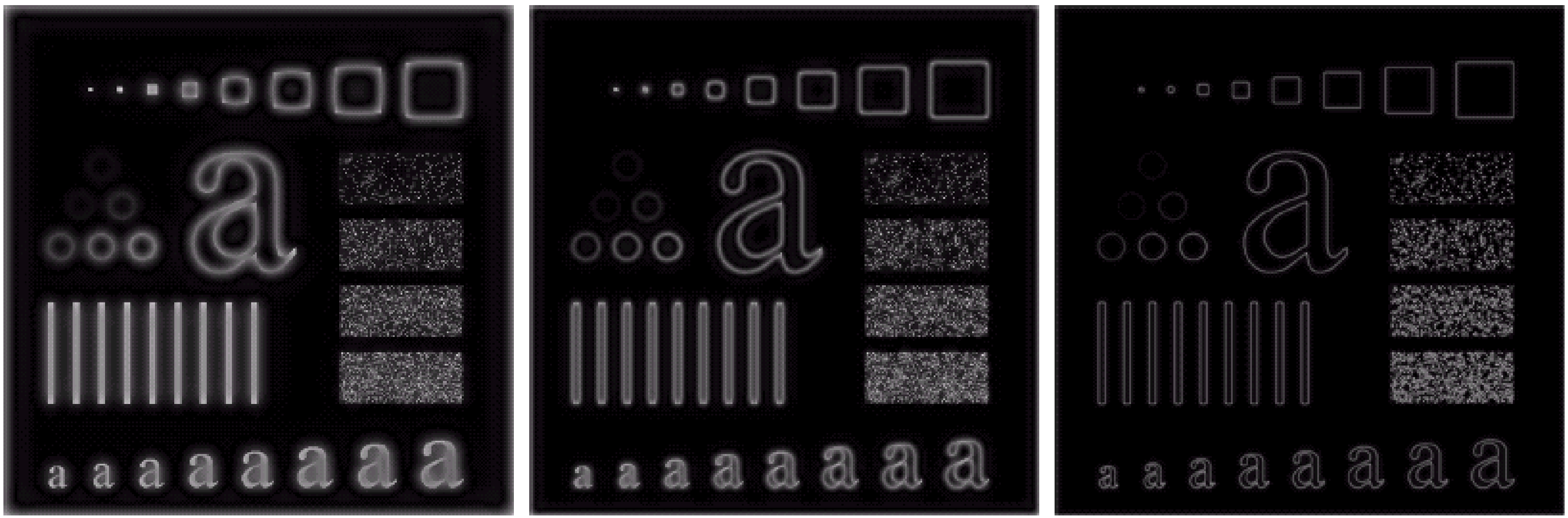
Result of Ideal Highpass Filtering



a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

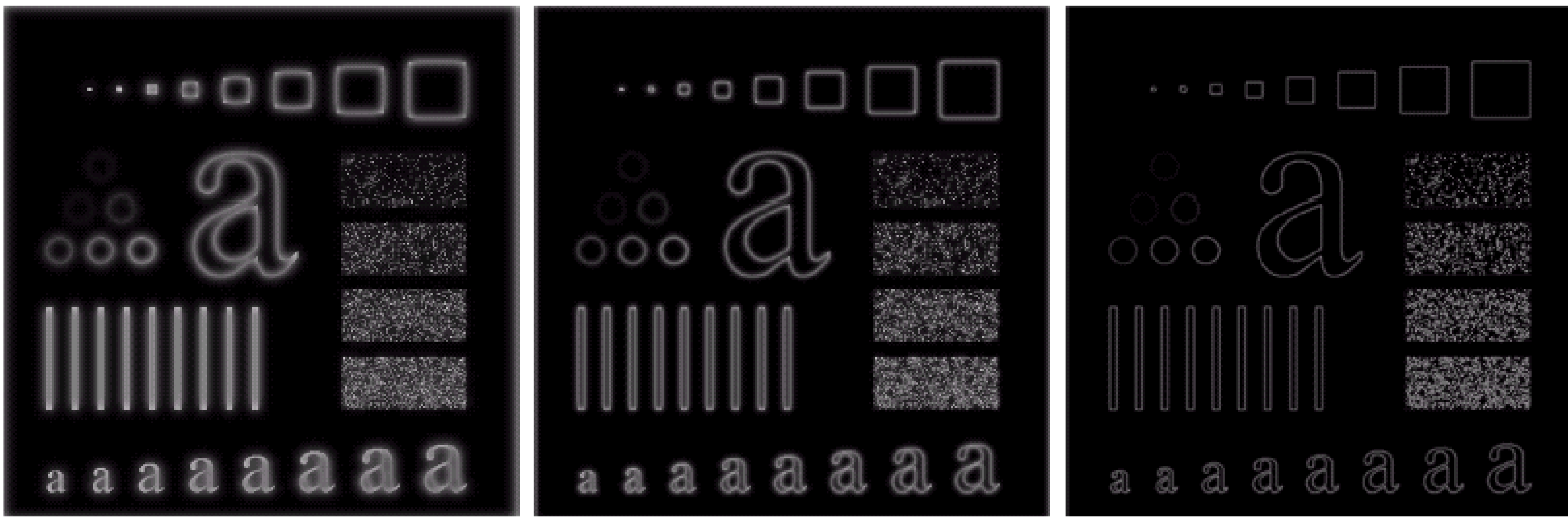
Result of Butterworth Highpass Filtering



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

Result of Gaussian Highpass Filtering



a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.