Small-World Minds Create the Illusion of Low-Dimensional Social Cognition Under Constrained Stimuli

Junsong Lu*[1] and Chujun Lin[1,2]

[1]Department of Psychology, University of California San Diego, La Jolla, U.S., ORCID: 0000-0001-6987-6228

[2]Department of Psychology, Columbia University, New York, U.S., ORCID: 0000-0002-7605-6508

1 Overview

This document provides supplementary information to accompany the main text and is organized into four sections. First, we examine the relationship between preferential attachment and the Chinese Restaurant Process (CRP). Second, we derive the probability that two nodes are connected at a given distance d. Third, we describe the procedures and parameters used in our simulation studies. Finally, we present two formal approaches for deriving the correlation between two nodes in a network.

The code for simulations is available at an online repository at GitHub repository.

2 The Connection Between a Growing Network and the CRP

The growing network is a Barabási–Albert model, which can be constructed using the two mechanisms mentioned in the main text:

- Growth: At each time step, a new node with m ($\leq m_0$) links that connect the new node to m existing nodes.
- Preferential Attachment: The probability Π that a link of the new node connects to node i depends on the degree of node i (k_i):

$$\Pi(i) = \frac{k_i}{\Sigma_j k_j}$$

**Corresponding author. E-mail: jul140@ucsd.edu

When $m = 1$, the growth of the network can be represented as

$$p = \begin{cases} \frac{k_i}{2t-1} & \text{if } 1 \leq i \leq t-1 \\ \frac{1}{2k-1} & \text{if } i = t \end{cases}$$

This pattern is mathematically similar to the CRP:

$$P(c_{nk}) = \begin{cases} \frac{n_i}{n-1+\alpha} & \text{if } k \leq K_n \\ \frac{\alpha}{n-1+\alpha} & \text{if } k = K_n + 1 \end{cases}$$

A graphical depict of the CRP is in Figure 1. The CRP is a stochastic process that describes how data points (customers) are assigned to clusters (tables) with a flexible number of groups [1]. Each new customer either joins an existing table with probability proportional to the number of people already seated there or starts a new table with probability proportional to the concentration parameter α. This process captures the intuition of rich-get-richer dynamics and is commonly used in Bayesian nonparametric models such as the Dirichlet Process. Note that there is a correspondence between the two formulas, as the denominator of the network formula can be reorganized as $2(k-1)+1$, while $2(k-1)$ corresponds to the $n-1$ in the CRP. Thus, the growing network model is equivalent to the CRP with a concentration parameter (α) of 1.

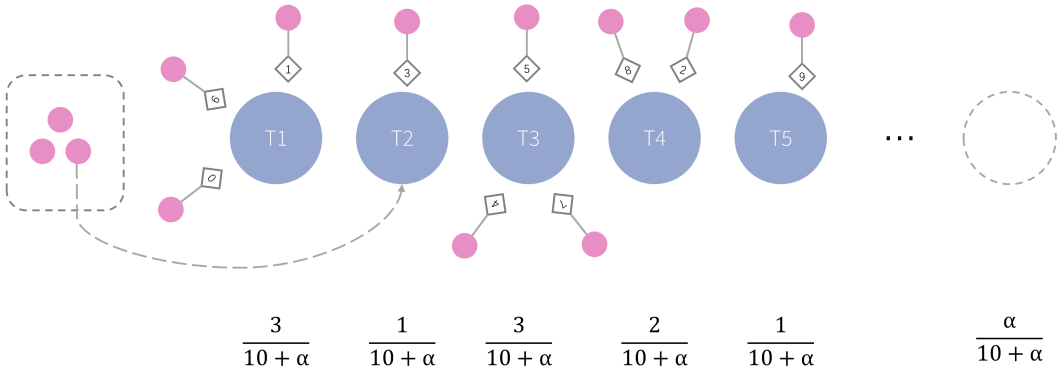

Figure 1: The generative process of the Chinese Restaurant Process, in which new customers (pink circles) are assigned to a table (light blue circles) in a probability proportional to the current guests in that table, while also with a small probability to occupy a new one.

3 Probability of Connections and Expected Number of Links Between Nodes

In this section, we derive the probability that two nodes are connected at a distance d. We also show that, in large networks, this probability approximates the expected number of links between the two

nodes.

3.1 Probability of Connections

Our derivation is based on the configuration model from complex network theory [2], which represents a generalized random network in which the degree of each node is specified in advance. In this model, each node is assigned a number of stubs (half-edges), resulting in a total of $\sum k_i = 2m$ stubs across the network. At each step, two stubs are selected uniformly at random and connected to form an edge. This process continues until all stubs are paired and the network is fully constructed.

Although this procedure may generate multiple edges between the same pair of nodes (i.e., multi-edges), such occurrences become negligible in large networks. Let k_i and k_j denote the degrees of two nodes. Under this model, we can derive the probability that at least one edge exists between them as follows:

$$p(edge \geq 1) = 1 - (1 - \frac{k_j}{N\langle k \rangle})^{k_i} \tag{1}$$

Based on the Taylor series, we have:

$$p(edge \geq 1) = 1 - (1 - k_i \frac{k_j}{N\langle k \rangle} + (\frac{k_i k_j}{N\langle k \rangle})^2 \frac{k_i(k_i - 1)}{2!} + \mathcal{O}(x^3)) \tag{2}$$

$$\approx \frac{k_i k_j}{2m} \tag{3}$$

where:

- N is the number of nodes,
- $\langle k \rangle$ is the average degree,
- $\langle k^2 \rangle$ is the second moment of the degree distribution.

Thus, in the limit of large m, the probability that an edge exists between two nodes converges to the expected number of edges between them.

Without loss of generality, we can extend this logic to derive the probability that two nodes are connected at a distance d in the network. Specifically, the probability that node i connects to an intermediate node l, which in turn connects to node j, can be expressed as:

$$p(edge_{il} \geq 1, edge_{lk} \geq 1) = \Sigma_l[1 - (1 - \frac{k_l}{N\langle k \rangle})^{k_i}][1 - (1 - \frac{k_j}{N\langle k \rangle - 1})^{k_l - 1}] \tag{4}$$

By expanding each expression using the Taylor series and discarding higher-order terms, we obtain:

$$p(\text{edge}_{il} \geq 1, \text{edge}_{lk} \geq 1) \approx \sum_l \left[1 - \left(1 - k_i \frac{k_l}{N\langle k \rangle} + \frac{1}{2}\left(k_i \frac{k_l}{N\langle k \rangle}\right)^2\right)\right] \tag{5}$$

$$\cdot \left[1 - \left(1 - (k_l - 1)\frac{k_j}{N\langle k \rangle} + \frac{1}{2}\left((k_l - 1)\frac{k_j}{N\langle k \rangle}\right)^2\right)\right] \tag{6}$$

$$= \sum_l \left[k_i \frac{k_l}{N\langle k \rangle} - \frac{1}{2}\left(k_i \frac{k_l}{N\langle k \rangle}\right)^2\right] \cdot \left[(k_l - 1)\frac{k_j}{N\langle k \rangle} - \frac{1}{2}\left((k_l - 1)\frac{k_j}{N\langle k \rangle}\right)^2\right] \tag{7}$$

$$= \sum_l \left[k_i (k_l - 1)\frac{k_l k_j}{(N\langle k \rangle)^2}\right] \tag{8}$$

$$= \frac{k_i k_j}{2m} \frac{\Sigma k_l (k_l - 1)}{2m - 1} \tag{9}$$

Based on this formulation, we can derive the probability that any two nodes are connected at a distance d in the network. Strictly speaking, by summing over the probabilities associated with all stubs at node i, we obtain the expected number of edges between nodes i and j, rather than the exact probability of an edge. However, in the limit of large m, this expected value becomes small for fixed degrees k_i and k_j. In this regime, the probability of a single edge and the expected number of edges converge, allowing us to treat them as approximately equal.

Next, we derive the expected number of links between two nodes at an arbitrary distance d.

3.2 Expected Number of Links

As a first step, we derive the expected number of links between two nodes with degrees k_i and k_j that are separated by a distance d. The simplest nontrivial case occurs when the two nodes are separated by two steps (i.e., distance $d = 2$).

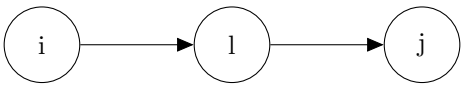

Figure 2: Two nodes at 2-step distance

This quantity is simply the mutiplication between the number of edges between i and l times that of between l and j:

$$\mathbb{E}[\#\text{edges}_{i\to j|d=2}] = \Sigma_l \frac{k_i k_l}{2m} \frac{(k_l-1)k_j}{2m-1} \tag{10}$$

$$= \frac{k_i k_j}{2m} \frac{\Sigma k_l(k_l-1)}{2m-1} \tag{11}$$

$$= p_{ij} \frac{\langle k^2\rangle - \langle k\rangle}{\langle k\rangle} \tag{12}$$

The derivation is similar to a 3-step pathway, in which node i passes through node l_1 and l_2 before reaching j.

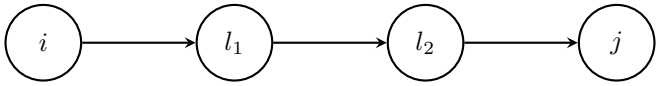

Figure 3: Two nodes at 3-step distance ($i \to l_1 \to l_2 \to j$)

The derivation is similar:

$$\mathbb{E}[\#\text{edges}_{i\to j|d=3}] = \Sigma_{l_2}\Sigma_{l_1} \frac{k_i k_{l_1}}{2m} \frac{(k_{l_1}-1)k_{l_2}}{2m} \frac{(k_{l_2}-1)k_j}{2m} \tag{13}$$

$$= \Sigma_{l_2}\Sigma_{l_1} \frac{k_i k_j}{2m} \frac{(k_{l_1}-1)k_{l_1}}{2m} \frac{(k_{l_2}-1)k_{l_2}}{2m} \tag{14}$$

$$= p_{ij}(\frac{\langle k^2\rangle - \langle k\rangle}{\langle k\rangle})^2 \tag{15}$$

Therefore, the formula is similar to that of a 2-step, so we can derive the general term:

$$\mathbb{E}[\#edges_{i\to j|d}] \approx \frac{k_i \cdot k_j}{N\langle k\rangle} \cdot \left(\frac{\langle k^2\rangle - \langle k\rangle}{\langle k\rangle}\right)^{d-1}, \tag{16}$$

This equation can be used to derive the expected number of links in Figure 3 in the main text.

4 Network Simulations

This section provides details underlying the network simulations presented in Figure 5 of the main text. Specifically, we describe how the network structure was generated, how activation propagates through the network, and how external inputs—such as experimental stimuli—interact with the network's structure.

4.1 Network Structure

We implemented a growing network model consisting of 100 nodes. The network was initialized with a single node. At each subsequent time step, a new node was added, bringing with it three links ($m = 3$), which were connected to existing nodes according to a specified attachment rule. The network was defined as undirected, and multiedges (i.e., multiple edges between the same pair of nodes) were not allowed.

Regarding the weighted links between nodes, one parameterization assumes an unweighted network. Alternatively, one can generate a weighted network. We adopted the second approach. We sampled weights from a uniform distribution between -0.2 and 0.4. The asymmetric distribution is inspired by our own empirical data [3]. The negative weights allow for inhibitory influence between nodes: when a node is activated, it can suppress the activation of negatively connected neighbors.

However, we note that negative links are not strictly necessary. Even if all weights in the network are positive, apparent negative relationships between concepts can emerge due to structural or contextual factors. For example, consider two nodes in the network, *Black* and *White*, representing two social categories. While these nodes might be positively connected in the network (i.e., activation of one spreads weakly to the other due to indirect or long-distance connections), their measured activation levels across a range of stimuli could still be negatively correlated.

To illustrate this, suppose that White faces are presented first as stimuli, strongly activating the *White* node (with values like 6, 6, 5), while the *Black* node receives only weak activation due to network distance (values like 3, 3, 2). Later, Black faces are presented, strongly activating the *Black* node (4, 4, 3), with the *White* node still weakly activated (2, 2, 1). Within each stimulus type (White or Black), the activations of the two nodes are positively correlated. However, across the full set of stimuli, the relationship becomes negative (r = -0.55), since the stimuli alternate between two conditions that preferentially activate one node over the other.

This example demonstrates that observed negative correlations between traits or concepts in behavioral data do not imply that the underlying network must contain negative weights. Instead, such correlations may emerge as a function of how activation patterns are elicited by specific sets of stimuli. We included negative weights in our simulations merely to explore a wider space of network dynamics, including both facilitatory and inhibitory influences.

4.2 Spreading Activations

Studies on spreading activation have assumed that the process is recursive [4, 5]. At each time step t, a node that has a nonzero activation would pass its activation to its neighbors. Although prior studies used different parameterizations (e.g., whether to include a reservoir parameter that controls how much activation is retained), the selection of parameterization should not influence our results

qualitatively.

We applied the parameterization of the dynamic interactive theory of person construal [5], as it is more related to social cognition. According to the theory, the net input to a node i at each time step is given by:

$$net_i = \sum_j w_{ij} o_j + ext_i + \varepsilon_\sigma \tag{17}$$

Here:

- w_{ij} is the connection weight from node j to node i,
- o_j is the output of node j at the previous time step,
- ext_i represents external input to node i (e.g., sensory input or task-relevant cues),
- ε_σ is Gaussian noise with standard deviation σ, capturing stochasticity in input integration.

The change in activation of node i, denoted Δa_i, depends on the sign of net_i, and follows a piecewise update rule:

- If $net_i > 0$:

$$\Delta a_i = I(M - a_i) \cdot net_i - D(a_i - r) \tag{18}$$

- If $net_i \leq 0$:

$$\Delta a_i = I(a_i - m) \cdot net_i - D(a_i - r) \tag{19}$$

In these equations:

- a_i is the current activation of node i,
- I is the integration rate (a scaling factor that controls how quickly a node responds to input),
- M and m are upper and lower bounds for activation, respectively,
- D is a decay rate parameter, capturing how quickly activations revert to a resting state,
- r is the resting activation level of the node.

Intuitively, when the net input to a node is positive, the node becomes more active, but this increase is limited by how close the node already is to the upper bound M. Conversely, if the net input is negative, the node's activation decreases, but this decrease is bounded by how close the activation is to the lower limit m. In both cases, the second term $D(a_i - r)$ pulls the activation back toward the resting state r, ensuring stability over time.

We simulated the spreading activation process over $n\_steps = 500$ time steps. The following parameter values were used:

- $I = 0.4$: the integration rate, determining the sensitivity of nodes to incoming activation.
- $D = 0.1$: the decay rate, controlling how quickly each node returns to its resting state.
- $r = 0$: the resting activation level.
- $M = 1$: the upper bound of activation.
- $m = 0$: the lower bound of activation.
- $\sigma = 0.05$: the standard deviation of the Gaussian noise term ε_σ, representing random fluctuations in input.

Each node was initialized with an activation level of zero, and activation was updated iteratively using the equations described above. At each time step, we computed the net input net_i for every node based on the current output of connected nodes and external input (if any), added Gaussian noise, and then updated the activation accordingly. This process simulates the temporal evolution of activation as it propagates through a network, reflecting the recursive nature of cognitive spreading processes.

In this implementation, the output o_j of each node was defined as a thresholded function of activation—e.g., $o_j = 1$ if $a_j > \theta$, and $o_j = 0$ otherwise, where θ is a small positive constant. This thresholding ensures that only sufficiently activated nodes can propagate activation to their neighbors, consistent with prior models of spreading activation in semantic or person-construal networks.

In sum, the simulation captures a continuous-time, stochastic, recursive spreading activation process, grounded in cognitive theory, and allows us to investigate how network structure and initial conditions shape the resulting dynamics.

4.3 Constrained Stimuli

Constrained stimuli can be operationalized as conditions in which only a small subset of nodes receives external input at the initial time step. In contrast, more diverse or naturalistic stimuli may activate a broader set of nodes, either simultaneously at the outset or progressively over time.

Formally, the pattern of external input can be represented by an $n \times T$ matrix $\mathbf{E}$, where n denotes the number of nodes in the network and T represents the number of simulation time steps. Each entry $E_{i,t}$ in this matrix indicates the magnitude of external input to node i at time t.

Under constrained stimulus conditions, the matrix $\mathbf{E}$ is sparse, with nonzero values restricted to one or a few rows (i.e., nodes) at the initial time step:

$$\mathbf{E}_{\text{constrained}} = \begin{bmatrix} e_1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & & \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

In contrast, under naturalistic stimulus conditions, external input may be distributed across many nodes either at the initial time step or over early steps:

$$\mathbf{E}_{\text{naturalistic}} = \begin{bmatrix} e_{1,1} & e_{1,2} & \cdots & 0 \\ e_{2,1} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_{n,1} & 0 & \cdots & 0 \end{bmatrix}$$

In our simulations, for simplicity and interpretability, we provided external input only at the first time step ($t = 1$), and left all subsequent time points with zero input. This setup allows us to isolate the effect of network structure and spreading dynamics from continued stimulus-driven activation. The external input at $t = 1$ thus serves as the initial perturbation from which activation propagates through the network.

4.4 Simulations

Based on the above specifications, we conducted two types of simulations. The first was a single-stimulus simulation (Figure 5a, 5c in the main text), in which a single stimulus (e.g., a face) was presented, and the corresponding nodes in the network were given an initial activation. The correlation between two behavioral measures was estimated by computing the correlation between node activations across a series of time steps (e.g., the first 50 steps). This simulation reflects the correlation between two perceptual judgments made by a single participant.

The second simulation more closely mirrors empirical studies of psychological dimensions (Figure 5b, 5d in the main text). We simulated 100 distinct stimuli, calculated the mean activation of each node across the first 50 time steps, and then computed the correlation of these mean activations across nodes. This approach aligns with standard practices in the field, where trait ratings (e.g., facial warmth) are aggregated across participants and stimuli. For simplicity, we assumed a shared network structure across participants.

5 Derivations of Correlation Between Activations

As a supplement to our simulations, we provided two approaches to analyze the correlation between the activations of two nodes.

5.1 Derivation Based on the Empirical Weight Distribution

This derivation builds upon our previous result for the expected number of links between nodes and proceeds in three steps. First, we derive the expected number of links between two nodes separated by a given distance. Second, we calculate the expected effect of these links. Third, we aggregate these effects to obtain the marginal effect across all distances.

5.1.1 Expected Number of Links

We have shown that the expected number of links between two nodes can be expressed as follows:

$$\mathbb{E}[N_{i\to j}^{(d)}] \approx \frac{k_i \cdot k_j}{N\langle k\rangle} \cdot \left(\frac{\langle k^2\rangle - \langle k\rangle}{\langle k\rangle}\right)^{d-1}, \tag{20}$$

5.1.2 The expected effect

We define the effect from node a to node i through all possible paths of length d as a weighted sum over those paths. If each edge contributes a correlation weight w, then a path of length d contributes w^d. Let $N_{a\to i}^{(d)}$ denote the expected number of such paths of length d from a to i in a configuration model.

The total expected effect from i to j is then given by:

$$\mathbb{E}[\text{Effect}_{i\to j} \mid w] = \sum_{d=1}^{n} w^d \cdot \mathbb{E}[N_{i\to j}^{(d)}]. \tag{21}$$

5.1.3 The marginal effect

In the final step, we can derive the marginal effect across all distances based on an assumed distribution of weights:

$$\mathbb{E}[\text{Effect}_{i\to j}] = \int \left(\sum_{d=1}^{n} w^d \cdot \mathbb{E}[N_{i\to j}^{(d)}]\right) p(w)\, dw \tag{22}$$

5.1.4 An example

This section provided a sample calculation. As shown above, the current method requires an assumption of the weight distribution. We tested two uniform distribution, $Uniform(-0.2, 0.4)$ and $Uniform(0, 0.5)$, and then tested . We also assumes k_i and k_j to be 10 and 15, respectively.

Recall that

$$\mathbb{E}[N_{i\to j}^{(d)}] = \frac{k_i\, k_j}{N\langle k\rangle}\left(\frac{\langle k^2\rangle - \langle k\rangle}{\langle k\rangle}\right)^{d-1} \equiv \frac{k_i\, k_j}{M}\, r^{\,d-1},$$

where $M = N\langle k\rangle$ and $r = (\langle k^2\rangle - \langle k\rangle)/\langle k\rangle$. If each path of length d contributes w^d, then

$$\mathbb{E}\big[\text{Effect}_{i\to j} \mid w\big] = \sum_{d=1}^{n} w^d\, \mathbb{E}\big[N_{i\to j}^{(d)}\big] = \sum_{d=1}^{n} w^d\, \frac{k_i\, k_j}{M}\, r^{\,d-1} = \frac{k_i\, k_j}{M} \sum_{d=1}^{n} w^d\, r^{\,d-1}. \tag{23}$$

Factor out one power of w:

$$\mathbb{E}\big[\text{Effect}_{i\to j} \mid w\big] = \frac{k_i\, k_j}{M}\; w \sum_{d=1}^{n} (w\, r)^{\,d-1} = \frac{k_i\, k_j}{M}\; w\; \frac{1-(w\, r)^n}{1-w\, r} \quad (\text{for } w\, r \neq 1). \tag{24}$$

To remove the conditioning on w, integrate over its density $p(w)$:

$$\mathbb{E}\big[\text{Effect}_{i\to j}\big] = \int_{-\infty}^{\infty} \mathbb{E}\big[\text{Effect}_{i\to j} \mid w\big]\; p(w)\, dw = \frac{k_i\, k_j}{M} \int w\, \frac{1-(w\, r)^n}{1-w\, r}\; p(w)\, dw. \tag{25}$$

Equivalently, exchanging sum and integral,

$$\mathbb{E}\big[\text{Effect}_{i\to j}\big] = \frac{k_i\, k_j}{M} \sum_{d=1}^{n} r^{\,d-1} \int w^d\, p(w)\, dw = \frac{k_i\, k_j}{M} \sum_{d=1}^{n} r^{\,d-1}\, \mathbb{E}[w^d]. \tag{26}$$

In particular, if $w \sim \text{Uniform}(a, b)$, then

$$\mathbb{E}[w^d] = \int_a^b w^d \cdot \frac{1}{b-a}\, dw \tag{27}$$

$$= \frac{1}{(d+1)(b-a)}\left(b^{d+1} - a^{d+1}\right) \tag{28}$$

and one may plug this into the sum above. Empirically, the largest distance represents the diameter of the network, which has an upper bound.

For a random network (the Erdős-Rényi network), the diameter can be calculated using

$$d \approx InN \tag{29}$$

For a scale-free network, the diameter d can be approximated using [6]

$$d \approx \frac{InN}{InInN} \tag{30}$$

Assuming a growing network of 100 nodes yields a network diameter of 3. Substituting these parameters along with a uniform weight distribution of $Uniform(-0.2, 0.4)$ produces a correlation of 0.283. When the weight distribution is adjusted to $Uniform(0, 0.5)$, the correlation increases substantially to 0.781.

5.2 Derivation Based on Unilateral Spreading Activation

While the previous derivation relies on knowing the weight distribution within the network, an alternative perspective is to analyze the spreading activation process directly. To examine the correlation between node activations, we begin with a simple bivariate unilateral relationship. Consider two nodes, A and B, where activation flows unilaterally from node A to node B:

$$b_1 = c + wa_0 + e \tag{31}$$

where c is the intercept and e is a random noise following a normal distribution $N(0, \sigma)$. The explanation of weight w in this formula is different from the above section (e.g., a path coefficient) and characterizes the proportion of activation that spreads to the downstream nodes. At time t_0, an external stimulus was provided and node A was activated with strength a_0. We can derive the correlation between the activation of A at $t0$ and of B at t_1:

$$\begin{aligned}
\rho_{AB} &= \frac{\mathrm{Cov}(A, B)}{\sigma_A \sigma_B} \\
&= \frac{\mathrm{Cov}(A, c + wA + e)}{\sigma_A \sigma_B} \\
&= \frac{wVar(A)}{\sqrt{Var(A)}\sqrt{w^2 Var(A) + Var(e)}} \\
&= \frac{w}{\sqrt{w^2 + \frac{Var(e)}{Var(A)}}}
\end{aligned} \tag{32}$$

Without loss of generality, we can extend this approach to derive the correlation between nodes separated by a distance d.

Let $X_0 = A \sim N(\mu_A, \sigma_A^2)$, and for $k = 1, 2, \ldots, d$,

$$X_k = \gamma_k + w\, X_{k-1} + \varepsilon_k, \quad \varepsilon_k \overset{\text{iid}}{\sim} N(0, \sigma_\varepsilon^2). \tag{33}$$

5.2.1 Expand X_d

The relationship between two nodes separated by distance d can be derived by expanding the linear equation recursively:

$$
\begin{aligned}
X_d &= \gamma_d + w\, X_{d-1} + \varepsilon_d && (34)\\
&= \gamma_d + w\big(\gamma_{d-1} + w\, X_{d-2} + \varepsilon_{d-1}\big) + \varepsilon_d && (35)\\
&\;\;\vdots && (36)\\
&= \underbrace{\gamma_d + w\gamma_{d-1} + \cdots + w^{d-1}\gamma_1}_{C_d} + w^d\, X_0 + \sum_{k=1}^{d} w^{\,d-k}\, \varepsilon_k && (37)\\
&= w^d A + \sum_{k=1}^{d} w^{\,d-k}\, \varepsilon_k. && (38)
\end{aligned}
$$

5.2.2 Compute $Cov(A, X_d)$

To calculate the correlation, we first derived their covariance:

$$
Cov\big(A, X_d\big) = Cov\Big(A,\ w^d A + \sum_{k=1}^{d} w^{\,d-k}\varepsilon_k\Big) = w^d\, Var(A) = w^d\, \sigma_A^2. \tag{39}
$$

5.2.3 Compute $Var(X_d)$

We next derived the variance of X_d:

$$
\begin{aligned}
Var(X_d) &= Var\Big(w^d A + \sum_{k=1}^{d} w^{\,d-k}\, \varepsilon_k\Big) && (40)\\
&= w^{2d}\, \sigma_A^2 + \sum_{k=1}^{d} w^{2(d-k)}\, \sigma_\varepsilon^2 && (41)\\
&= w^{2d}\, \sigma_A^2 + \sigma_\varepsilon^2 \sum_{j=0}^{d-1} w^{2j} = w^{2d}\, \sigma_A^2 + \sigma_\varepsilon^2\, \frac{1 - w^{2d}}{1 - w^2}, \quad (w^2 \neq 1). && (42)
\end{aligned}
$$

Special case $w^2 = 1$: $\sum_{j=0}^{d-1} w^{2j} = d$, so $Var(X_d) = \sigma_A^2 + d\, \sigma_\varepsilon^2$.

5.2.4 Compute $\mathrm{Corr}(A, X_d)$

Based on the covariance and variance, the correlation can be derived:

$$\mathrm{Corr}(A, X_d) = \frac{Cov(A, X_d)}{\sqrt{Var(A)\, Var(X_d)}} \tag{43}$$

$$= \frac{w^d\, \sigma_A^2}{\sigma_A \sqrt{w^{2d}\sigma_A^2 + \sigma_\varepsilon^2 \dfrac{1 - w^{2d}}{1 - w^2}}} \tag{44}$$

$$= \frac{w^d\, \sigma_A}{\sqrt{w^{2d}\sigma_A^2 + \sigma_\varepsilon^2 \dfrac{1 - w^{2d}}{1 - w^2}}}. \tag{45}$$

$$\text{If } w^2 = 1, \quad \mathrm{Corr}(A, X_d) = \frac{\sigma_A}{\sqrt{\sigma_A^2 + d\, \sigma_\varepsilon^2}}.$$

5.2.5 An example

To illustrate the influence of path length on the correlation between the source node A and the downstream signal X_d, we provide a numerical example. We assumed a weight parameter $w = 0.5$, source standard deviation $\sigma_A = 1$, and noise standard deviation $\sigma_\varepsilon = 0.25$. When the path length is $d = 2$, the resulting correlation is $\mathrm{Corr}(A, X_2) = 0.67$. However, increasing the path length to $d = 3$ reduces the correlation to $\mathrm{Corr}(A, X_3) = 0.4$. This demonstrates that, even with moderate weights, increasing the number of transmission steps attenuates the association between the original input and the downstream output.

5.3 Implications for a recurrent system

The above analysis is based on the assumption of unilateral influence. However, spreading activation can be recursive. For example, an initial perception of attractiveness may influence judgments of intelligence, and once an impression of intelligence is formed, it may, in turn, bias the perception of attractiveness. This kind of mutual reinforcement can be incorporated into the analysis by abstracting all intermediary pathways into a single total weight w, allowing the system to be modeled as a simple linear recurrence.

We consider a simple linear recurrent system with two nodes A and B, where the activation vector at time t is

$$\mathbf{x}_t = \begin{pmatrix} a_t \\ b_t \end{pmatrix} \tag{46}$$

and the recurrence relation is given by

$$\mathbf{x}_t = W\mathbf{x}_{t-1} + \mathbf{e}_t, \quad \text{with} \quad W = \begin{pmatrix} 0 & w \\ w & 0 \end{pmatrix} \tag{47}$$

and $\mathbf{e}_t = (e_t^A, e_t^B)^T$, where $e_t^A, e_t^B \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2)$, and independent across time and variables.

To derive the correlation between the activations of two nodes, we must compute both their variances at equilibrium and their lagged covariance. Importantly, the relevant measure is the lagged correlation, rather than the stationary correlation, as our focus is on the directional influence of an initially activated node, such as attractiveness, on subsequent activations of other nodes, such as intelligence. Since activation propagates with a delay, the effect of the initial node manifests in neighboring nodes after one time step.

5.3.1 Stationary Distribution and Variance

Assuming $|w| < 1$, the system converges to a stationary distribution. The stationary covariance matrix Σ satisfies the discrete Lyapunov equation:

$$\Sigma = W\Sigma W^T + \sigma^2 I \tag{48}$$

Since $WW^T = w^2 I$, we have

$$\Sigma = w^2\Sigma + \sigma^2 I \tag{49}$$

$$\Rightarrow \quad (1 - w^2)\Sigma = \sigma^2 I \tag{50}$$

$$\Rightarrow \quad \Sigma = \frac{\sigma^2}{1 - w^2} I. \tag{51}$$

Therefore, in the stationary state:

$$\text{Var}(a_t) = \text{Var}(b_t) = \frac{\sigma^2}{1 - w^2}, \quad \text{Cov}(a_t, b_t) = 0. \tag{52}$$

5.3.2 Lagged Correlation: $\rho(a_t, b_{t+1})$

From the recurrence relation:

$$b_{t+1} = wa_t + e^B_{t+1}, \tag{53}$$

taking covariance with a_t gives:

$$\mathrm{Cov}(a_t, b_{t+1}) = \mathrm{Cov}(a_t, wa_t + e^B_{t+1}) = w\,\mathrm{Var}(a_t), \tag{54}$$

since a_t and e^B_{t+1} are independent. Therefore,

$$\mathrm{Cov}(a_t, b_{t+1}) = w \cdot \frac{\sigma^2}{1-w^2}. \tag{55}$$

Using the standard deviations:

$$\mathrm{Std}(a_t) = \mathrm{Std}(b_{t+1}) = \sqrt{\frac{\sigma^2}{1-w^2}}, \tag{56}$$

we get the lag-1 correlation:

$$\rho(a_t, b_{t+1}) = \frac{\mathrm{Cov}(a_t, b_{t+1})}{\mathrm{Std}(a_t)\ \mathrm{Std}(b_{t+1})} \tag{57}$$

$$= \frac{w \cdot \frac{\sigma^2}{1-w^2}}{\frac{\sigma^2}{1-w^2}} = w. \tag{58}$$

References

[1] David J Aldous. Exchangeability and related topics. In *École d'Été de Probabilités de Saint-Flour XIII—1983*, pages 1–198. Springer, 2006.

[2] Mark Newman. *Networks*. Oxford university press, 2018.

[3] Junsong Lu and Chujun Lin. Network models reveal high-dimensional social inferences in naturalistic settings beyond latent construct models. *Communications Psychology*, 3(1), July 2025.

[4] Allan M. Collins and Elizabeth F. Loftus. A spreading-activation theory of semantic processing. *Psychological review*, 82(6):407, 1975.

[5] Jonathan B. Freeman and Nalini Ambady. A dynamic interactive theory of person construal. *Psychological Review*, 118(2):247–279, 2011.

[6] Reuven Cohen and Shlomo Havlin. Scale-free networks are ultrasmall. *Physical review letters*, 90(5):058701, 2003.