Integrated Navigation of Interferometric Radar Altimeter-aided Terrain Referenced Navigation and Inertial Navigation

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Junwoo Park *[1], Youngjoo Kim [2], and Hyochoong Bang [3]

Abstract—This paper proposes a method of utilizing measurement of interferometric radar altimeter (IRA) into integration of inertial navigation and terrain referenced navigation (TRN). While conventional TRN is supported by radar altimeter that measures vertical channel terrain clearance and estimates horizontal position, proposed method can estimate 3 dimensional navigational states of an unmanned air vehicles (UAV) by utilizing inertial navigation system (INS) in accordance with IRA-aided TRN. Rather than combining the output of TRN with INS in a separated form, integrated navigation that fuses the two in a tightly coupled manner is suggested. To deal with highly nonlinear characteristics of measurement of IRA, particle filter is utilized. Particularly, Rao-Blackwellized particle filter (RBPF) that benefits from state space domain system structure is applicable. This paper aims at step-wise linearization procedure of IRA's measurement so that the system can take advantage of linear substructure. Numerical simulation was conducted under certain condition, and results show that proposed method can estimate UAV's 3-dimension navigational state accurately even when inertial measurement units (IMU) and IRA are the only supporting sensors.

Keywords—terrain referenced navigation, interferometric radar altimeter, integrated navigation, inertial navigation system, rao-blackwellized particle filter.

I. Introduction

Although use of inertial navigation system (INS) is a primary beginning of navigation, stand-alone use of INS is vulnerable to inertial measurement units' (IMU) biases, random noises, misalignment or cross-coupling errors, and eventually the solution diverges in a long term use. Therefore, it is common to use INS in accordance with other supportive sources like global navigation satellite system (GNSS) to achieve robust and reliable navigation solution of (unmanned) air vehicles. However, GNSS, mostly represented as GPS, is vulnerable to external signal interference and to intended jamming or spoofing of enemy especially in military point of view. Namely it is suffered from signal reception problem in a various condition. Terrain referenced navigation (TRN) is suggested as one of good reliable alternatives of GNSS in terms of INS supporting source. Principle of TRN is to estimate position of an aircraft by comparing terrain information with onboard terrain database that mostly exist as a form of digital elevation map (DEM). Since it is supported by terrain sensor, e.g. radar altimeter, that is equipped along with aircraft itself, it can be used as independent navigation solution without being disturbed by (intended) external jamming. One of key factors that determine performance of TRN is terrain roughness and uniqueness. In other words, TRN can have more accurate positioning solution when UAV is flying over more uneven terrain. In this context, however, conventional radar altimeter that measures vertical channel terrain clearance is not adequate when utilized under a fast varying condition or highly maneuvering condition as it cannot capture the terrain features. Interferometric radar altimeter (IRA) is suggested as one good alternative of radar altimeter in that it can precisely measure the range and side looking angle to the closest point among those that lie within zero-Doppler line. Here zero-Doppler line is a intersecting curve of a plane perpendicular to aircraft velocity vector and terrain surface. It is known that range measurement of IRA to the closest terrain point is more accurate than terrain clearance measurement of radar altimeter under various conditions. Moreover, it is known that side looking angle is accurately achievable through multi receiving antennas and interference effect. Geometric depiction of IRA's measurement is given in Figure 1, while details of underlying signal processing of IRA is omitted here in this paper.

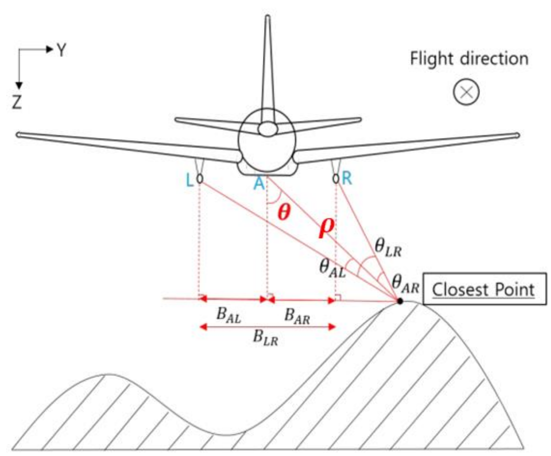

Figure 1 Rearview of IRA measurement. B stands for length of baseline(s) between antennas,

Apart from terrain sensor, fusion algorithm is also important in TRN and its relevant issues. Typically, TRN suffers from highly nonlinear characteristics in measurement as terrain cannot be represented in an explicit function, and as it exists as

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database in a look-up table form. Therefore, linear/Gaussian filter such as Kalman filter or extended Kalman filter (EKF) is not suitable for TRN problems. Many of the literatures [1], [2] report that use of EKF diverges frequently. Nonlinear Bayesian filter such as point mass filter (PMF) or particle filter (PF) must instead be applied to consider nonlinear characteristics of terrain measurement model. Here in this paper PF is considered as primary approach since PMF cannot be practically applied to problems with more than 3 states, and integrated navigation problem consists of at least 8 or 9 number of states including position, velocity, and attitude of an aircraft. In addition to that, Rao-Blackwellization technique is introduced which utilizes linear substructure of state space model so that optimal estimator like EKF is applicable for some portion of state space. Step-wise linearization procedure of IRA's measurement with respect to velocity and attitude of an aircraft is derived followed by result of simulation result.

II. Rao-Blackwellized Particle Filter

A. Particle filter practicality

Although particle filter is known to be able to solve any nonlinear tracking problem when equipped with tons of particles and that convergence has nothing to do with state dimension, practical application of particle filter is limited. Problems with small process noise can cause the particles cluster within small region and this make the navigation filter diverge loosing particle diversity, and alternative approaches are required to solve such problems. Linearizing both process and measurement model with respect to some portion of state that possesses small process noise enables us to utilize deterministic estimator which is EKF.

B. System representation and filtering procedure

Rao-Blackwellized particle filter assumes the following state space domain system representation.

$$x_{t+1}^n = f_t^n(x_t^n) + F_t^n(x_t^n) \cdot x_t^l + \omega_t^n \quad (1)$$

$$x_{t+1}^l = f_t^l(x_t^n) + F_t^l(x_t^n) \cdot x_t^l + \omega_t^l \quad (2)$$

$$y_t = h_t(x_t^n) + H_t(x_t^n) \cdot x_t^l + e_t \quad (3)$$

here x^n denotes nonlinear portion of state, x^l denotes linear part, f and h denotes nonlinear function, F and H are matrix of proper dimension for linear relationship, ω and e denotes white Gaussian noise. (1) and (2) are system process model and (3) is system measurement model. From above system representation, one can notice that x^l relates the both process and measurement model in a linear manner and x^n in a nonlinear way. Rao-Blackwellized particle filter benefits from this linear substructure. Key of Rao-Blackwellized particle filter is to fully utilize following Bayesian inference to the portion of state whose analytic and optimal estimation are available.

$$p(x_k|y_{0:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|y_{0:k-1})dx_{k-1} \quad (3)$$

$$p(x_k|y_{0:k}) = \frac{p(y_k|x_k)p(x_k|y_{0:k-1})}{\int p(y_k|x_k)p(x_k|y_{0:k-1})dx_k} \quad (4)$$

Following represents the prediction and correction procedure of Rao-Blackwellized particle filter based on (1)~(3).

$$x_{t|t-1}^n \sim \mathcal{N}\left(f_{t-1}^n(x_{t-1|t-1}^n) + F_{t-1}^n \hat{x}_{t-1|t-1}^l, F_{t-1}^n P_{t-1|t-1} {F_{t-1}^n}^T + Q_{t-1}^n\right) \quad (5)$$

$$\hat{x}_{t|t-1}^l = f_{t-1}^l(x_{t|t-1}^n) + F_{t-1}^l \hat{x}^l_{t-1|t-1} + L_{t-1}\left(z_{t-1} - F_{t-1}^n \hat{x}^l_{t-1|t-1}\right) \quad (6)$$

$$P_{t|t-1} = F_{t-1}^l P_{t-1|t-1} {F_{t-1}^l}^T + Q_{t-1}^l - L_{t-1} N_{t-1} L_{t-1}^T \quad (7)$$

$$x_{t|t-1}^l \sim \mathcal{N}\left(\hat{x}_{t|t-1}^l, P_{t|t-1}\right) \quad (8)$$

$$y_t \sim \mathcal{N}\left(h_t(x_{t|t-1}^n) + H_t \hat{x}_{t|t-1}^l, H_t P_{t|t-1} H_t^T + R_t\right) \quad (9)$$

$$\hat{x}_{t|t}^l = \hat{x}_{t|t-1}^l + K_t\left((y_t - h_t) - H_t \hat{x}_{t|t-1}^l\right) \quad (10)$$

$$P_{t|t} = P_{t|t-1} - K_t M_t K_t^T \quad (11)$$

$$x_{t|t}^l \sim \mathcal{N}\left(\hat{x}_{t|t}^l, P_{t|t}\right) \quad (12)$$

where Q, R are covariance matrix with no off-diagonal terms. x^n is predicted through (5) and corrected by (4) and (9). x^l is propagated through (6)~(8) and corrected by (10)~(12). Especially, K, M in (10) and (11) follow the same notation for EKF case. N, L and z of (6)~(8) are represented as follows.

$$N_{t-1} = F_{t-1}^n P_{t-1|t-1} {F_{t-1}^n}^T + Q_{t-1}^n \quad (13)$$

$$L_{t-1} = F_{t-1}^l P_{t-1|t-1} {F_{t-1}^n}^T N_{t-1}^{-1} \quad (14)$$

$$z_t = x_t^n - f_{t-1}^n \quad (15)$$

One can consult with [3], [4] for detailed derivation of above equations. What's good about Rao-Blackwellized particle filter is that it can estimate the state at given time more accurately than stand-alone particle filter since more particles are occupied by small dimension of state space and the rest are estimated through EKF which is an optimal estimator for linear case.

III. Integrated Navigation

A. System mode

Integrated navigation problem can be represented as following state space model in a linearized form.

$$\dot{x}(t) = Ax(t) + Bw(t) \tag{16}$$

where w is system noise x, A, and B denotes following

$$x = [\underbrace{\delta L \quad \delta\lambda \quad \delta h}_{\text{Position}} \quad \underbrace{\delta v_n \quad \delta v_e \quad \delta v_d}_{\text{Velocity}} \quad \underbrace{\epsilon_n \quad \epsilon_e \quad \epsilon_d}_{\text{Attitude}} \quad \underbrace{\delta f_x^b \quad \delta f_y^b \quad \delta f_z^b \quad \delta\omega_x^b \quad \delta\omega_y^b \quad \delta\omega_z^b}_{\text{Accel/Gyro Bias}}]^T$$
(17)

$$A(t) = \begin{bmatrix} A_{rr} & A_{rv} & 0 & 0 & 0 \\ A_{vr} & A_{vv} & A_{v\varepsilon} & -C_b^n & 0 \\ A_{\varepsilon r} & A_{\varepsilon v} & A_{\varepsilon\varepsilon} & 0 & C_b^n \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$$A_{rr} = \begin{bmatrix} 0 & 0 & -\frac{v_n}{(r_L+h)^2} \\ \frac{v_e \sin L}{(r_\lambda+h)\cos^2 L} & 0 & -\frac{v_e}{(r_L+h)^2 \cos L} \\ 0 & 0 & 0 \end{bmatrix} \quad A_{rv} = \begin{bmatrix} \frac{1}{r_L+h} & 0 & 0 \\ 0 & \frac{1}{(r_\lambda+h)\cos L} & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{19}$$

$$A_{vr} = \begin{bmatrix} -2v_e\omega_{ie}\cos L - \frac{v_e^2}{(r_\lambda+h)\cos^2 L} & 0 & -\frac{v_n v_d}{(r_L+h)^2} + \frac{v_e^2 \tan^2 L}{(r_\lambda+h)^2} \\ 2\omega_{ie}(v_n \cos L - v_d \sin L) + \frac{v_e v_n}{(r_\lambda+h)\cos^2 L} & 0 & -\frac{v_e v_d}{(r_\lambda+h)^2} - \frac{v_n v_e \tan^2 L}{(r_\lambda+h)^2} \\ 2v_e\omega_{ie}\sin L & 0 & \frac{v_e^2}{(r_\lambda+h)^2} + \frac{v_n^2}{(r_L+h)^2} \end{bmatrix}$$

$$A_{vv} = \begin{bmatrix} \frac{v_d}{r_L+h} & -2\omega_{ie}\sin L - \frac{2v_e \tan L}{r_\lambda+h} & \frac{v_n}{r_L+h} \\ 2\omega_{ie}\sin L + \frac{v_e \tan L}{r_\lambda+h} & \frac{v_d + v_n \tan L}{r_\lambda+h} & 2\omega_{ie}\cos L + \frac{v_e}{r_\lambda+h} \\ -\frac{2v_n}{r_L+h} & -2\omega_{ie}\cos L - \frac{2v_e}{r_\lambda+h} & 0 \end{bmatrix} \tag{20}$$

$$A_{v\varepsilon} = \begin{bmatrix} 0 & -f_z^n & f_y^n \\ f_z^n & 0 & -f_x^n \\ -f_y^n & f_x^n & 0 \end{bmatrix} \quad A_{\varepsilon r} = \begin{bmatrix} -\omega_{ie}\sin L & 0 & -\frac{v_e}{(r_\lambda+h)^2} \\ 0 & 0 & \frac{v_n}{(r_L+h)^2} \\ -\omega_{ie}\cos L - \frac{v_e}{(r_\lambda+h)\cos^2 L} & 0 & \frac{v_e \tan L}{(r_\lambda+h)^2} \end{bmatrix}$$

$$A_{\varepsilon v} = \begin{bmatrix} 0 & \frac{1}{r_\lambda+h} & 0 \\ -\frac{1}{r_L+h} & 0 & 0 \\ 0 & \frac{\tan L}{r_\lambda+h} & 0 \end{bmatrix} \quad A_{\varepsilon\varepsilon} = \begin{bmatrix} 0 & \omega_{in,d}^n & -\omega_{in,e}^n \\ -\omega_{in,d}^n & 0 & \omega_{in,n}^n \\ \omega_{in,e}^n & -\omega_{in,n}^n & 0 \end{bmatrix} \tag{21}$$

$$B = \begin{bmatrix} 0 & 0 \\ C_b^n & 0 \\ 0 & -C_b^n \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{22}$$

They are linearized INS error kinematic equations and one can consult with [5] for the details of above equations.

B. *Rao-Blackwellization*

In order to fully benefit from linear substructure of Rao-Blackwellized particle filter, equation (3) for IRA's measurement must be derived. This paper proposes following measurement model.

$$y_t = \begin{bmatrix} \min_{(L,\lambda)\in Z.D.L} \left\| (L,\lambda,h_{DTED}(L,\lambda))_{ECEF} - (L_t,\lambda_t,h_t)_{ECEF} \right\|_2 + \begin{bmatrix} \frac{\partial\rho}{\partial v_n} & \frac{\partial\rho}{\partial v_e} & \frac{\partial\rho}{\partial v_d} \end{bmatrix}_{(L^*,\lambda^*)} \begin{bmatrix} dv_n \\ dv_e \\ dv_d \end{bmatrix} \\ \tan^{-1}\left(\frac{y_b}{\sqrt{x_b^2+z_b^2}}\right) + \begin{bmatrix} \frac{\partial\theta}{\partial\epsilon_n} & \frac{\partial\theta}{\partial\epsilon_e} & \frac{\partial\theta}{\partial\epsilon_d} \end{bmatrix}_{(L^*,\lambda^*)} \begin{bmatrix} \epsilon_n \\ \epsilon_e \\ \epsilon_d \end{bmatrix} \end{bmatrix} + \begin{bmatrix} e_{\rho,t} \\ e_{\theta,t} \end{bmatrix} \tag{23}$$

where upper right part is IRA range perturbation induced by velocity perturbation, and lower right part is IRA side looking angle perturbation induced by attitude peryurbation. Each of them can be calculated at each given state as follows.

$$\begin{bmatrix} \frac{\partial\rho}{\partial v_n} & \frac{\partial\rho}{\partial v_e} & \frac{\partial\rho}{\partial v_d} \end{bmatrix}_{(L^*,\lambda^*)} = \begin{bmatrix} \frac{\partial\rho_{DB}}{\partial\theta_{zdl}} & \frac{\partial\rho_{DB}}{\partial\psi_{zdl}} \end{bmatrix}_{(L^*,\lambda^*)} \begin{bmatrix} 0 & 0 & \frac{-\sqrt{v_n^2+v_e^2}}{v_n^2+v_e^2+v_d^2} \\ -\frac{1}{v_e} & \frac{1}{v_n} & 0 \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} \frac{\partial\theta}{\partial\epsilon_n} & \frac{\partial\theta}{\partial\epsilon_e} & \frac{\partial\theta}{\partial\epsilon_d} \end{bmatrix}_{(L^*,\lambda^*)} = \begin{bmatrix} \frac{z_b}{\sqrt{x_b^2+z_b^2}} & 0 & -\frac{x_b}{\sqrt{x_b^2+z_b^2}} \end{bmatrix} \tag{25}$$

For (24), a step-wise linearization procedure is proposed. Second term of (24)'s right hand side represents how much zero-Doppler line is shifted from given velocity perturbation when zero-Doppler line is parametrized with two angles; azimuth, and elevation with respect to west-east line. First term of (24)'s right hand side represents how much the IRA's range measurement would change when zero-Doppler line is shifted assuming that the new closest terrain point will be present in the vicinity of true closest errain point. It represents range gradient along the line where zero-Doppler is shifted. Figure 2 represents such procedure. (25) is linearized form of side looking angle with respect to an aircraft's attitude. Side looking angle can be represented as follow,

$$\theta = \tan^{-1}\left(\frac{y_b}{\sqrt{x_b^2+z_b^2}}\right) \tag{26}$$

where x_b, y_b, z_b denotes coordinates of closest terrain point resolved in aircraft body frame.

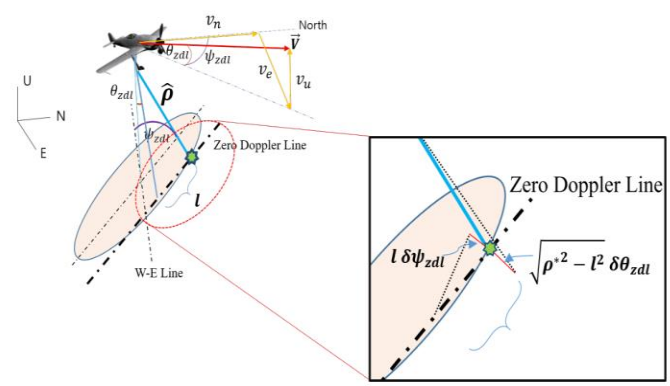

Figure 2 Closest terrain point shift due to velocity perturbation

IV. Simulation Results

A. *Simulation condition*

Simulation conditions are listed in Table 1

B. *Numerical Result*

Under condition presented in Table 1, Figure 3 represent the navigation performance of suggested method. One can find that all of the states that include 3 dimensional position, velocity and attitude angles are accurately estimated through the entire simulation.

Table 1 Simulation condition

Items		Value
Initial Position	(L_0, λ_0, h_0)	(35.7488°, 128.1381°, 2500m)
Initial Velocity	$(v_{n,0}, v_{e,0}, v_{d,0})$	(228m/s, 75m/s, 0m/s), 240m/s in norm
Initial Attitude	$(\phi_0, \theta_0, \psi_0)$	(0°, 0°, 18.1575°)
Flight Condition		Level flight
DEM Resolution		3"(=90m)
Initial Distribution	$p(x_0^n)$	$\mathcal{N}(\frac{50}{R_0}, \frac{50^2}{R_0}) \cdot \mathcal{N}(-\frac{50}{R_0\cos(L_0)}, \frac{50}{R_0\cos(L_0)}^2) \cdot \mathcal{N}(20, 20^2)$
Initial Distribution	$p(x_0^l)$	$\mathcal{N}(\mathbf{0}, diag\left(1/\sqrt{2}, 1/\sqrt{2}, 0.5, 0.1\pi/180, 0.1\pi/180, 0.1\pi/180, 10^{-4}\rvert_{1\times3}, 10^{-5}\rvert_{1\times3}\right)^2)$
Process Noise	$p(u_t^n)$	$\mathcal{N}(\mathbf{0}, diag\left(0.5/R_0, 0.5/R_0\cos(L_0), 0.5\right)^2)$
Process Noise	$p(u_t^l)$	$\mathcal{N}(\mathbf{0}, diag\left(10^{-4}\rvert_{1\times3}, 10^{-6}\rvert_{1\times3}, 10^{-7}\rvert_{1\times6}\right)^2)$
INS Sampling Rate		50Hz
TRN Sampling Rate		1Hz
Measurement Noise	$(\sigma_\rho, \sigma_\theta)$	(3.5m, 0.1°)
# of Particles in RBPF		1000

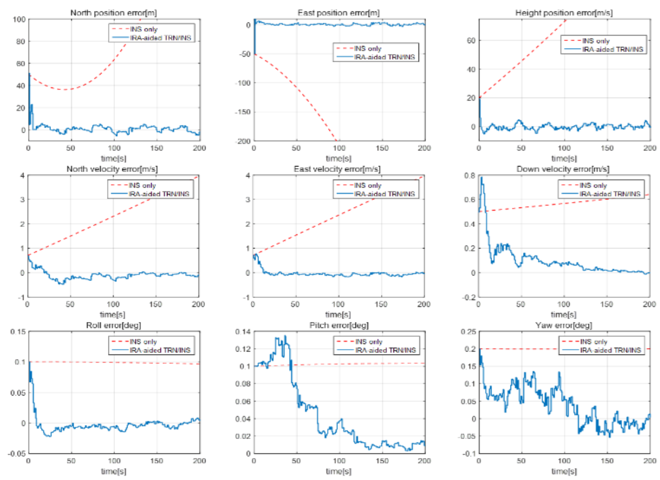

Figure 3 Navigation error of proposed method, position (first row), velocity (second row), and attitude (third row)

V. Concluding Remarks

It was possible to apply Rao-Blackwellized particle filter in integrated navigation problem, particularly that utilizes interferometric radar altimeter. Linearization of measurement of interferometric radar altimeter was proposed. Numerical result shows that one can construct navigation system for entire 3 dimensional solutions that is supported only by interferometric radar altimeter and inertial measurement units.

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