A New Measurement Model of Interferometric Radar Altimeter for Terrain Referenced Navigation using Particle Filter

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Abstract—**This paper proposes a novel method of grafting one of the measurements of interferometric radar altimeter (IRA) onto particle filter that uses sequential importance resampling (SIR) to enhance the performance of terrain referenced navigation (TRN). IRA which is a terrain sensor that measures slant range, slant angle and look angle to the closest terrain point from the aircraft is introduced. The procedure of numerically replicating the measurements of IRA when the position, velocity, and attitude of aircraft, and the terrain database is given is provided herein this paper. Although the degree of accuracy of IRA measurements is higher than radar altimeter (RA), performance of TRN that utilizes IRA stays in a similar level with RA-aided TRN or becomes even worse because the way of utilizing IRA measurements remains unchanged. Proposed method estimates the slant range from each of the particles to their closest terrain point to update the position of an aircraft. Numerical simulations that use actual terrain database is presented to observe the performance of proposed method in comparison with typical TRN that utilizes RA and another that makes use of IRA measurements in a conventional manner. Performance of TRN with proposed method gets improved in terms of root mean square error of aircraft's 2D position and initial position error correction.**

Keywords—Interferometric radar altimeter; Terrain referenced navigation; Particle filter;

I. INTRODUCTION

Use of inertial navigation system (INS) in cooperation with global navigation satellite system (GNSS) is one of the most typical methods that have been utilized to achieve navigation solution of an aircraft as it works as a mutual complementation to integrate the strong points of each. Even though the INS data is regarded as principal beginning of achieving aircraft's navigation solution, crucial defects of INS like drifting over time are still unresolved. Accordingly, use of an INS is aided by other sources to supplement its drawbacks. As GNSS can keep the error of vehicle's position confined within certain boundary, GNSS is used as standard supporting method of INS. GNSS, however, is vulnerable to the intended attack and jamming since the structure of its signal is widely known. Terrain referenced navigation (TRN) has been suggested as an effective alternative of GNSS as TRN can operate with no support of external system, yet keeping the navigation solution of INS from drifting over time. The principle of TRN is to gather information of terrain through terrain sensor(s) placed on the aircraft and to estimate the state, mostly position, of an aircraft by comparing the measurements from the terrain sensor(s) with terrain elevation database called digital elevation model (DEM).

To deal with nonlinearity of terrain elevation database, there have been many approaches of utilizing extended Kalman filter (EKF), or bank of Kalman filters (BKF) [6] in TRN. EKF is regarded as standard method for many nonlinear estimation problems, however, such linearized filters often fail when they face the intractable challenges of TRN like multimodality. Therefore approaches of using grid-based methods like point mass filter (PMF) and particle filter (PF) which is nonparametric Bayesian approach can result to a better performance of TRN in many cases [7-9] by accommodating not only nonlinear characteristics but also multimodal probability distribution of TRN. Apart from the fusion algorithms of TRN, terrain sensor itself is also a critical part that affects the performance of current positioning problem. Typical TRN utilizes radar altimeter (RA) as its tool of measuring terrain elevation. RA gives user the distance to the terrain point right beneath the vehicle. Ground clearance measured from RA, however, is not suitable to reflect the roughness of terrain when the aircraft is flying over inclined terrain or when the flight altitude is relatively high since RA provides distance by taking average of measured values within certain swath. Interferometric radar altimeter (IRA) therefore has been suggested as an alternative terrain sensor to the RA.

IRA can measure the range to the closest terrain point from the aircraft which is called slant range. In addition to the slant range, IRA measures two more angles to the closest point. First one, called slant angle, is measured in alongtrack direction relative to the perpendicular plane of aircraft's velocity vector. Another one, called look angle, is measured in crosstrack direction relative to the aircraft's x-z plane. It is known that IRA is more accurate in terms of noise included within measurements as it is based on differential(s) of two or more signals.

Reference [3], however, utilizes a conventional measurement model that RA-aided TRN adopted and inserted IRA measurements within existing framework. Thereby the positioning performance of IRA-aided TRN remained as it was. Since IRA differs from RA in terms of measurements and has different error characteristics with RA, a distinct way of grafting measurements onto filter algorithm should be taken into account.

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Therefore, this paper proposes a new measurement model of IRA and focuses on verifying an improvement of new measurement model that independently utilizes the measurements of IRA, especially slant range. Throughout the numerical simulations, comparison between the proposed measurement model and existing approach that makes use of IRA's measurement in a way that RA-aided TRN did has been conducted, and results will be discussed at the end of this paper.

II. INTERFEROMETRIC RADAR ALTIMETER

A. Modeling of interferometric radar altimeter (IRA)

In this subsection, IRA will briefly be introduced as descripted in [1]. Details of processing signals will be omitted here, yet providing geometry of IRA measurements. Unlike the conventional radar altimeter (RA), IRA measures the slant range ρ, slant angle γ, and look angle θ to the nearest terrain point. IRA consists of one antenna for transmitting and receiving signal and one or more antennas for receiving signal as it utilizes the phase difference of two or more returned signals. Geometric depiction of IRA measurements is shown in Fig. 1, where the distance between antennas is called baseline B. Toward both crosstrack and alongtrack direction, IRA system receives signals reflected from the terrain with wide aperture. However, if an IRA keeps measuring the closest terrain point taking the entire wide aperture as its pool of search, terrain points detected by IRA will be limited within few cusps. Therefore, IRA adopts Doppler bandwidth filter to put restriction on searching swath along the alongtrack direction as Doppler bandwidth filter can filter out the signals with Doppler shift included. Thereby the searching swath of IRA is limited and subdivided by isodops, as shown in Fig.2, and current searching swath is the region denoted by zero Doppler domains. IRA would find the earliest returned signal among the signals reflected from the terrain and passed the Doppler bandwidth filter. Slant range ρ can be inversely calculated from the detected signal and will be elaborated by undergoing two phase approach. Slant angle γ denotes an alongtrack angle between aircraft's velocity vector and relative position vector to the closest terrain point subtracted from π/2. It is measured in alongtrack direction relative to the perpendicular plane of aircraft's velocity vector. One can refer to (1) and (2) to calculate the slant angle.

$$\gamma = \sin^{-1}(\frac{\lambda}{2V} f_{DC}) \tag{1}$$

$$f_{DC} = \sin^{-1}(\frac{V}{\lambda H} f_{DC}) \tag{2}$$

where λ, V, H, and f_{DC} denotes the wavelength of the signal, speed of an aircraft, altitude of an aircraft, and Doppler center frequency.

Look angle θ can be measured by making use of interference and phase difference between two or more signals. Since the aircraft is staying in relatively distant position, it is assumed that look angles measured on both antennas is equivalent. Thereby differential(s) between two or more signals' path length can be calculated from phase difference of the signals which can be

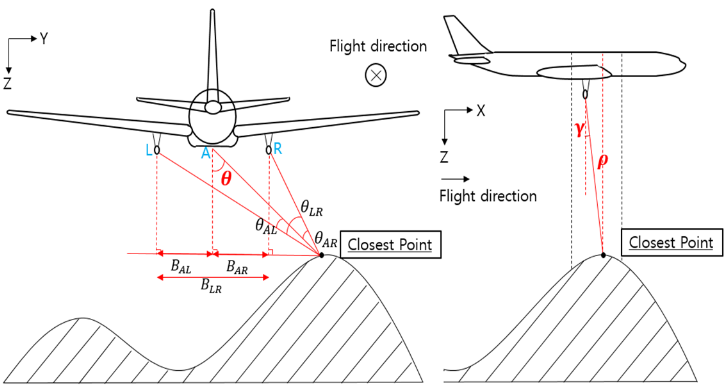


Fig. 1. Rearview and sideview of IRA measurements

detected by sensor. Differential of two path lengths can then be utilized in solving for look angle. When IRA is equipped with two antennas, however, there resides an ambiguity in calculating path length gap between two returned signals as it is not uniquely determined by phase difference without knowing the number of phase repetition. Therefore, IRA mostly utilizes 3 antennas, one for both transmitting and receiving and two others for receiving signals to eliminate the ambiguity integer. One can refer to (3) to determine look angle in 3 antennas case.

$$\theta = \sin^{-1}\left(\frac{B_{AR}\sin\overline{\theta_{AR}}+B_{LA}\sin\overline{\theta_{LA}}+B_{AR}\sin\overline{\theta_{LR}}}{B_{AR}+B_{LA}+B_{LR}}\right) \tag{3}$$

where B, and θ with subscripts denote the baselines between two antennas, and look angle measured using only two antennas. Subscript A denotes ambiguity antenna additionally set up for resolving ambiguity, L denotes left antenna, and R denotes right antenna. $\sin\overline{\theta_{AR}}$ can then be rewritten as follow

$$\sin\overline{\theta_{AR}} = \frac{\lambda(\phi_{AR}+2\pi\overline{n_{AR}})}{2\pi B_{AR}} \tag{4}$$

where ϕ, $\bar{n}$ and with subscripts denote the phase difference measured between two antennas designated by subscripts, and $\sin\overline{\theta_{LA}}$ and $\sin\overline{\theta_{LR}}$ can be rewritten in a similar manner. Ambiguity integer can be selected to one that possess the least error.

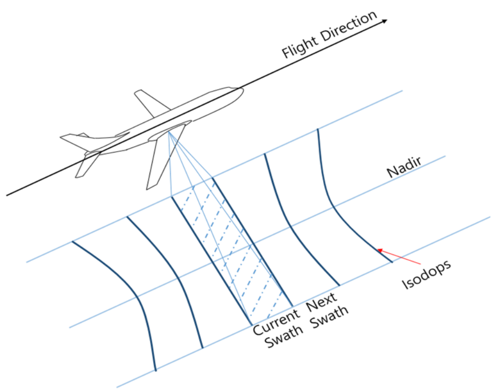


Fig. 2. Isodops and swath selection using Doppler bandwidth filter

B. Numerically replicating the measurements of IRA

In order to compare the navigation performance of proposed method with navigation performance of others that include typical RA-aided TRN, and IRA-aided TRN which utilizes conventional framework, copy of IRA measurements is required. Therefore a procedure of numerically replicating the measurements of IRA when aircraft's position, attitude, and velocity, and database is given is suggested herein this subsection. Resolution of 3arcsec level (≈90m) terrain database was utilized in this measurement reproduction and the procedure includes effort to raise the resolution of database up by using bilinear interpolation. Fig. 3 depicts the procedure, and one can get IRA measurements with following procedure; First, establish a region of interest (ROI) onto the terrain database based on the position and attitude of an aircraft by taking certain field of view (FOV) as a specification of the apertures. Secondly, find a near-zero Doppler shifted region by taking aircraft's velocity into account. As one can regard intersection line of terrain plane and plane perpendicular to velocity vector as zero Doppler line, narrow region bounded within ROI that includes the zero Doppler line and is parallel to the zero Doppler line can be searching swath for IRA. Then calculate the distance from current position to every grid points included within searching swath, and find the point with minimum distance by using linear search. In order to reflect the realities of terrain and to well replicate the IRA measurements, resolution of database then gets raised up higher using bilinear interpolation. By taking the closest point found in rough linear search as the center, eight surrounding terrain grid points get utilized to acquire a finer version of IRA measurements. Bilinear interpolation consists of two independent linearization procedures in two directions which are mutually perpendicular, latitude and longitude directions. One can refer to (5) to calculate the terrain elevation in a finer resolution.

$$h_{approx} = h_{lat,1} \cdot \frac{d-a}{d} + h_{\mathrm{lat},2} \cdot \frac{a}{d} \tag{5}$$

where h_{approx} is terrain elevation calculated in finer resolution, $h_{lat,1} = h_1 \cdot \frac{d-b}{d} + h_4 \cdot \frac{b}{d}$, $h_{lat,2} = h_2 \cdot \frac{d-b}{d} + h_3 \cdot \frac{b}{d}$, (a, b) is longitudinal and latitudinal relative position in finer resolution grid centered at closest point found in rough search, and d is grid size. Once the closest terrain point been chosen through numerical search, ideal measurements of IRA can be obtained as

$$\vec{\rho} = \boldsymbol{C}_{\boldsymbol{NED}}^{\boldsymbol{body}} \cdot \boldsymbol{C}_{\boldsymbol{ECEF}}^{\boldsymbol{NED}} [\vec{P} - \vec{X}_{aircraft}] \tag{6}$$

$$\rho = \|\vec{\rho}\| \tag{7}$$

$$\gamma = \frac{\pi}{2} - \cos^{-1}(\frac{\vec{V} \cdot \vec{\rho}}{\|\vec{V}\| \rho}) \tag{8}$$

$$\theta = \cos^{-1}(\frac{\|\vec{\rho}_{proj.y}\|}{\rho}) \tag{9}$$

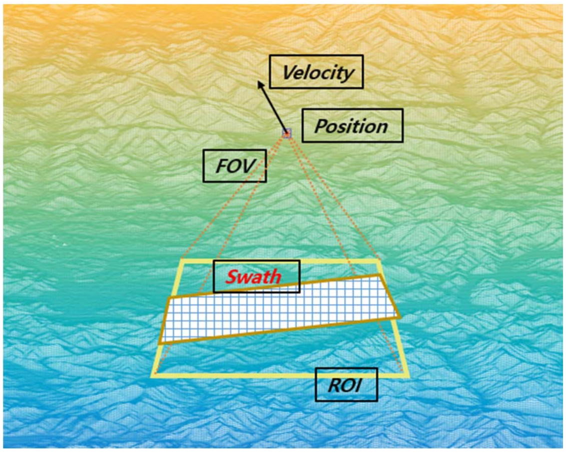


Fig. 3. Procedure of numerically replicating the IRA's measurements

where $\boldsymbol{C}_{\boldsymbol{A}}^{\boldsymbol{B}}$ denotes a coordinate transformation matrix from frame A to frame B, $\vec{P}$ denotes the closest terrain point, $\vec{X}_{aircraft}$ denotes the 3D position of an aircraft, $\vec{V}$ denotes the velocity vector of an aircraft, and subscript $proj.y$ denotes a projection of a vector onto y plane. Although it is possible to raise the resolution up to infinite, 10m level resolution was selected since the effect of DEM resolution to the navigation performance enhancement gets decreased as resolution of DEM goes up. At current reproduction step, 60° of FOV and 5° of swath width were selected to reflect the realistic aperture value of IRA. One can narrow the swath width down, since IRA's actual pool of search is zero Doppler region. However if it becomes too small, there will be no grid points included within the establish swath. 5° of swath width at 2.5km altitude includes two or three grid points in alongtrack direction.

Result of [1], in which the signal level simulations were conducted, shows IRA's error characteristics. As a set of slant range, slant angle, and look angle can uniquely determine a closest terrain point, those measurements can then be converted into Cartesian coordinates to establish a relative position vector to the nearest terrain point. According to [1], measurements of IRA carry 3.40m, 0.40m, and 4.61m level of error standard deviation when converted into aircraft's conventional body frame. In order to estimate the error budget of IRA's measurements, 400 reference points that are evenly distributed in the sky were selected and closest terrain points from each of the reference points were calculated by following the above procedure. Those closest terrain points then get converted into body frame relative position represented in Cartesian coordinate form. By adding Gaussian noise to the relative position and converting them into measurements of IRA using (6) ~ (9), it is possible to estimate the error statistics of IRA's measurements

III. Problem Description

A. RA-aided TRN

Let x_k be the kth state of an aircraft which is a 2D vector that includes latitudinal and longitudinal position of an aircraft.

$$x_k = [x_{k,latitude} \quad x_{k,longitude}]^T \tag{10}$$

The movement of an aircraft can be modeled by using Markov process represented as following prediction model.

$$x_{k+1} = x_k + u_k + w_k \tag{11}$$

Where x_k, u_k, w_k denotes the position state vector, the relative movement, and the additive process noise at kth time step respectively. It is assumed that position incremental u_k is obtained from inertial navigation system (INS). Reference [5] deals with algorithm of INS/TRN fusion, however, this paper will focus on the measurement models without addressing the further detailed model of INS as the purpose of this paper is to verify the performance of navigation with new measurement model of IRA. In RA-aided TRN case, measurement z_k denotes the terrain elevation at the horizontal position x_k. As TRN algorithm refers to the terrain database to update the aircraft's state, measurement model of RA-aided TRN with particle filter can be represented as

$$z_k = h(x_k) + v_k \tag{12}$$

Where $h(\cdot)$ denotes a function that computes the elevation of terrain using DEM when the aircraft's state 2D position x_k is given. Since RA, however, measures the distance from the aircraft to the terrain beneath it which is called ground clearance, actual elevation measurement can be obtained by subtracting ground clearance measured by RA from the absolute altitude of an aircraft measured by barometer. It is assumed that v_k includes errors of RA, barometer, and DEM.

B. Particle Filter

In this subsection, brief introduction to the particle filter algorithm that utilizes sequential importance resampling (SIR) will be given. To overcome the degeneracy problems of sequential importance sampling (SIS), SIR is widely used in resampling procedure when dealing with particle filter [10]. As particle filter is a Bayesian approach to recursively estimate the posterior probability $p(x_k|z_{1:k})$, where $z_{1:k}$ is the entire measurements up to the kth step, it follows the following two steps; Prediction and measurement update represented as follow

$$p(x_k|z_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1})dx_{k-1} \tag{13}$$

$$p(x_k|z_{1:k}) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{\int p(z_k|x_k)p(x_k|z_{1:k-1})dx_k} \tag{14}$$

where the probabilistic model of the state transition is defined by the prediction model (11) with known statistics of w_{k-1}, and $p(z_k|x_k)$ denotes a likelihood function that relates the current observation to the state distribution which is defined by the measurement model (12) with known statistics of v_k. It is mostly modeled as unimodal normal distribution. Measurement update, however, is hardly implementable in practical uses due to the integral over continuous domains. Therefore, the posterior probability distribution is then approximated by using a set of randomly chosen weighted particles through Dirac mixture approximation as

$$p(x_k|z_{1:k}) \approx \textstyle\sum_{i=1}^{N} \omega_k^i \, \delta(x_k - x_k^i) \tag{15}$$

where $\delta(\cdot), N, \omega_k^i, x_k^i$ denotes the Dirac delta function, number of particles, the importance weight of i-th particle, and state of i-th particle respectively. ω_k^i can be obtained as

$$\tilde{\omega}_k^i = \omega_k^i \cdot \frac{p(z_k|\tilde{x}_k^i)p(x_k|x_{k-1}^i)}{q(x_k^i|x_{k-1}^i,\ z_{1:k})} \tag{16}$$

$$\omega_k^i = \frac{\tilde{\omega}_k^i}{\sum_{j=1}^{N} \tilde{\omega}_k^j} \tag{17}$$

where $q(x_k^i|x_{k-1}^i,\ z_{1:k})$ is the importance density, and $\tilde{x}_k^i$ denotes the state of i-th particle after the prediction and before the measurement update. In SIR particle filter, however, all the ω_k^i gets equalized by resampling the particles based on the $\tilde{\omega}_k^i$ value. The larger the $\tilde{\omega}_k^i$ value is, the higher probability that i-th particle would likely to be resampled. Therefore, there's no need for particles to preserve its previous weight. Overall description of the particle filter with SIR is given as following in pseudo-code version as descripted in [11].

TABLE I. PSUEO-CODE DESCRIPTION OF SIR PARTICLE FILTER ALGORITHM

SIR Particle Filter Algorithm: $\left[\{x_k^i\}_{i=1}^N, \{\tilde{x}_k^i\}_{i=1}^N, \{\tilde{\omega}_k^i\}_{i=1}^N\right] = \mathrm{SIR}\left(\{x_{k-1}^i\}_{i=1}^N, z_k\right)$
- FOR $i = 1{:}N$ Draw $\tilde{x}_k^i \sim p(x_k\|x_{k-1}^i)$ Calculate the weight $\tilde{\omega}_k^i = p(z_k\|\tilde{x}_k^i)$ END FOR - FOR $i = 1{:}N$ Normalize $\omega_k^i = \frac{\tilde{\omega}_k^i}{\sum_{j=1}^N \tilde{\omega}_k^j}$ END FOR - Calculate CDF P using $\{\omega_k^i\}_{i=1}^N$ - FOR $i = 1{:}N$ $u \sim U(0,1]$ $j = P^{-1}(u)$ $x_k^i = \tilde{x}_k^j$ END FOR

In case of RA-aided TRN, the likelihood function is specified as

$$p(z_k|\tilde{x}_k^i) = \frac{1}{\sigma_v\sqrt{2\pi}} e^{-\frac{\eta^2}{2\sigma_v}} \tag{18}$$

where $\eta = (h_b - h_a) - h(\tilde{x}_k^i)$ denotes the measurement residual with h_b a barometer measurement and h_a a RA measurement, $h(\tilde{x}_k^i)$ denotes the terrain elevation calculated at the current state estimation $\tilde{x}_k^i$, and σ_v denotes standard deviation of normal distribution. The SIR particle filter computes the cumulative distribution function (CDF) of the importance weights after normalizing them and resamples the particles based on the CDF. P^{-1} denotes the inverse function of CDF and uniformly generated random number u can indicate an index of a particle to be resampled. It follows the principle of inverse transform sampling (ITS), and one can refer to [10] for further details of resampling procedure.

C. Current status of IRA-aided TRN

While maintaining the overall filtering algorithm as described in section III. A, and III. B [3-4] used following measurement model to utilize IRA's accurate measurements for particle filter algorithm.

$$z_k = h(x_k + \Delta p_k) + v_k \tag{19}$$

where $\Delta p_k = [\rho_k \sin\gamma_k \quad \rho_k \sin\theta_k]^T$ denotes the horizontal displacement from the aircraft to the closest terrain point by using IRA's measurements ρ, θ, and γ. Likelihood function in this case remains same as (18), however, measurement residual η gets changed as follow

$$\eta = (h_b - \Delta z_k) - h(\tilde{x}_k^i + \Delta p_k) \tag{20}$$

where $\Delta z_k = \sqrt{\rho_k^2 - \Delta p_k \cdot \Delta p_k^T}$ denotes the vertical displacement from the aircraft to the closest terrain point by using IRA's measurements ρ, γ, and θ, h_b remains same as (18), and $h(\tilde{x}_k^i + \Delta p_k)$ denotes the terrain elevation calculated at the position $\tilde{x}_k^i + \Delta p_k$ which is a simple summation of current particle state estimation $\tilde{x}_k^i$ after the prediction and horizontal displacement to the closest point Δp_k. Such measurement model retains same problems that RA-aided TRN possesses since the framework of measurement model is equivalent to that of RA-aided TRN. As likelihood function in this case reflects the shape of terrain nearby the position $\tilde{x}_k^i + \Delta p_k$, filter algorithm may encounter low convergence rate problem when the terrain around the position $\tilde{x}_k^i + \Delta p_k$ is too smooth to be distinguished by terrain elevation. It is also hard to track down the characteristics of terrain on which the aircraft is flying since the direction to the closest point keeps varying over time.

Results of [3-4] shows that utilizing (19) and (20) which inserts IRA's measurement into existing framework brings no enhancement in positioning performance compared to the one that utilizes RA with (12) even though IRA itself is more accurate than RA. Therefore, this paper proposes a novel measurement model that would effectively reflect the error characteristics of an IRA, especially in terms of slant range, and that improve the positioning performance.

IV. PROPOSED MEASUREMENT MODEL

As addressed in section III. C, current measurement model of IRA for particle filter algorithm reutilizes the framework of RA-aided TRN. It makes use of IRA's accurate measurements in a blended form without distinguishing the measurements. Thereby it inserts the IRA's measurements into improper framework that has been used ever since the RA-aided TRN was suggested.

The proposed measurement model estimates the closest terrain point from the particle and calculate the slant range from the filter's point of view. During the measurement update at each time step, weight of each particles will be updated based on the difference between IRA's slant range measurement and estimated slant range from each of the particles. Proposed measurement model can be represented as

$$z_k = \rho_{\text{closest}}(DEM, x_k^i, \overrightarrow{V_k}) + v_k \tag{21}$$

where $\rho_{\text{closest}}(\cdot)$ returns the closest terrain point among the ones that are included within DEM and bounded by searching swath denoted by velocity of an aircraft $\overrightarrow{V_k}$ from the particle's position state x_k^i assuming no attitude changes. Section II. B introduces such procedure. It can be rewritten as

$$z_k = min(\|X_k^i - X\|) + v_k \tag{22}$$

where $X_k^i = [x_k^{i\,T} \quad H]^T$ denotes the current estimated 3D position of the aircraft, H denotes the known altitude of the aircraft, X denotes the terrain points that satisfy $X \in \{X | X = [x_{lat} \quad x_{long} \quad h([x_{lat} \quad x_{long}]^T)]^T,\ X \in (DEM \cap ROI \cap near\ zero\ Doppler\ swath)\}$, x_{lat} denotes latitudinal position of the terrain point, and x_{long} denotes longitudinal position of the terrain point. Likelihood function remains same as (18), however, measurement residual η in this case follows

$$\eta = \rho_k - \rho_k^i \tag{23}$$

where ρ_k^i denotes the slant range estimation using (21) or (22) calculated from the estimate particle state $\tilde{x}_k^i$ after the prediction step, and ρ_k denotes the IRA's slant range measurement. This model makes use of the feature that IRA gives the user 3D information to the terrain point and the fact that it is the closest point from the aircraft. Filter itself estimates the closest point of each particles, and thereby it can result to a better positioning performance because the state error is detectable by observing slant range. Slant range can be written as follow,

$$\rho = \sqrt{\Delta p^2 + \Delta z^2} \tag{24}$$

where Δp, and Δz denotes the horizontal displacement and vertical displacement to the point designated by $\vec{\rho}$ respectively, and partial differential of slant range in terms of horizontal 2D position error can be represented as follow

$$\frac{\partial \rho}{\partial \Delta p} = \Delta p \Big/ \sqrt{\Delta p^2 + \Delta z^2} \tag{25}$$

It is expected that there resides a tendency in between aircraft's 2D position state error Δp and measurement residual that smaller Δp significantly decreases measurement residual as (24) is monotonically increasing function in terms of horizontal displacement Δp. (25) converges to a constant value which induces a linear function between Δp and measurement residual.

V. SIMULATION

In this section, the simulation results that verify the positioning performance of proposed method are presented. Comparison of positioning performance between proposed measurement model of IRA with conventional measurement model of either RA or IRA is conducted. An aircraft moving along the straight path at a speed of 40m/s at an altitude of 2.5km was considered. It is assumed that the aircraft is equipped with IRA, RA, and barometer along with an INS. All of the equipment is assumed to provide its measurements every 1 second. Transitional movement measured by INS is assumed to include zero-mean Gaussian error with standard deviation of 4m. DEM onboard contains grid-wise terrain profile with resolution of 3” (≈90m). IRA slant range error is sampled from a zero-mean Gaussian distribution with standard deviation of 4.54m, and overall measurement error from RA, barometer and DEM is sampled from a zero-mean Gaussian distribution with standard deviation of 15m. 1000 particles are used for SIR particle filter. It is assumed that the aircraft is flying along the straight path directed to the North, and is flying with constant height, velocity, and attitude. TABLE II contains the detailed conditions of simulation, and TABLE III represents the specifications of sensors used.

Firstly, numerical simulation was carried out on both smooth terrain and rough terrain. Time history of root mean square error (RMSE) is presented in Fig. 4, and Fig. 5. For the comparison, results from two different measurement models of IRA, and one from conventional measurement model of RA is presented in a same figure; IRA-aided TRN that utilizes (21) (IRA proposed), IRA-aided TRN that utilizes (19) (IRA typical), and RA-aided TRN that utilizes (12) (RA typical). 100 times of Monte-Carlo simulation was conducted, and time history of RMSE was obtained. Fig. 6 represents result of 5 times of independently repeated simulation to depict the tendency of positioning. Entire simulation was done under equivalent simulation setting.

TABLE II. SIMULATION CONDITIONS

Items	Value
Initial Position (latitude, longitude)	Rough terrain: (37.20°, 128.50°) Smooth terrain: (37.16 °, 128.47 °)
Altitude and speed	2.5km, 40m/s (heading North)
Flight condition	Constant attitude, constant altitude, and constant velocity
DEM resolution	3” (≈90m)
Initial error	50m, and 100m in both latitude and longitude direction
Sampling rate & Duration	1Hz, 100s
Number of particles	1000
Process noise, σ	4m
IRA ρ measurement noise, σ_v	10m
RA & IRA elevation measurement noise, σ_v	15m
Number of Monte-Carlo simulations	100

TABLE III. SENSOR SPECIFICATION

Interferometric Radar Altimeter (IRA)	
ρ error, σ	4.54m
γ error, σ	0.0013rad
θ error, σ	3.60e-4rad
Radar Altimeter (RA)	
Ground Clearance h_a error, σ	10m
Barometer	
Absolute altitude error, σ	3m
Inertial Navigation System (INS)	
Latitude velocity error, σ	1m/s
Longitude velocity error, σ	1m/s

Error standard deviations used for IRA are results from statistical estimation in section II. B.

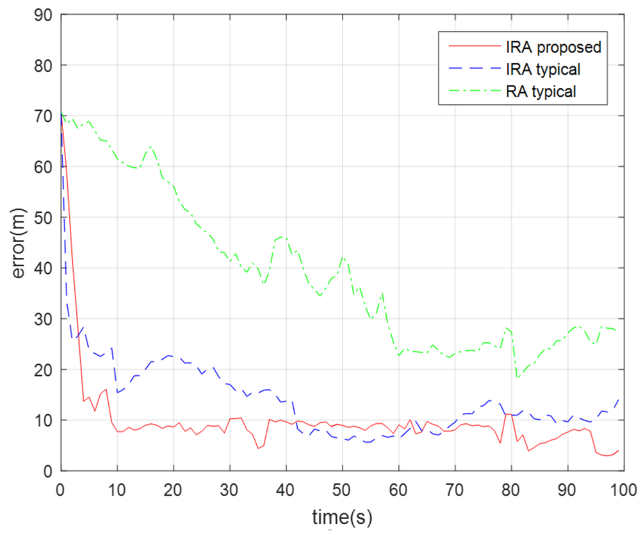


Fig. 4. Times history of root mean square error of the Monte-Carlo simulation for the smooth terrain

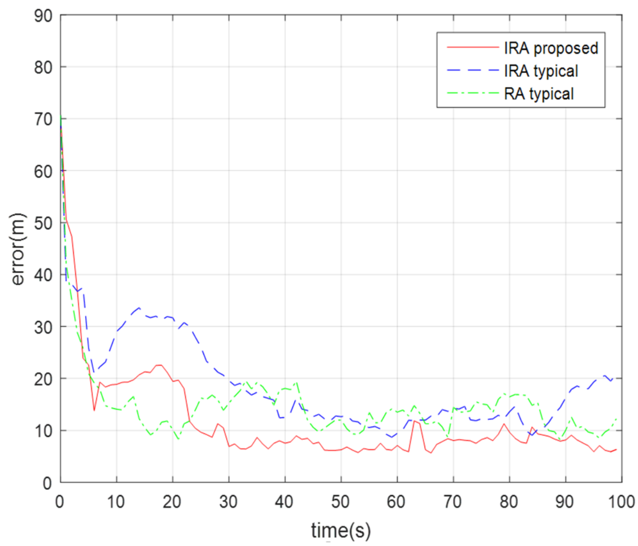
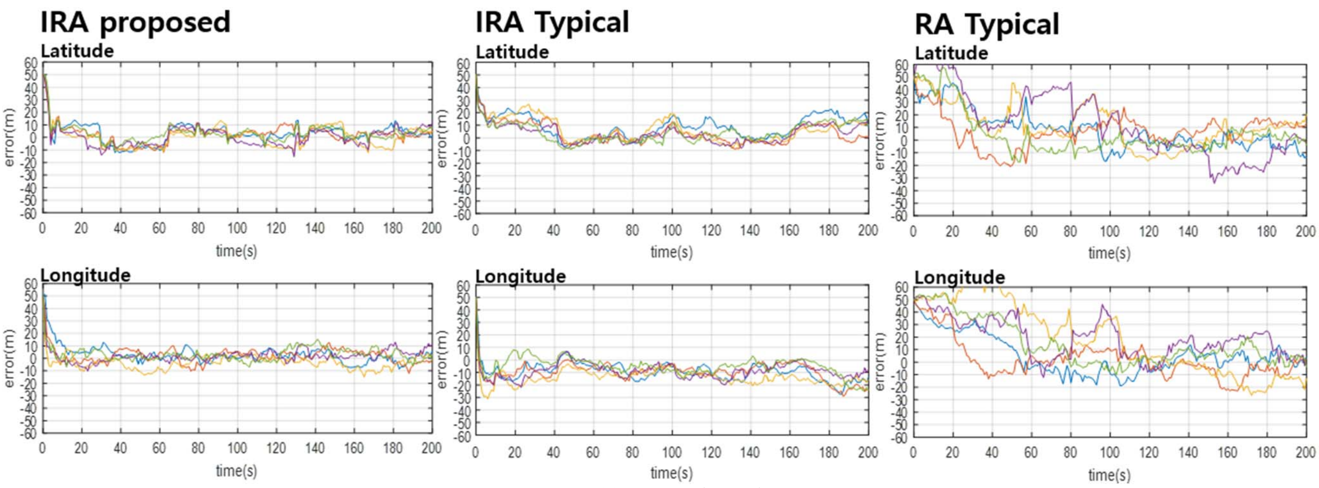
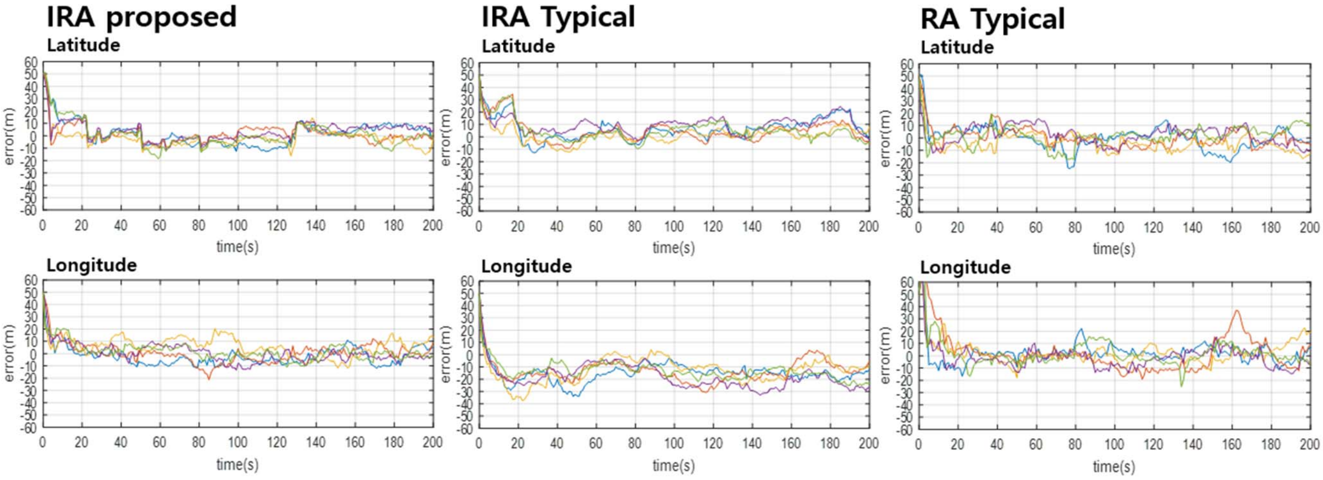


Fig. 5. Times history of root mean square error of the Monte-Carlo simulation for the rough terrain



(a) Smooth terrain



(b) Rough terrain

Fig. 6. Times history of latidinal and longitudinal position error of 5 time simulations for the (a) smooth terrain, and (b) rough terrain.

As particle filter started with an initial position error of (50m, 50m), every graphs from Fig.4 to Fig. 6 starts from 50m. It has been observed that the positioning performance of filter that utilizes proposed IRA's measurement model gets improved in comparison with typical one that inserts measurement of IRA into existing framework in terms of RMSE for both the rough and smooth terrain. Moreover, the proposed method less tend to fluctuate as IRA measurements vary along the time. IRA-aided TRN with proposed measurement model showed better performance in position error comparison with RA-aided TRN for both rough and smooth terrain, whereas IRA-aided TRN with typical measurement model did not for the rough terrain.

Secondly, numerical simulation was carried out in a single rough terrain to compare the tendency of correcting the initial error in proposed IRA measurement model, and conventional IRA measurement model. An additional simulation was conducted under different initial position error. 100 times of Monte-Carlo simulation was conducted, and time history of RMSE was obtained. Fig. 7 represents the positioning performance of two IRA-aided TRNs when initial position error is doubled to (100m, 100m).

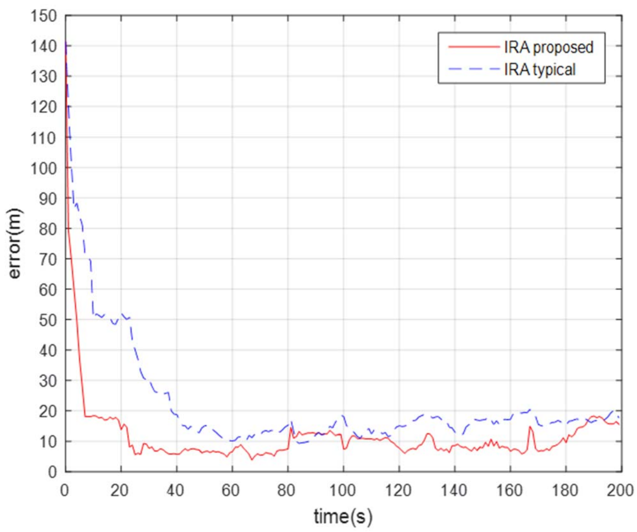


Fig. 7. Times history of RMSE of IRA-aided TRNs when the initial position error is (100m, 100m).

IRA-aided TRN with proposed measurement model showed a better performance in terms of initial error correction. Filter with proposed IRA measurement model tends to converge faster than typical IRA measurement model. This is because proposed measurement model shows more linearity in between 2D state error and measurement residual even when there resides large error in position estimation.

VI. CONCLUSION

In this paper, a novel measurement model of interferometric radar altimeter (IRA) for particle filter was suggested to well reflect the error characteristics of IRA and to enhance the performance of positioning problem. Instead of fitting the IRA's measurements into existing framework of typical terrain referenced navigation (TRN) and correcting the weight of particles based on terrain height at a point that IRA measurements' horizontal component induces, each particle can directly estimate the measurement of IRA, slant range. It has been observed that inserting IRA measurements into existing framework of radar altimeter (RA)-aided TRN brings little improvements in positioning performance, while positioning performance could benefit from IRA's accurate measurements when they are independently utilized and estimated from the point of view of filter. Based on numerical simulations that compare performance of new measurement model with conventional ones, suggested measurement model shows better performance in terms of not only RMSE of 2D position error, but also initial error correction.

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