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Balanced Cooperative Target Search of Mobile Sensor Fleet under Localization Uncertainty

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Abstract— **This paper deals with the problem of multiple sensors installed on mobile platforms cooperating to search for the target while each sensor is subject to localization uncertainty. The balance between a couple of conflicting objectives: i) suppressing the growth of self-localization uncertainty as an individual mobile agent; ii) reducing target uncertainty as a team, is sought under the information-theoretic framework. The conventional particle filter approach that concerns only the target state is augmented with self-localization by jointly estimating both. The self-localization is facilitated by introducing additional radio signals emitted from the origin, which should be a practical assumption considering limited resources and the restricted capability of small mobile sensors in case of a GNSS outage. The proposed method is featured with the numerical calculation of mutual information based on the principle of kernel conditional density estimation, and the unified comparison scheme between conflicting objectives. The numerical experiment shows that the proposed probing control law can automatically switch between target ascertaining and observability enhancing based on the mutual information-based utility function in a distributed fashion.**

I. INTRODUCTION

A fleet of mobile sensors installed on the flying vehicles serves as an agile information-gathering framework. Especially when the platform gets smaller, cheaper, and nimbler, such automated information search becomes more reliable and redundant by deploying a multitude of them. However, due to the limited size, weight and power (SWAP) condition [1] of micro aerial vehicles (MAV), the sensing capability of the available sensors is often degenerated and the duration of sensor operation is restricted. Therefore, the goal of the information gathering framework enabled by many cheap, low-powered, and possibly semi-disposable [2] flying vehicles is commonly posed as minimizing the time consumed [13] until the target is localized using sensors of only moderate accuracy.

Since the performance of target localization varies not only upon the sensor accuracy but also on the sensor geometry [9], the problem is also featured with actively controlling vehicles in a way that the whole group of them can cooperatively capture the target. The processes that solution approaches share in common are to represent some notion of target information as an utility function, either in myopic [12] or in far-sighted sense [14], and to maximize it with respect to the admissible control inputs, either in centralized [16] or distributed [7] way.

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Both studies [9], [16] adopt Fisher information matrix (FIM) as their optimization cost and the group of vehicles are driven in a collective fashion that maximizes D-optimality [23] criteria. By deriving analytic representation for sensor-specific FIM, [16] optimizes the entire path of the vehicles to the target taking final time t_f into account. [7], [8] introduce decentralized cooperation framework while the former is featured with the utilization of information form Kalman filter (KF). The approach eases the integration of observed information from multiple vehicles by transforming it into simple addition. [13], [15] make use of the particle filter (PF) [4] for the general nonlinear non-Gaussian problem. Especially the former approach effectively approximates the calculation of mutual information, from which distributed probing law is derived. Considering the scalability, redundancy and complexity of the target searching problems, distributed approach applicable to the general sensor suits should be favored.

One blind spot of these information-theoretic approaches, however, is that the location of mobile sensors is assumed to be known both accurately and deterministically with the help of position-fixing tools such as a global navigation satellite system (GNSS). It apparently makes sense because sensors being unable to localize themselves are not well suited for locating the other important targets. Such targets often include victims of natural disasters, military assets of enemy forces, and endangered wildlife, whose exact locations are essential. At the same time, however, the sorts of applications being referred to as search and rescue (SaR) mission [10], [11] are mostly environed by a rugged mountain, dense forest, or even contaminated with radiation in cases of avalanche, earthquake, wildfire or accident of power plants. Thus, the external aid that can accurately localize mobile sensors is less likely to be available. Situations, where only partial and noisy information about the mobile sensor's location is available are more practical in that sense. But then what immediately follows is that mobile sensors are subject to the localization drift, and the growing uncertainty.

This study deals with the problem placed under two conflicting goals: not to let the localization uncertainty of each vehicle grows over time, and to ascertain the target state. The approach directly extends the PF-based Hoffmann's work [13], which only focuses on the target state, by simultaneously estimating the joint distribution of the target and the sensor state and by deriving distributed probing control law of mobile sensors. Yet, naively introducing additional variables will make the system unobservable and yield an ill-posed problem. Therefore, we slightly relax the condition

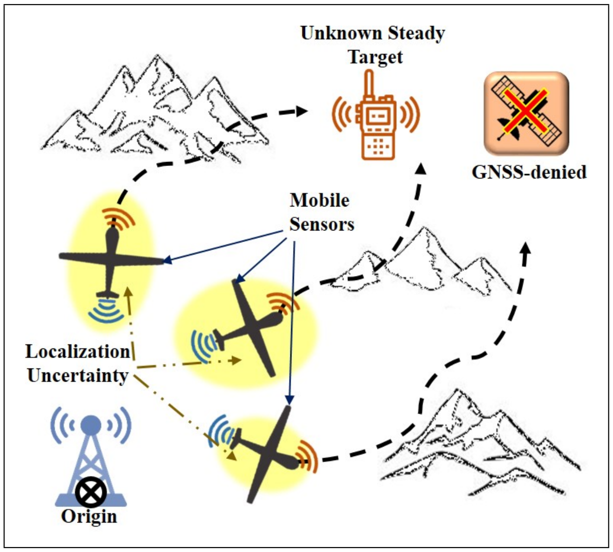

Fig. 1. Mission sketch of multiple mobile sensors searching for a steady target. The proposed probing law looks for the balance between target ascertaining and suppressing the growth of self-localization uncertainty. The obstacles in the figure are not considered in this study.

by adopting the commonly known origin (or the rendezvous point as a formal term) and by observing it. Instead of letting the SWAP-limited vehicles travel all the way by themselves to the remote operational area, where the target is supposed to be hidden, a situation where the sensor fleet is transported to the safe place in the vicinity of the operational area by stable ground mobility is considered. Furthermore, for the cooperative target search, vehicles are assumed to be able to communicate with each other in a fully connected fashion. The scenario is sketched in Fig. 1.

II. Problem Formulation

A. Dynamic and Measurement Model of Mobile Sensor

Throughout this paper, a notation $s_k^{a,[j]}$ symbolizes j^{th} particle (see Section III) of PF that represents a physical signal s of a at time step k. Note that when a becomes an integer, $i \in \{1, \cdots, n_s\}$ given the total number of the sensors n_s, the notation refers to s of i^{th} sensor, and otherwise it is clear from the context. Without a superscript $[j]$, it denotes the variable itself.

The target, represented by its 2-dimensional position $\mathrm{x}_k^\tau = [x_k^\tau, y_k^\tau]^T$, is assumed to be stationary over time in this study. The state of the i^{th} mobile sensor is considered as $\mathrm{x}_k^i = [x_k^i, y_k^i, \psi_k^i]^T$ where ψ denotes heading angle. The fixed-wing flying vehicle whose discrete-time dynamic model is given as

$$\begin{aligned} \mathrm{x}_{k+1}^i &= f(\mathrm{x}_k^i, u_k^i) + v_{k+1} \\ &= \mathrm{x}_k^i + \begin{bmatrix} V\cos\left(\psi_k^i + u_k^i \Delta t\right) \\ V\sin\left(\psi_k^i + u_k^i \Delta t\right) \\ u_k^i \Delta t \end{bmatrix} + v_{k+1}, \end{aligned} \tag{1}$$

is considered for the mobile platform. Here, the bounded control input, $u_k^i \in \mathbb{U}^i := [u_{\min}, u_{\max}]$, is the turn rate, Δt denotes the time difference between adjacent discrete steps, and v_k denotes additive Gaussian noise. Note that the heading alters before the propagation of position. Such the model is utilized in order to disambiguate the utility function (especially (6), see Section II-B) with respect to the variation of u_k^i. The fact that attitude dynamics of an aerial vehicle is much faster than that of velocity or position rationalizes the model.

The target in this study is considered to emit a radio signal and is observed via the angle of arrival (AOA) sensor [20]. Therefore, the target measurement, ${}_\tau z$, is modelled as

$$\begin{aligned} {}_\tau z_k^i &= h_a(\mathrm{x}_k^\tau, \mathrm{x}_k^i) + e_k^\tau \\ &= \operatorname{atan2}\left(y_k^\tau - y_k^i, x_k^\tau - x_k^i\right) - \psi_k^i + e_k^\tau, \end{aligned} \tag{2}$$

where atan2 denotes quadrant arctan function.

At the same time in order not to lose self-localization accuracy, mobile sensors keep observing the origin. It is assumed that the transporter, for instance, located at the origin resting motionlessly can emit the same sort of radio signals as those emitted from the target so that the present sensor suit can be reused as it is. The origin measurement, ${}_o z$, is modelled as a combination of AOA and bearing angle from the origin,

$$\begin{aligned} {}_o z_k^i &= \begin{bmatrix} h_a(\vec{0}_{2\times 1}, \mathrm{x}_k^i) \\ h_b(\mathrm{x}_k^i) \end{bmatrix} + e_k^o \\ &= \begin{bmatrix} \operatorname{atan2}\left(-y_k^i, -x_k^i\right) - \psi_k^i \\ \operatorname{atan2}\left(y_k^i, x_k^i\right) \end{bmatrix} + e_k^o. \end{aligned} \tag{3}$$

Here, e_k^τ and e_k^o denote measurement noises of appropriate dimension added to the respective measurements. The rationale behind the capability of sensing bearing angle is that azimuthally distinct, e.g., phased, radio signals are emitted that the sensors are aware of. Note that the noise distributions need not to be Gaussian, yet in this study e_k^τ and e_k^o are modelled as $\mathcal{N}(\mathbf{0}, R_k^\tau)$ and $\mathcal{N}(\mathbf{0}, R_k^o)$ respectively.

B. Mutual Information-based Balanced Target Search

Each i^{th} mobile sensor keeps tracking the distribution of the joint state, $\mathbf{x}_k^i = [(\mathrm{x}_k^\tau)^T, (\mathrm{x}_k^i)^T]^T$, i.e., $p(\mathrm{x}_k^\tau, \mathrm{x}_k^i)$, using all the available information either locally inferred or transferred from the other sensors. It is a collection of history of the local target measurements ${}_\tau z_{1:k}^i, {}_o z_{1:k}^i$ up to the time, and the the other mobile sensors' target measurements, ${}_\tau z_k^{\bar{i}}$, and minimum mean square error (MMSE) estimate of their respective joint state, $\hat{\mathbf{x}}_k^{\bar{i}}$. Here, $\bar{i}$ denotes the set of all vehicles but i. It is assumed that the fleet of mobile sensors is fully connected and that local communication is available. In this study, as analogous to [12], [14], the PF is considered as a method of estimating state distribution from which a useful notion of information and the control law can be derived.

It is well-known [3], [7] that maximizing mutual information between the distributions of some state of interest, x, and the observation, z, naturally minimize the expected uncertainty embodied in $p(\mathrm{x}|\mathrm{z})$ in a Bayesian filtering framework. Such uncertainties are represented as entropy through the relation:

$$H(\mathrm{x}_k|\mathrm{z}_k) = H(\mathrm{x}_k) - \mathcal{I}(\mathrm{x}_k; \mathrm{z}_k), \tag{4}$$

where the mutual information, $\mathcal{I}(\mathrm{x}_k; \mathrm{z}_k)$, provides a measure of non-negative gain that a certain observation can imply about the state of interest. They become interdependent over time steps as the exertion of control input, u_k^i, through (1) influences the vehicle state, x_{k+1}^i, and the effect is indicated by the observations, z_{k+1}^i, as in (3) and/or (2).

The commutative nature [21], [22] of the mutual information, which can be rewritten as

$$\begin{aligned} \mathcal{I}(\mathrm{x}_k; \mathrm{z}_k) &= H(\mathrm{x}_k) - H(\mathrm{x}_k|\mathrm{z}_k) \\ &= H(\mathrm{z}_k) - H(\mathrm{z}_k|\mathrm{x}_k), \end{aligned} \tag{5}$$

provides us a rationale for the equivalence of the minimum posterior uncertainty and the maximum mutual information through the first row, as the uncertainty inherent in the prior distribution is independent of the action of our choosing. The second row on the other hand, provides us a way of calculating it as in [12]. Moreover, it has been shown [13] that considering only the own one-step future observation of each sensor, namely single-node approximation, suffices achieving *cooperative* behaviors provided the observations of all the other sensors up to the time.

Then a mobile sensor, as pointed out in the early section, is placed under a couple of conflicting goals: to keep self localization uncertainty as low as possible standing as a single unit, and to minimize uncertainty of the target standing as a member of the fleet. This study seeks the optimal choice of control input that does not betray the both. The term *balanced* in this study thus represents that the derived control law takes importance of both roles into account.

As long as reducing the uncertainty of localization estimate is concerned, i.e. $\mathrm{x}_k = \mathrm{x}_k^i$ and $\mathrm{z}_k = {}_o z_k^i$ in (5), the utility function of the form:

$$Q_1\left(p(\mathrm{x}_k^\tau, \mathrm{x}_k^i), u_k^i\right) = \mathcal{I}\left({}_o z_{k+1}^i; \mathrm{x}_{k+1}^i\right), \tag{6}$$

will suffice. Note that by assuming independence between AOA, ${}_o^A z_{k+1}^i$, and bearing angle, ${}_o^B z_{k+1}^i$, of ${}_o z_{k+1}^i$, (6) is equivalent to adding two individual mutual information as

$$\mathcal{I}\left({}_o^A z_{k+1}^i; \mathrm{x}_{k+1}^i\right) + \mathcal{I}\left({}_o^B z_{k+1}^i; \mathrm{x}_{k+1}^i\right). \tag{7}$$

Meanwhile, when ascertaining the future target is the sole purpose of the fleet of mobile sensors, i.e., $\mathrm{x}_k = \mathrm{x}_k^\tau$, $\mathrm{z}_k = {}_\tau z_k^i$, the utility function of the form:

$$Q_2\left(p(\mathrm{x}_k^\tau, \mathrm{x}_k^i), u_k^i\right) = \mathcal{I}\left({}_\tau z_{k+1}^i; \mathrm{x}_{k+1}^\tau \,\middle|\, \mathrm{x}_{k+1}^i = \hat{\mathrm{x}}_{k+1}^i\right), \tag{8}$$

is adequate. Note that the dependency is not upon the variable but a specific estimate for the practicality of evaluation, which can be easily done by PF.

As a conservative approach, this study proposes a control law of each i^{th} mobile sensor ${u^*}_k^i$ such that,

$${u^*}_k^i = \begin{cases} u_1, & \text{if} \quad \begin{aligned} &\gamma_1^{-1} Q_1\left(p(\mathrm{x}_k^\tau, \mathrm{x}_k^i), u_1\right) \\ &\quad < \gamma_2^{-1} Q_2\left(p(\mathrm{x}_k^\tau, \mathrm{x}_k^i), u_2\right) \end{aligned} \\ u_2, & \text{otherwise}, \end{cases} \tag{9}$$

where $u_j, j \in \{1, 2\}$ is respective optimal choice based on (6) and (8), i.e., $u_j = \operatorname{argmax}_{u_k^i} Q_j(p(\mathrm{x}_k^\tau, \mathrm{x}_k^i), u_k^i)$, while the detailed calculation is given in Section III-B. Moreover, γ_j for $j \in \{1, 2\}$ are normalizing factors for Q_j so that the both are similarly sensitive to the control input variation. The rational choice of γ_j would be the maximum value of Q_j as the reciprocals of them unify the scale of different Q_j. Since the mutual information tends to has its maximum value when the state of interest is in its most uncertain form, the Q_j calculated from the initial setup of PF using associated noise parameters are suited for the purpose.

Let $\mathbf{x}_0^i$ be approximated as Gaussian, i.e., $\mathbf{x}_0^i \sim \mathcal{N}([(\hat{\mathrm{x}}_0^\tau)^T, (\hat{\mathrm{x}}_0^i)^T]^T, \mathbf{\Sigma}_0^i)$, such that

$$\begin{aligned} \hat{\mathrm{x}}_0^\tau &= \begin{bmatrix} \hat{x}_0^\tau & \hat{y}_0^\tau \end{bmatrix}^T, \\ \hat{\mathrm{x}}_0^i &= \begin{bmatrix} \hat{x}_0^i & \hat{y}_0^i & \hat{\psi}_0^i \end{bmatrix}^T, \\ \mathbf{\Sigma}_0^i &= \begin{bmatrix} \Sigma_0^\tau & \mathbf{0} \\ \mathbf{0} & \Sigma_0^i \end{bmatrix}. \end{aligned} \tag{10}$$

Then the Jacobian of (3) with respect to x_0^i is

$${}_o J^i = \begin{bmatrix} -\hat{y}_0^i/(r^i)^2 & \hat{x}_0^i/(r^i)^2 & -1 \\ -\hat{y}_0^i/(r^i)^2 & \hat{x}_0^i/(r^i)^2 & 0 \end{bmatrix}, \tag{11}$$

where $r^i = \sqrt{(\hat{x}_0^i)^2 + (\hat{y}_0^i)^2}$, while the Jacobian of (2) with respect to x_0^τ is

$${}_\tau J^i = \frac{1}{(r^\tau)^2}\begin{bmatrix} -(\hat{y}_0^\tau - \hat{y}_0^i) & \hat{x}_0^\tau - \hat{x}_0^i \end{bmatrix}, \tag{12}$$

where $r^\tau = \sqrt{(\hat{x}_0^\tau - \hat{x}_0^i)^2 + (\hat{y}_0^\tau - \hat{y}_0^i)^2}$.

Then, using (5) and the entropy formula for Gaussians [21], γ_1 and γ_2 for the i^{th} mobile sensor is approximated as

$$\begin{aligned} \gamma_1 &= \frac{1}{2}\log\left(\frac{\left|\Sigma_0^i\right|}{\left|\left(\Lambda_0^i + {}_o J^{i^T} R_0^{o^{-1}} {}_o J^i\right)^{-1}\right|}\right), \\ \gamma_2 &= \frac{1}{2}\log\left(\frac{\left|\Sigma_0^\tau\right|}{\left|\left(\Lambda_0^\tau + \sum_{k=1}^{n_s} {}_\tau J^{k^T} R_0^{\tau^{-1}} {}_\tau J^k\right)^{-1}\right|}\right) \end{aligned} \tag{13}$$

where $\Lambda_0^i = \left(\Sigma_0^i\right)^{-1}$ and $\Lambda_0^\tau = (\Sigma_0^\tau)^{-1}$ which should be designed by user. Note that in (13), γ_1 is calculated in an individual sense while γ_2 is calculated in a collective fashion. This is due to the way data are assimilated across the sensors so that only target observation is shared among the sensors. What follows are the details of the particle filter update using communicated measurement.

III. Particle Filter Approach

A. Communication Among Sensors and Filter Update

In tracking the distribution of the joint state, $\mathbf{x}_k^i$, the PF gets utilized. As it is not restricted to a specific setup, it is also scalable to a wide range of sensors and vehicles combination. The PF embodied in i^{th} vehicle approximates the probability distribution of $\mathbf{x}_k^i$ using N stochastically drawn samples and their respective weights, i.e., $\{\mathbf{x}_k^{i,[j]}, w_k^{i,[j]}\}_{j=1}^N$. These particles navigate through the state space based on the sampling strategy and are weighted based on the system model and noise characteristics using the principle of importance sampling. For the sake of conciseness, readers are referred to [5], [6] and references therein for the details of particle filtering.

Each particle propagates across both the state space and the time by simulating the stochastic model,

$$\begin{aligned}\mathbf{x}_k^i &= \mathbf{f}(\mathbf{x}_{k-1}^i, u_{k-1}^i) + \mathbf{v}_k \\ &= \begin{bmatrix} \mathrm{x}_{k-1}^\tau + \nu_k \\ f(\mathrm{x}_{k-1}^i, u_{k-1}^i) + v_k \end{bmatrix},\end{aligned} \tag{14}$$

which should be an augmentation of stationary motion and (1). Here, ν_k denotes an artificial zero-mean noise added to the target position at each time step to prevent sample degeneration. Sampling through an alternative proposal distribution is not considered.

What immediately follows is the weight update using local measurement ${}_o z_k^i$ as

$$w_k^{i,[j]} \propto w_{k-1}^{i,[j]} \mathcal{N}\left({}_o z_k^i - \begin{bmatrix} h_a(\vec{0}_{2\times 1}, \mathrm{x}_k^{i,[j]}) \\ h_b(\mathrm{x}_k^{i,[j]}) \end{bmatrix}; \mathbf{0}, R_k^o\right), \tag{15}$$

where the proportion means relative importance among the particles. Thus, weights should be normalized using their sum, $\sum_{j=1}^N w_k^{i,[j]}$. Note that up to here are the general steps of the PF.

For the cooperative and consented target localization, each vehicle state's MMSE estimate at the moment,

$$\hat{\mathrm{x}}_k^i = \sum_{j=1}^N w_k^{i,[j]} \mathrm{x}_k^{i,[j]}, \tag{16}$$

and target measurement, ${}_\tau z_k^i$, are intercommunicated. Then PF of i^{th} vehicle is corrected first by using its local target measurement as

$$w_k^{i,[j]} \propto w_k^{i,[j]} \mathcal{N}\left({}_\tau z_k^i - h_a(\mathrm{x}_k^{\tau,[j]}, \mathrm{x}_k^{i,[j]}); \mathbf{0}, R_k^\tau\right), \tag{17}$$

and in succession, using the received information from sensors $l \in \bar{i}$ as

$$w_k^{i,[j]} \propto w_k^{i,[j]} \mathcal{N}\left({}_\tau z_k^l - h_a(\mathrm{x}_k^{\tau,[j]}, \hat{\mathrm{x}}_k^l); \mathbf{0}, R_k^\tau\right). \tag{18}$$

The common implementation of resampling [5] goes after.

B. Approximation of Mutual Information

One of the most powerful features of PF is that it can approximate an expectation of a function, $g(\theta)$, of a vector $\theta \in \mathbb{R}^{n_\theta}$ over its posterior distribution by using a set of N particles of the state and their associated weights, $\{\theta^{[j]}, \mathrm{w}^{[j]}\}_{j=1}^N$, which are sampled from $p(\theta|\mathrm{z})$, as

$$\int_\Theta g(\theta) p(\theta|\mathrm{z}) dx \approx \sum_{j=1}^N \mathrm{w}^{[j]} g(\theta^{[j]}). \tag{19}$$

Considering (5), calculating the mutual information of (6) and (7) involves calculating $H({}_o z_{k+1}^i)$ and $H({}_o z_{k+1}^i | \mathrm{x}_{k+1}^i)$. Firstly, propagating $p(\mathbf{x}_k^i)$ one step forward temporarily for the given u_k^i yields $p(\mathbf{x}_{k+1}^i) \equiv \{\mathbf{x}_{k+1}^{i,[j]}, w_{k+1}^{i,[j]}\}_{j=1}^N$. What follows is to approximate the two entropy values using particles and their associated weights as introduced in [12].

For the sake of simplicity, all time stamps hereafter until (24), which should be $k+1$, are omitted as well as the separation of A with B, and the superscript i.

By definition, it holds that

$$H({}_o z) = -\int_{{}_o\mathcal{Z}} p({}_o z) \log\left(p({}_o z)\right) d_o z, \tag{20}$$

where

$$\begin{aligned} p({}_o z) &= \int_{\mathbf{X}} p({}_o z|\mathrm{x}) p(\mathrm{x}) d\mathrm{x} \\ &\approx \sum_{j=1}^N w^{[j]} p({}_o z|\mathrm{x} = \mathrm{x}^{[j]}) \end{aligned} \tag{21}$$

The rationale behind (21) is to consider $p({}_o z)$ as $\mathbb{E}_{\mathbf{X}}[p({}_o z|\mathrm{x})]$ and to realize it by using Monte Carlo approximation (19).

Then, (20) immediately turns into

$$\begin{aligned} H({}_o z) = -\int_{{}_o\mathcal{Z}} &\left(\sum_{j=1}^N w^{[j]} p({}_o z|\mathrm{x} = \mathrm{x}^{[j]})\right) \\ &\cdot \log\left(\sum_{j=1}^N w^{[j]} p({}_o z|\mathrm{x} = \mathrm{x}^{[j]})\right) d_o z. \end{aligned} \tag{22}$$

Next, conditional entropy

$$H({}_o z|\mathrm{x}) = -\int_{{}_o\mathcal{Z}, \mathbf{X}} p({}_o z, \mathrm{x}) \log\left(p({}_o z|\mathrm{x})\right) d\mathrm{x} d_o z \tag{23}$$

can be approximated as

$$\begin{aligned} H({}_o z|\mathrm{x}) &\approx -\int_{{}_o\mathcal{Z}} g({}_o z) d_o z, \\ g({}_o z) &= \sum_{j=1}^N \left\{ \begin{array}{c} w^{[j]} p({}_o z|\mathrm{x} = \mathrm{x}^{[j]}) \\ \cdot \log\left(p({}_o z|\mathrm{x} = \mathrm{x}^{[j]})\right) \end{array} \right\}, \end{aligned} \tag{24}$$

by using the chain rule, $p({}_o z, \mathrm{x}) = p({}_o z|\mathrm{x}) p(\mathrm{x})$, and as analogous to (21), by considering inner integral of (23) as $\mathbb{E}_{\mathbf{X}}[p({}_o z|\mathrm{x}) \log(p({}_o z|\mathrm{x}))]$

The evaluations of (22), (24) can be implemented using numerical quadrature technique. Note that $\{\mathbf{x}_{k+1}^{i,[j]}, w_{k+1}^{i,[j]}\}_{j=1}^N$ which describes the distribution, is the only ingredient for the realization. One can repeat from (20) to (24) for each AOA and bearing angle to complete (7). For in-depth explanations and error analysis of the approximation, readers are referred to [13].

C. Kernel Conditional Density Estimation

Then, calculating (8) now requires probability density of x_k^τ conditioned on some estimate of sensor state $\hat{\mathrm{x}}_k^i$, e.g., MMSE (16).

Consider a situation where θ is partitioned into $\theta = [\theta_1^T, \theta_2^T]^T$, where θ_1 and θ_2 are vectors of appropriate dimensions. Then, conditional probability density $p_{\Theta_1|\Theta_2=\theta_2}(\theta_1)$ can be approximated by using N independent samples $\{(\theta_1^{[j]}, \theta_2^{[j]})\}_{j=1}^N$ drawn from the joint distribution $p_\Theta(\theta)$ and the principle of the kernel conditional density estimation (KCDE) [18], [19] as

$$p_{\Theta_1|\Theta_2=\theta_2}(\theta_1) \approx \frac{\sum_{j=1}^{N} K_2(\theta_2, \theta_2^{[j]}) K_1(\theta_1, \theta_1^{[j]})}{\sum_{j'=1}^{N} K_2(\theta_2, \theta_2^{[j']})} \tag{25}$$

where K_1 and K_2 are the kernel functions that weight each sample in their respective domain based on the Euclidean distance between arguments.

Now, let the samples are drawn from some other proposal distributions and weighted according to the principle of importance sampling, i.e., $\{(\theta_1^{[j]}, \theta_2^{[j]}), w^{[j]}\}_{j=1}^{N}$, then (25) turns into

$$p_{\Theta_1|\Theta_2=\theta_2}(\theta_1) \approx \frac{\sum_{j=1}^{N} w^{[j]} K_2(\theta_2, \theta_2^{[j]}) K_1(\theta_1, \theta_1^{[j]})}{\sum_{j'=1}^{N} w^{[j']} K_2(\theta_2, \theta_2^{[j']})}. \tag{26}$$

The principle directly applies to estimating $p(\mathrm{x}_k^\tau | \mathrm{x}_k^i = \hat{\mathrm{x}}_k^i)$. Substituting dirac delta function for the K_1 and replacing θ with $\mathbf{x}_k^i$ yields

$$\begin{aligned} & p_{\mathbf{x}_k^\tau | \mathbf{x}_k^i = \hat{\mathbf{x}}_k^i}(\mathrm{x}_k^\tau) \\ & \approx \frac{\sum_{j=1}^{N} w_k^{i,[j]} K_2(\hat{\mathrm{x}}_k^i, \mathrm{x}_k^{i,[j]}) \delta(\mathrm{x}_k^\tau - \mathrm{x}_k^{\tau,[j]})}{\sum_{j'=1}^{N} w_k^{i,[j']} K_2(\hat{\mathrm{x}}_k^i, \mathrm{x}_k^{i,[j']})} \\ & = \sum_{j=1}^{N} \hat{w}_k(\mathrm{x}_k^i, w_k^i, j) \delta(\mathrm{x}_k^\tau - \mathrm{x}_k^{\tau,[j]}), \end{aligned} \tag{27}$$

where

$$\hat{w}_k(\mathrm{x}_k^i, w_k^i, j) = \frac{w_k^{i,[j]} K_2(\hat{\mathrm{x}}_k^i, \mathrm{x}_k^{i,[j]})}{\sum_{j'=1}^{N} w_k^{i,[j']} K_2(\hat{\mathrm{x}}_k^i, \mathrm{x}_k^{i,[j']})}. \tag{28}$$

In this way, each particle $\mathbf{x}_k^{i,[j]}$ is projected to a hyper space $\mathrm{x}_k^i = \hat{\mathrm{x}}_k^i$ and reweighted according to (28). Then, $p(\mathrm{x}_k^\tau | \mathrm{x}_k^i = \hat{\mathrm{x}}_k^i)$ is approximated using the particle-weight pairs, $\{\mathrm{x}_k^{\tau,[j]}, \hat{w}_k(\mathrm{x}_k^i, w_k^i, j)\}_{j=1}^{N}$, and application of the same process in Section III-B using this set yields (8).

For K_2, a product of two radial-symmetric Gaussian kernels is utilized: one for spatial domain and the other for azimuthal domain, considering the different scale between the two as

$$\begin{aligned} K_2(\hat{\mathrm{x}}_k^i, \mathrm{x}_k^{i,[j]}) = K_{h_{xy}} & \left(\begin{bmatrix} \hat{x}_j^i \\ \hat{y}_k^i \end{bmatrix} - \begin{bmatrix} x_k^{i,[j]} \\ y_k^{i,[j]} \end{bmatrix} \right) \\ & \cdot K_{h_\psi} \left(\hat{\psi}_k^i - \psi_k^{i,[j]} \right) \end{aligned} \tag{29}$$

Here, K_h for a vector input, $x \in \mathbb{R}^{n_x}$, is a Gaussian kernel, K, of bandwidth $h > 0$,

$$\begin{aligned} K(x) &= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \|x\|^2 \right), \\ K_h(x) &= \frac{1}{h^{n_x}} K\left(\frac{x}{h} \right). \end{aligned} \tag{30}$$

The particles close to the conditioned value automatically contribute more to the resultant conditional distribution while otherwise less weighted.

For the selection of bandwidth, covariance matrix at the moment is utilized as

$$\begin{aligned} h_{xy} &= \sqrt{\sum_{i=1}^{2} C_{ii}^2} \\ h_\psi &= C_{33}, \end{aligned} \tag{31}$$

where

$$\begin{aligned} C &= \mathrm{cov}\left(\mathrm{x}_k^i\right)^{\frac{1}{2}} \\ &\approx \left(\sum_{j=1}^{N} (\mathrm{x}_k^{i,[j]} - \hat{x}_k^i) w_k^{i,[j]} (\mathrm{x}_k^{i,[j]} - \hat{\mathrm{x}}_k^i)^T \right)^{\frac{1}{2}}. \end{aligned} \tag{32}$$

Note that it can efficiently approximate the distribution even without the Gaussian assumption.

IV. Numerical Simulation

In addition to the (6), (8), penalty function based on the minimum separation, d, is considered to keep any pair of vehicles from colliding each other. The proximity penalty for the i^{th} sensor is posed as

$$P(\mathrm{x}_k^i, u_k^i | \mathrm{x}_k^{\bar{i}}, u_k^{\bar{i}}) = \sum_{l = \in \bar{i}} \max\left(0, d - \|\mathrm{x}_{k+1}^i - \mathrm{x}_{k+1}^l\|\right)^2 \tag{33}$$

which should be zero when the constraint is satisfied. Then, when maximizing (6) and (8), the penalty function is added and solved based on iterative algorithm [8].

The $u_j, j \in \{1, 2\}$, based on which (9) is realized, now should be the solution to the optimization problem:

$$\begin{aligned} \underset{u^i \in \mathbb{U}^i}{\text{maximize}} \quad & Q_j\left(p(\mathrm{x}_k^\tau, \mathrm{x}_k^i), u_k^i\right) - \frac{1}{\beta} P(\mathrm{x}_k^i, u_k^i | \mathrm{x}_k^{\bar{i}}, u_k^{\bar{i}}) \\ \text{subject to} \quad & \mathbf{x}_{k+1}^i = \mathbf{f}(\mathbf{x}_k^i, u_k^i) + \mathbf{v}_k, \\ & {}_\tau z_k^i = h_a(\mathrm{x}_k^\tau, \mathrm{x}_k^i) + e_k^\tau, \\ & {}_o z_k^i = \begin{bmatrix} h_a(\vec{0}_{2\times 1}, \mathrm{x}_k^i) \\ h_b(\mathrm{x}_k^i) \end{bmatrix} + e_k^o, \end{aligned} \tag{34}$$

where β is a penalty parameter which should be scaled every iteration as $\beta := \alpha\beta$ until the solution converges. Readers are referred to Algorithm 1 of [13] for the detailed optimization process.

Fig. 2 shows simulation result around 50×50 unitless square. The directions that each sensor is kicked off are evenly spaced since there's no prior information where the target would be located. Solid lines denote the true trajectory following ${u^*}_{1:k}^i$ and dashed lines are the estimates of it. Blue dots represent the target particles sized according to their weights. The arrow-shaped patches are added to the beginning and the end of each trajectory and are aligned to the heading angles. Magenta squares are the history of target estimates.

Initially, the particles are spread uniformly over the target space. As the vehicles advance, target particle distribution is becoming more accurate and less spread out. Until $t = 8s$, vehicles seem to probe target cooperatively without concerning their self-localization accuracy, which is similar to the result shown in [13]. However, bearing angle measurement gets less accurate when the range from the origin and thus around $t = 15s$ the estimates of some vehicles begin to drift. Nevertheless, a tangential maneuver that can enhance the radial observability immediately follows and the vehicles get back on track. The proposed method can *automatically switches* between u_1 and u_2 based on (9).

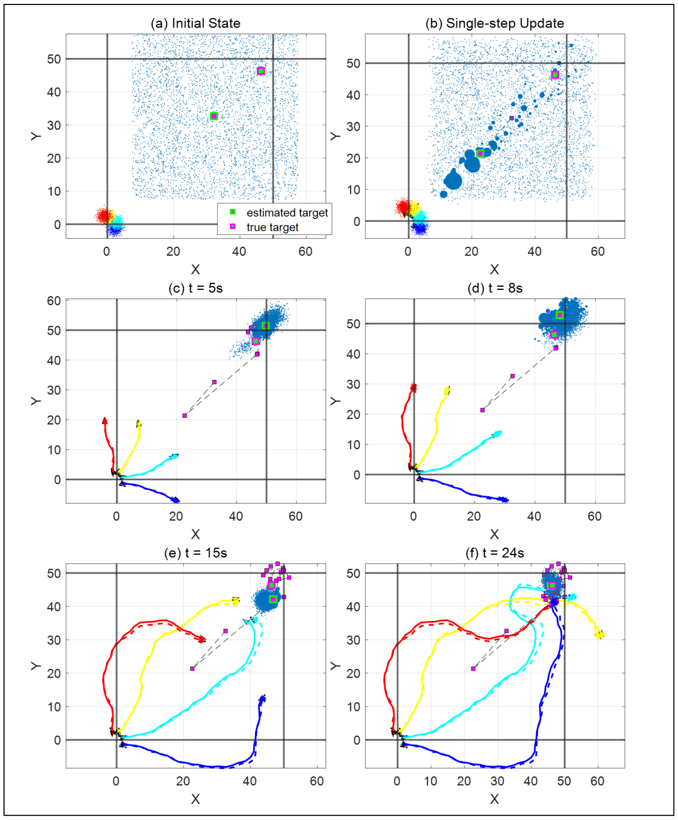

Fig. 2. Simulation with four mobile sensors. Target uncertainty minimizing maneuvers resulted from (8) is dominant, while weaving motions due to (6) are often observed.

V. Conclusion

This study focuses on probing the control of a fleet of mobile sensors which can balance between target ascertaining and suppressing the growth of self-localization uncertainty when both are subject to noisy observations. Information-theoretic method of accomplishing it supported by the particle filter was proposed and the simulation results show that the proposed method can effectively mitigate divergence of either estimate by choosing a more urgent action. As the particle filter is, the proposed approach is not restricted to the Gaussian sensor noise, nor the specific suit of sensors or vehicles. The heterogeneous setup is one of the interesting extensions of this study as long as the dynamic and measurement models along with noise characteristics are known. Moreover, since (6) gets a higher value as the vehicle approaches the origin, another interesting extension will be appending the return of each vehicle to the rendezvous point (the origin) to the scenario, as soon as the target is well localized.

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