# COMPARISON OF DIFFERENT MARKET MAKING STRATEGIES FOR HIGH FREQUENCY TRADERS

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#### **ABSTRACT**

This paper utilizes agent-based simulation to compare different market making strategies for high frequency traders (HFTs). After proposing a model representing HFTs' activities in financial market when they act as market makers, we carry out simulations to explore how different quoting strategies affect their profit. The results show that combination of (i) offering prices based on the latest trading price, and (ii) using the information about market volatility and order imbalance, increase market makers' daily returns. In addition, other scenarios including the competition environment of increased competitors and decreased latencies are incorporated in the model, in order to find out how these factors change the performance of market making strategy.

### 1 INTRODUCTION

Market making refers to a trading strategy that seeks to profit by providing liquidity to other traders and gaining the ask/bid spread, while avoiding accumulating a large net position in a stock. Market making is not a new trading strategy, but the wide spread of high frequency trading (HFT), a form of algorithmic trading which use sophisticated technologies to rapidly trade securities, has given it new features. It is now involved in higher trading speed, and greater trading volume. The Securities and Exchange Commission (SEC) generalized four types of trading strategies that often utilized by HFTs (SEC 2010). Among them, market making is the most transparent one and constitutes more than 60% of HFT volume (Hagstromer and Norden 2013). Menkveld (2013) carefully studies the profit and net position of a large HFT who acts as a modern market maker. But the strategy under the performance of this HFT, as well as the relationship between strategy and market condition, are not described. The extensive usage and considerable profit of this strategy have attracted many scholars (Guilbaud and Pham 2013, Guéant et al. 2013).

The formidable challenge to get to better understanding of trading strategies, is obtaining comprehensive and detailed transaction data and testing the strategy in different market conditions. In this regard, agent-based simulation provides an effective way to solve these problems, and has already been used to design trading strategies.

There have been some approaches focused on marker making and try to find optimal strategies (Nevmyvaka et al. 2003, Kendall and Su 2004, Wang et al. 2013, Wah and Wellman 2015). However, the key difference between the traditional market making and the high frequency market marking, is that the

latter is assumed to gain market information faster, thus submitting and adjusting orders much quicker. Hence, market makers' orders will affect market price, and the competition between market makers becomes intense. Contemplating these facts, this paper differs from the previous papers in the following aspects:

- Previous research considers the trading volume of market marker is small and his order to be executed fully and immediately without market impact. Contrarily, we consider high frequency market makers conducting considerable trading volumes, thus the situation of order execution is quite different and the activity of market makers are likely to affect the whole market.
- Previous research mainly encompasses only one market maker, while in this paper, multiple market makers are included and we test the strategy under competition environments.

In this paper, we utilize an agent-based model to analyze the performance of different market making strategies. Our main contributions are as follows:

- We build an artificial transaction system to represent HFTs' activities in stock market when they act as market makers. This system fits with main statistical properties of financial markets and is used to compare the performance of different market making strategies.
- We find one market making strategy which increases daily return, is offering prices based on the latest trading price, as well as utilizing the information of market volatility and order imbalance.
- We further introduce the environment of increased competitors and decreased latencies in the model, in order to test the strategy under different market conditions.

The rest of the paper proceeds as follows. The next section reviews the relevant literature in HFT and market making. Section 3 describes the design of the transaction system and its validatio. Section 4 presents the results of simulation experiments to compare different kinds of market making strategies for HFTs. In section 5, we further analyze how the number and the trading speed of market makers affect the performance of this strategy, and section 6 concludes.

# 2 RELATED WORK

## 2.1 High Frequency Trading

The rapid development of HFT raises many concerns among scholars on its strategy and profit. For instance, Brogaard (2010) analyzes a dataset of 26 HFT firms to study their strategies and profitability. He finds that HFTs tend to follow a price reversal strategy driven by order imbalances, and provide the best bid and offer quotes for a significant portion in a trading day. In another study, Menkveld (2013) studies a large HFT who has an annualized Sharpe ratio of 9.35, acting as a modern market-maker of a new market, chi-X. The gross profit of the HFT have been decomposed into spread profit and position loss, and the position loss has been decomposed into profit less than five seconds and loss on longer duration positions. In this study, the single HFT firm has an aggressiveness ratio of only 22%. Differently, in other studies, Carrion (2013), as well as Brogaard et al. (2013) analyze NASDAQ Datasets and find that more than 50% of HFT activity is aggressive. Since these HFTs use combination of strategies and are involved in different market conditions, the conclusions may lack of generality. But they can be a great help for the design of agent-based HFT model, which is able to focus on a certain type of strategy and test its performance in different market environments.

Regarding the complexity of HFT, agent-based models have been used to represent HFTs' activity and explore their role on financial market. In this stream of research, Paddrik et al. (2012) propose a zero-intelligence agent-based model of the <u>E-Mini S&P 500 futures market</u>, and suggest that HFTs do contribute to generate the May 6th flash crash. Also, Leal et al. (2014) build an agent-based model to

study how the interplay between low- and high-frequency trading affects asset price dynamics, and find that the presence of high-frequency trading increases market volatility and plays a fundamental role in the generation of flash crashes. In their approaches, HFTs' strategies are relatively simple and they focus on HFT's market impact. On the other hand, our model assumes HFTs adjust their strategy according to the market conditions, and we focus on the performance of different trading strategies.

# 2.2 Market Making Strategy

Market making is considered as a trading strategy which providing limit orders on both sides of the midprice. How these limit orders are placed in terms of volume and distance from the mid-price, are the concerns of this strategy. Relatively, Nevmyvaka et al. (2003) use a simple class of non-predictive trading strategies to test electronic market making, and examine the impact of various parameters on the market maker's performance. Kendall and Su (2004) also use an agent-based model to evolve successful trading strategies by integrating individual learning and social learning. Wang et al. (2013) implement a learning algorithm for market makers to search the optimal trading frequency, and they study how different trading frequencies of market makers affect the market. In another paper, Wah and Wellman (2015) employ simulation based methods to evaluate heuristic strategies for market makers and find the presence of the market maker is benefit to both impatient investors and overall market. However, the high frequency market market market information and conduct their transactions. So in our model, we take into account the huge trading volume of high frequency market makers, and consider how the competition between market makers affects the performance of strategy.

# 3 MODELLING OF HIGH FREQUENCY MARKET MAKING

In this section, we build a continuous double auction stock market in which agents trade using information on the limit order book (LOB), a place records unexecuted limit orders. Agents in the model are classified into two categories according to their goals and strategies. The one is Low Frequency Traders (LFTs), who evaluate the value of the asset and try to earn the profit by using an integrated strategy of fundamental value-based and trend-follow. The other is High Frequency Traders (HFTs), who ignore the value of the asset but only pay attention to the trading environment itself, and they mainly try to accumulate the profit on the spread using the market making strategy. We stimulate the intra-day transaction scenario where both agent categories trade on one single asset. The framework of the model and more details are presented in following subsections.

#### 3.1 Framework

The model is an extension of the one presented by Leal et al. (2014). For the totally T trading sessions, trading procedure is as follows:

- Active LFTs decide whether to enter the market according to their expected returns. If enter, they submit either a sell or a buy order with size and price based on their expectations.
- Knowing the orders submitted by LFTs, HFTs decide whether to enter the market. If enter, they usually submit both a sell and a buy order with size and price in order to absorb the orders of LFTs and earn the profit on the spread.
- LFTs and HFTs' orders are matched and executed according to their price and arrival time. The latest trading price is determined then and unexecuted orders rest in the LOB for the next trading session.
- After each session, LFTs and HFTs decide whether to update their trading parameters according to their performances.

# 3.2 Low Frequency Traders Activity

For each trading session, LFT i acts as following:

- Decides whether to be active according to his active probability LF<sup>i</sup>\_ap.
   LF<sup>i</sup>\_ap is drawn from a uniform distribution with support [α<sup>L</sup><sub>min</sub>, α<sup>L</sup><sub>max</sub>] and can be changed according to individual profit.
- 2. If being active, LFT i first calculates the expected price of the asset LF<sup>i</sup>\_EP based on the expected return LF<sup>i</sup>\_ER, then generates the ask price LF<sup>i</sup>\_AP<sub>t</sub> and bid price LF<sup>i</sup>\_BP<sub>t</sub> at time t based on the latest trading price p<sub>t</sub>.

The return at time t is defined as

$$R_t = \log(\frac{p_t}{p_{t-1}}). \tag{1}$$

Utilizing the idea of LeBaron and Yamamoto (2007), LFTs form their weighted forecasts on the future returns by combining fundamental-, chart-, and noise-based forecasts as follows:

$$LF^{i} _{ER} = n_{1}^{i} \times \log(\frac{p^{f}}{p_{t}}) + n_{2}^{i} \times \frac{1}{l_{i}} \sum_{j=1}^{l_{i}} \log(\frac{p_{t-1}}{p_{t-j-1}}) + n_{3}^{i} \times N(0,1) .$$
 (2)

And the expected price  $LF^{i}$ \_EP is calculated as  $p_{t} \times e^{\wedge} LF^{i}$ \_ER.

 $l_i$  represents the memory length of LFT i, and  $l_i \sim U(1, \, l_{max}).$   $n_1^i, \, n_2^i, \, n_3^i$  are the weights for fundamentalist, chartist, and noise-induced components for LFT i, respectively, and randomly assigned according to normal distributions:  $n_1^i \sim |N(0, \, \sigma_1)|, \, n_2^i \sim N(0, \, \sigma_2)$  and  $n_3^i \sim N(0, \, \sigma_3)$ .

And price for sell/buy orders at time t are formed as follows:

$$LF^{i}_{-}AP_{t} = p_{t-1} \times (1 - \kappa_{i}^{L})$$

$$LF^{i}_{-}BP_{t} = p_{t-1} \times (1 + \kappa_{i}^{L})$$
(3)

Where  $\kappa_i \sim U(\kappa_{min}, \kappa_{max})$  represents the price fluctuation parameter.

3. If the ask price of LFT i is higher than its expected future price, LFT i will submit a sell order at price LF<sup>i</sup>\_AP<sub>t</sub> with size LF<sup>i</sup>\_AS; if the bid price of LFT i is lower than its expected future price, he will submit a buy order at price LF<sup>i</sup>\_BP<sub>t</sub> with size LF<sup>i</sup>\_BS. The valid time of the order is γ<sup>L</sup>. The size of the orders are proportional to expected returns and are formed as:

$$LF^{i} \_AS = LF^{i} \_ER \times \eta_{i}^{L}$$

$$LF^{i} \_BS = -LF^{i} \_ER \times \eta_{i}^{L}$$
(4)

where  $\eta_i^L$  represents the size fluctuation parameter of LFTs and  $\eta_i^L \sim U(\eta_{min}^L, \eta_{max}^L)$ , it can be changed according to individual profit. And if the order has not been fully executed within  $\gamma^L$ , the rest of it will be removed from the LOB.

4. After  $\tau$  sessions, LFT i decides whether to update his trading parameters based on his profit LF<sup>i</sup> P.

If  $LF^i$   $P_t > 0$ , LFT i will update some of his parameters as:

$$LF^{i}$$
\_ap ~  $U(LF^{i}$ \_ap,  $\alpha_{max}$ ),  $\eta^{L}_{i}$  ~  $U(\eta^{L}_{i}$ ,  $\eta^{L}_{max}$ )

If LF<sup>i</sup>  $P_t \le 0$ , it becomes:

 $LF_{\_ap}^{i} \sim U(\alpha_{min} \ , LF_{\_ap}^{i}), \ \eta_{\scriptscriptstyle I}^{\scriptscriptstyle L} \sim U(\eta_{\scriptscriptstyle min}^{\scriptscriptstyle L} \ , \ \eta_{\scriptscriptstyle i}^{\scriptscriptstyle L})$  If  $LF_{\_}^{i}P_{\scriptscriptstyle t} \leq 0$  and a random number  $\sim U[0,1] < \lambda$ , then the component-weighted parameters and memory length will be renewed based on the distributions:  $n_{\scriptscriptstyle 1}^{i} \sim |N(0,\sigma_{\scriptscriptstyle 1})|, \ n_{\scriptscriptstyle 2}^{i} \sim N(0,\sigma_{\scriptscriptstyle 2}), \ n_{\scriptscriptstyle 3}^{i} \sim N(0,\sigma_{\scriptscriptstyle 3}) \ and \ l_{\scriptscriptstyle i} \sim U(1,\ l_{\scriptscriptstyle max}).$ 

## 3.3 High Frequency Market Makers Activity

Concerning market making strategies of high frequency traders, their activities are quite different from the original model since they submit both ask and bid orders. Each trader aiming to profit from the ask/bid spread and needs to control his inventory. For each trading session, HFT j acts as follows:

1. HFT j decides whether to be active based on the price fluctuation  $P_t^{\text{flu}}(bps)$  at time t and his action threshold  $HF^j$  at.

Since evidences suggest HFT activities prefer higher volatility. We calculate

$$P_{t}^{flu} = \left(\frac{p_{t-1} - p_{t-2}}{p_{t-1}}\right) \mid \times 10000.$$
 (5)

And  $HF^j\_at \sim U(\alpha^H_{min}$ ,  $\alpha^H_{max})$ . If  $P_t^{nu} \ge HF^j\_at$ , then HFT j becomes active.

- 2. If being active, HFT j submits both a sell order at price  $HF^i\_AP$  with size  $HF^i\_AS$  and a buy order at price  $HF^i\_BP$  with size  $HF^i\_BS$ . All orders from HFTs are submitted in a random order, and the valid time of the orders is  $\gamma^H$ . Under the default setting,  $HF^i\_AP = P_t + \kappa_i^H$ ,  $HF^i\_BP = P_t \kappa_i^H$ ,  $\kappa^H$  refers to price fluctuation. While HFTs decide the order quantity based on the quotes in the LOB.  $HF^i\_AS = HF^i\_BS = 0.5$   $\times (q_b + q_s) \times \eta^H$ . Where  $q_b(q_s)$  refers to the total size of buy (sell) orders in the LOB at this session, and  $\eta^H$  refers to order absorption rate.
- 3. Like LFTs, after  $\tau$  sessions, HFT j decides whether to update  $\eta^H$  based on his performance.

### 3.4 Model Validation

Table 1 lists the default value of all the parameters. The number of sessions is set as 400 for intra-day trading. There are 400 traders, and 2% (8) of them are HFTs (Hagstromer and Norden 2013). Price and volume related parameters are calibrated to fit with market volatility and liquidity condition respectively, while keeping the diversity of agents.

Table 1: Parameters in initial simulation.

Description	Symbol	Value
Number of trading sessions	T	400
Number of traders	N	400
Fundamental value	$p_{\mathrm{f}}$	50
Tick size	$t_{s}$	0.01
LFT initial active possibility	$[lpha^{ m L}_{ m min},lpha^{ m L}_{ m max}]$	[0.01, 0.1]
LFT max memory length	$l_{max}$	30
LFT order price fluctuation	$\left[\kappa^{\mathrm{L}}_{\mathrm{min}},\kappa^{\mathrm{L}}_{\mathrm{max}}\right]$	[-0.002, 0.01]
LFT order size fluctuation	$[\eta^{ ext{L}}_{ ext{min}},\eta^{ ext{L}}_{ ext{max}}]$	[200, 1000]
LFT order life	$\gamma^{ m L}$	10
LFT parameter evolution circle	τ	30
LFT parameter evolution rate	λ	0.3
Std of fundamental component	$\sigma_1$	0.3
Std of chartists component	$\sigma_2$	0.6
Std of noise-trader component	$\sigma_3$	0.1
HFT percentage	HFT <sub>per</sub>	2%
HFT active threshold	$[\alpha^{\rm H}_{\rm min}, \alpha^{\rm H}_{\rm max}]$	[5, 5]
HFT order price fluctuation	$\kappa^{\mathrm{H}}$	0.01
HFT order absorption rate	$[\eta^{H}_{min}, \eta^{H}_{max}]$	[0.1, 0.5]
HFT order life	$\gamma^{\rm H}$	1

We check whether the model is able to account for the main stylized facts of financial markets. The price movement generated by the model is in line with the empirical evidence as absence of autocorrelation, see as Figure 1. In contrast, the autocorrelation function of absolute return display a slow decaying pattern, see as Figure 2. In addition, the existence of fat tails in the distribution of return is shown in Figure 3, which is also fit with observing evidence.

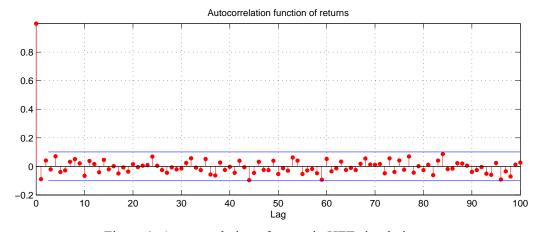


Figure 1: Autocorrelation of return in HFT simulations.

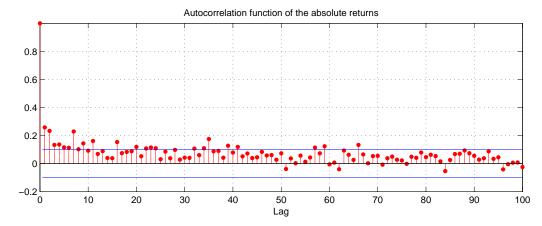


Figure 2: Volatility clustering in HFT simulations.

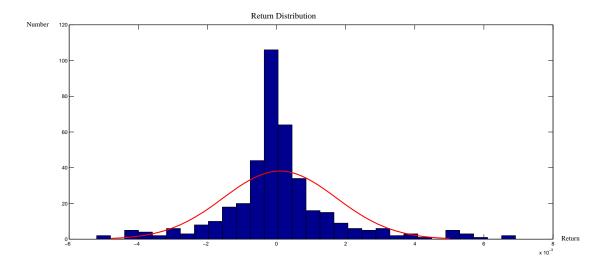


Figure 3: Return distribution in HFT simulations.

# 4 COMPARISON OF MARKET MAKING STRATEGY

In this section, some experiments that compare the performance of different market making strategies are discussed. For more clarification, we mainly discuss the relationship between different quoting prices and daily return.

# 4.1 Strategies considering quoting position and spread

Market makers usually submit their orders in both buy and sell sides in order to earn the spread. A common approach is to submit limit buy/sell orders at the current best bid/ask prices (denoted as  $P_t^b/P_t^a$ ). This implies that the spread position is relatively constant. Hence the <u>mid-quote is taken as the base price</u>. Another alternative is using the <u>latest trading price</u> as the base <u>price</u>. This approach considers <u>trading price</u> in next few sessions may evenly distributed around the latest trading <u>price</u>.

Regarding the ask/bid order spread of market makers, we first look into the situation which taking mid-quote as the base price. If a market maker places his orders at the current best bid/ask prices, denoted as "ask/bid" strategy, then his ask/bid order spread is equal to the spread in the LOB. One can also place his ask (bid) orders at a marginal lower (higher) price than the current ask (bid), in order to get priority in

the order execution. However, spreads in the LOB for a liquid stock are usually extremely tight and leave little profit margin. Therefore, such submission strategy may hit the opposite side of the LOB and in fact bring a loss to the market maker. Moreover, a market maker can place his orders deeper into the LOB, such as one tick away from current best ask/bid price, denoted as "ask/bid-" strategy. This strategy results in higher margins as well as lower order execution rate for market makers.

The previously described mechanisms are expected when a market maker uses the latest trading price as the base price. We assume he places ask/bid quotes one or two ticks away from the base price. These scenarios are denoted as "last" and "last-" strategy respectively. Quoting prices of each strategy are illustrated in Table 2.

Strategy	Ask order price	Bid order price
ask/bid	$\mathbf{P}_{t}^{a}$	$\mathbf{P}_{t}^{b}$
ask/bid-	$P_t^a + t_s$	$P_{t}^{b}+t_{s}$
last	$P_t + t_s$	$P_t$ - $t_s$
last-	$P_t + t_s \times 2$	$P_t$ - $t_s \times 2$

Table 2: Quoting price of market making strategies(1).

In addition to quoting strategies, 2% of the traders are considered to be HFTs. Moreover, all of those traders use a same quoting strategy, selected from the Table 2, and their orders are submitted in a random order. We test the performance of these strategies evaluating the daily return and end-of-day inventory. (end-of-day inventory for strategy "last" is seen as 1.) Each strategy is simulated for 200 times, and the average return and end-of-day inventory are calculated. The comparative results are shown in Figure 4.



Figure 4: Comparison of market making strategies(1).

Figure 4 shows that quoting based on the <u>latest trading price helps to gain more daily return than</u> quoting based on mid-quote price. This strategy is also likely to result in higher end-of-day inventory.

## 4.2 Strategies considering volatility and order imbalance

The prior market making strategy can work well when the trajectory of the price is similar to a random walk, but it may meet with difficulties if things happen in another way. For example, if there comes the

period that price keeps going up, the market maker can hardly get his bid orders executed. Hence he accumulates a large short position and loses a lot of money. So the information of market volatility and the order book imbalance need to be considered to avoid such situation.

According to empirical study (Raman et al. 2014), market makers are likely to reduce their participation and their liquidity provision in periods of significantly high volatility. In this case, we assume the market maker will place his orders deeper from standard situation, denoted as strategy "v".

Moreover, market makers tend to adjust their quoting price when observing an order imbalance in the LOB (Hendershott and Menkveld 2010). When the difference between the volume of ask quotes and bid quotes becomes significant in the LOB, i.e. when  $(\underline{q} - \underline{b} - \underline{q} \underline{s})/(\underline{q} - \underline{b} + \underline{q} \underline{s}) > \lambda_1$ , market makers expect an uptrend in the price movement and adjust their ask/bid quotes with an increment  $\lambda_2$ . They can also lower the ask/bid quotes when observing there are much more sells than buys. We set  $\lambda_1$  equals to 0.5 and  $\lambda_2$  equals to  $t_s$ , and name this order imbalance concerned strategy as "im".

We assume a market maker can use either one or the combination of these strategies. The quoting prices of these strategies are listed in Table 3, and the "last" strategy is taken as the baseline.

Strategy	Ask order price	Bid order price
last	$P_t + t_s$	$P_t$ - $t_s$
last+v	$P_{t} + ( P_{t} - P_{t-1}  + 1)t_{s}$	$P_{t} - ( P_{t} - P_{t-1}  + 1)t_{s}$
	$P_t + t_s \times 2$ , if $(q_b - q_s)/(q_b + q_s) > 0.5$	$P_t$ , if $(q_b - q_s)/(q_b + q_s) > 0.5$
last+im	$P_t$ , if $(q_s - q_b)/(q_b + q_s) > 0.5$	$P_{t}$ - $t_{s} \times 2$ , if $(q_{s} - q_{b})/(q_{b} + q_{s}) > 0.5$
	$P_t + t_s$ , if others	$P_t$ - $t_s$ , if others
	$P_t + ( P_t - P_{t-1}  + 2)t_s$ , if $(q_b - q_s)/(q_b + q_s) > 0.5$	$  P_t -   P_t - P_{t-1}   t_s,$ if $(q_b - q_s)/(q_b + q_s) >$
last+im+v	$P_t +  P_t - P_{t-1} t_s$ , if $(q_s - q_b)/(q_b + q_s) > 0.5$	0.5
	$P_{t} + ( P_{t}-P_{t-1} +1)t_{s}$ , if others	$P_t - ( P_t - P_{t-1}  + 2)t_s$ , if $(q_s - q_b)/(q_b + q_s) > 0$
		0.5
		$P_{t} - ( P_{t}-P_{t-1} +1)t_{s}$ , if others

Table 3: quoting price of market making strategies(2).

After simulating each strategy for 200 times, the average daily return and average inventory are calculated. The results are shown in Figure 5.



Figure 5: Comparison of market making strategies(2).

From Figure 5, we see that the strategy concerning the order book imbalance increases market makers' daily return and end-of-day inventory. In addition, adjustment of the ask/bid spread according to market volatility helps to increase return and decrease inventory. Combining these two strategies together can reach the highest daily return.

#### 5 EXPERIMENTS ON COMPETITION

In the following simulations, the percentage of HFTs (2% in previous experiments) is adjusted to see how the total number of HFTs affect the performance of strategy. On the other hand, in past experiments, all HFTs submit their orders in a random order, which means they have similar latencies. Considering trading speed plays an important role in the order execution of market makers, we arrange different latencies for HFTs. In this scenario HFTs submit their orders one after another in a fixed order, so a HFT with lower latency submits its orders quicker and is likely to have higher order execution probabilities.

We discuss how HFTs' profits will be affected by increased competitors and decreased latencies. In the experiments, all HFTs using the "last+im+v" strategy, but different HFTs have different order submitting latencies. We adjust the number of HFTs, and run 200 simulations for each different HFT percentage (0.5%, 1%, 1.5%, ... 5%). The return differences among LFTs, normal HFTs and the fastest HFT are illustrated in Figure 6.

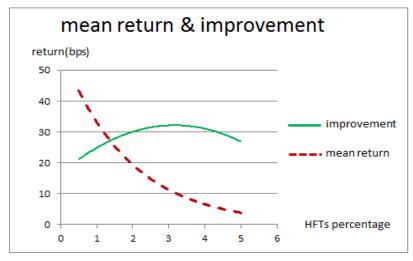


Figure 6: Return in competition environment.

In Figure 6, the dash line shows the average return of HFTs. It decreases with the increase of HFTs percentage and can be seen as the return of a normal HFT. The solid line, on the other hand, is calculated as the difference between the average and the highest return (return of the HFT with the lowest latency). This chart suggests that trading speed plays an important role in the performance of this market making strategy. If a market maker is faster than others, he is likely to earn much more than the average. On the contrary, when the number of market maker becomes larger, losing the competition of speed may bring one zero or even negative return.

#### 6 CONCLUSIONS

This paper focuses on comparing the performance of different market making strategies for high frequency traders. We find one market making strategy which offering prices based on the latest trading price, adjusting ask/bid spread according to market volatility, as well as adopting a trend-follow strategy

<u>driven by order imbalance</u>, helps to earn more profits than others. In addition, simulation results show that <u>trading speed plays an important role in this strategy</u>, especially when the number of HFTs increases.

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