Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2218714

A New Approach of Portfolio Investment

Summary

Everyone wants to make a fortune in the financial market, which is why there are so many actuaries, investment institutions, and even finance experts and professors. We have studied many ways to make more money in the financial market.

Methods are never perfect. With the idea of coming up with a more complete theory, we built the following models to make **portfolio investment** based on the historical values of gold and bitcoin:

Model I: Gold Price Prediction Model Based on BP Neural Network; Model II: Bitcoin Price Prediction Model Based on Grey System; Model III: Trading Decision Model Based on the Exhaustive Algorithm, etc.

Before building time series forecasting models, we first **normalize** the data to be studied. For normalized values, we can do better analysis. In addition, we used **Graphical** methods to make the results more intuitive.

For Model I: Based on references and our own experimental results, we find that traditional planning methods are not accurate for gold price forecasts, and do not meet the needs of these methods based on the amount of data provided. And because the neural network model is very developed now, and it is very suitable for the prediction of **nonlinear sequences**. We have established a gold price prediction model based on the **BP neural network** time series prediction model. Results and rationality analysis are discussed in detail in Section 5.3 .

For Model II: According to various references and our verification, we found that the neural network-based model is not **accurate** enough for gold price prediction, and is not suitable for the Bitcoin market that fluctuates rapidly in a short period of time, especially in the case of insufficient data. It has been verified that the **time series prediction model** based on the **grey system** has higher accuracy in the short term when predicting the price of Bitcoin, which is more in line with the characteristics of Bitcoin. Results and plausibility analysis are discussed in detail in Section 6.3.

For Model III: We independently created the Trading Decision Model Based on the **Exhaustive Algorithm**, based on the predicted values in the model I and model II, and **VaR** theoretical risk analysis. We took the acceptable buy-sell ratio and the VaR value as **input** to the algorithm, each type of input corresponds to its own unique rate of return. Finally, we divide different types of input into aggressive, ordinary and moderate, and calculate their rate of return as 20% to 120%, about 60% and about 60%.

Furthermore, since the above three models are also applicable to any abstract financial market, our model is highly **adaptable**. It can also be easily used in the case of other portfolio investments.

Finally, **sensitivity analysis** is used to demonstrate that our model is the best. Afterward, we write a memorandum for traders to introduce our model.

Keywords: portfolio investment, data normalization, time series prediction model, BP neural network, grey system, Exhaustive Algorithm, VaR

Contents

1 Introduction

1.1 Background

Investment is an inescapable aspect of every modern person's life. We know that the biggest difference between modern society and ancient society is that everyone has great economic freedom - that is to say, private property is sacrosanct, which is guaranteed by law. Our quality of life, social status, etc. are greatly related to everyone's economic ability.

For a long time, **gold** has attracted much attention as a measure of value, a means of circulation, a way of payment, and a world currency. Gold is almost the oldest, most value-preserving, and most important means of payment and financial management in existence. Since gold itself has utility as a metal and the stock is limited, it is almost impossible to have a great loss of value. Therefore, investing in gold is a very effective method of risk hedging.

From a global perspective, digital cryptocurrencies, both in terms of type and market value, have experienced substantial growth in the past ten years, gradually forming an influential digital cryptocurrency market. **Bitcoin** is the leader in cryptocurrencies, and its 24-hour tradability and drastic changes in value make Bitcoin extremely attractive for investment, but at the same time, it is accompanied by great risks.

Based on the above points, the research on gold and bitcoin is of great significance to anyone who needs to invest.

1.2 Restatement of the Problem

Through in-depth analysis and research on the background of the problem, combined with the specific constraints given, the restate of the problem can be expressed as follows:

• Develop a model that provides the best daily trading strategy based only on price data up to that day. Then answer the question that how much is the $1000 initial investment worth on 09/10/2021 using the model and strategy?

• Prove that the model built provides the best strategy.

• Detect the sensitivity of trading strategy to transaction costs. How will transaction costs affect the strategy as well as results?

• Teach the strategies, models, and results to the trader in a memorandum of up to two pages.

1.3 Literature Review

This topic is a portfolio investment of gold and bitcoin on the basis of only focusing on their respective past prices. Generally speaking, it can be divided into two parts, the first is to **predict the price trend of gold and bitcoin**, and the second is to **invest in a combination** of these two investment methods. This section mainly discusses the models that have been proposed.

•**First of all**, to predict the price of gold:

Wang Chengbiao (2007) [1] believes that the price of gold is affected by many factors such as the US dollar exchange rate, world economic situation, inflation, international situation, international trade, crude oil price, foreign exchange policy, stock market, fiscal deficit, turmoil, war, etc. Most of these elements are ambiguous.

Wang Zhongxiang (2009) [2] pointed out that the gold futures market is an extremely complex nonlinear dynamic system, and established a BP neural network model to predict the price of gold futures.

Li Jingyang (2017) [3] pointed out that for a large number of nonlinear and non-stationary complex dynamic system problems in reality, it is difficult to determine the appropriate model order in the application of the nonlinear ARIMA model, which is difficult to solve in traditional prediction methods. In the case of ineffectiveness of traditional time series forecasting methods, applying intelligent prediction methods such as neural networks can often be more effective.

Cheng Ming (2020) [4] believes that non-statistical methods are powerful tools for predicting nonlinear time series. Both artificial neural networks and grey system theory are non-statistical methods widely used to predict nonlinear time series.

•**Secondly**, to forecast the price of the bitcoin:

According to the existing research, the accuracy of predicting Bitcoin price by neural network method is about 50% [5] [6] [7].

Jalali (2020) adopts a first-order grey model (GM(1,1)). It uses first-order differential equations to model trends in time series. The results show that the GM(1,1) model is able to accurately predict the price of Bitcoin with a confidence level of about 98% maximum profit by choosing the appropriate time frame and managing investment assets. The proposed method is compared with RNN and BNN to demonstrate the accuracy and robustness of the method proposed in this paper. The comparison results confirm that the grey system theory is superior to RNN and BNN [8].

•**Finally**, in terms of portfolio investment:

Baur and Lucey (2010) [9] defined when distinguishing gold hedging utility, risk diversification utility and hedging utility: if an asset has a significant negative correlation with another asset in the long-term, it can be considered that the asset has a strong risk hedging effect; if an asset has no obvious correlation with another asset in the long-term, it can be considered that the asset has a weak risk hedging effect; if an asset has a When there is a weak positive correlation in the long-term dimension, the asset can be considered to have a risk diversification effect.

Bouri et al. (2017) [10] used quantile regression to analyze the relationship between digital cryptocurrency and global uncertainty. The results show that digital cryptocurrency assets can hedge against global uncertainty, but only in a short period of time. The investment period is valid, and the effect is not obvious in the bear market.

Eisl et al. (2015) [11] studied the role of Bitcoin in spreading the risk of investment portfolios, improved the mean-variance model, and concluded that adding Bitcoin to a portfolio can improve the risk-reward ratio of the portfolio, it has the value of diversifying risk.

1.4 Our Work

This question requires us to determine an optimal investment scenario based solely on the historical prices of gold and bitcoin. Our work mainly includes the following aspects:

1) Based on the gold price data, we established a bp neural network time series prediction model;

2) Based on Bitcoin price data, we established a time series prediction model based on grey system;

3) We established a trading decision model based on the programming exhaustive algorithm, this paper effectively proves the effectiveness and applicability of the method.

In order to avoid complicated descriptions and intuitively reflect our workflow, the flowchart is shown in the figure:

2 Assumptions and Justifications

Considering that practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation:

Assumption 1: When trading on the Nth day, the gold and Bitcoin prices until the Nth day are known, and after this price is bought, the profit is calculated on the N+1th day.

Explanation: In the given data, there is only one price value for each date, so we need to determine the cut-off point between the two dates. This processing method is equivalent to taking the last moment of the price change on that day as the cut-off point.

Assumption 2: Assume that the respective transaction rates of gold and bitcoin are charged on the same day, and income is generated from the next day.

Explanation: Since the given conditions do not have a settlement time for a given transaction rate, we set it to settle immediately upon purchase. Make sure that this settlement time does not contradict the first assumption.

Assumption 3: There is no fixed annual interest rate increase for holding cash.

Explanation: We make this assumption because it simplifies the calculation and hardly affects the results. Because the interest rate gains are very small compared to changes in the price of bitcoin and gold.

Assumption 4: There is no minimum transaction amount limit, the gold and bitcoin purchased do not necessarily need to be in integers, and any small amount of investment can be made.

Explanation: Since the principal of $1,000 is too small, it is difficult to buy gold and bitcoin in integer multiples. At the same time, if we can make an arbitrarily small investment, it is also convenient for the algorithm.

Additional assumptions are made to simplify analysis for individual sections. These assumptions will be discussed at the appropriate locations.

3 Notations

The primary notations used in this paper are listed in Table 1.

Table 1:Notations

Symbol	Description
p_{gi}	Gold price on day i
p_{bi}	Bitcoin price on day i
fp_{gi}	Gold price forecast for day i
fp_{bi}	Bitcoin price forecast for day i

4 data preprocessing

4.1 Missing Values Handling

In the given table **LBMA-GOLD.csv**, some values are missing. Since we believe that the price of gold on a certain day is closely related to the price of the days before and after it, it is a random loss (**MAR, Missing at Random**). Random loss means the probability of missing data is not related to the missing data itself, but only to some of the observed data. That is, the absence of data is not completely random, and the absence of such data depends on other complete variables.

Here, since the daily gold price changes based on the previous day's price, and the next day's gold price changes on the basis of the current day's price, we believe that the missing data and the adjacent points satisfy a certain **Linear fitting relationship,** seen in the equation (1).

$$p_{gi} = \frac{1}{2}(p_{g(i+1)} + p_{g(i-1)}) \tag{1}$$

Where p_{gi} represents the gold price on day i.

4.2 Data Normalization

Because in the bp neural network time series prediction model, the direct use of the original data will cause the function to converge very slowly; at the same time, no matter in the bp neural network time series prediction model or the time series prediction model based on the grey system, the direct use of the original data will lead to a decrease in the **prediction accuracy**. Therefore, it is necessary to normalize the data.

The first advantage of normalization is that it can map data to a **specified range** for processing, which is more convenient. The second benefit of normalization is that it can turn a dimensional expression into a **dimensionless** expression, so that indicators of different units or magnitudes can be compared and weighed. After normalization, the dimensional data set is turned into a pure digital quantity, which can also simplify the calculation.

Normalization is essentially a **linear transformation**. The linear transformation has many good properties that determine that it will not cause "failure" after changing the data, but can improve the performance of the data. These properties are the premise of normalization.

Data normalization refers to mapping data to a specified range, such as mapping data to a range of zero to one or minus one to one for processing, the transformation equation we use is listed in equation (2).

$$y=\frac{y_{max}-y_{min}}{x_{max}-x_{min}}\bullet(x-x_{min})+y_{min} \tag{2}$$

When the output value is obtained, the predictor variable needs to be treated with an inverse normalization method, as shown in equation (3)

$$x=\frac{(y-y_{min})(x_{max}-x_{min})}{y_{max}-y_{min}}+x_{min} \tag{3}$$

Where x and y represent the mapped value and the result after mapping, y_{max} and y_{min} indicate the upper and the lower bound of the mapping, x_{max} and x_{min} express the maximum and the minimum value among all x.

The normalization range we finally choose is **zero to one**, and normalize the price of gold and the price of bitcoin respectively.

In the same plane rectangular coordinate system, draw the image of the price of gold and the price of bitcoin over time, Shown in Figure 1.

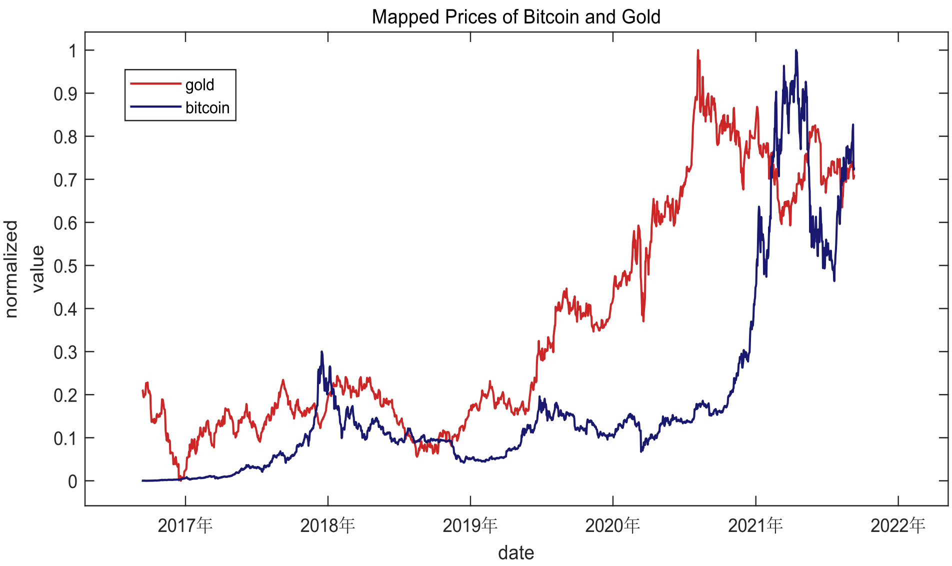

Figure 1 :mapped prices of bitcoin and gold

5 Gold Price Prediction Model Based on BP Neural Network

5.1 Review of Models that Forecast Gold Prices

At present, the traditional methods for forecasting the gold futures market mainly include **investment analysis** and **time series analysis**. The former makes predictions based on theory and cannot be quantified; while the latter if the traditional model mentioned above is used, relies too much on the linear relationship between gold prices and has limitations.

There are few empirical analyses of gold futures in the existing literature, and most of the analysis of gold futures price is realized by linear analysis, which is difficult to analyze with econometric models.

Cheng Ming (2020) pointed out that when using the **ARIMA model** or other traditional time series forecasting models such as the **arch model**, it is usually necessary to comprehensively consider the influence of factors such as **other currencies** in order to obtain satisfactory results.

If we use the ARIMA model, or model such as arch to directly perform time series forecasting. With the increase of forecast days, the relative error shows an increasing trend, which is only suitable for **short-term** forecasting, and the long-term forecasting effect is not ideal.

The **backpropagation neural network** is one of the most widely used neural network models at present. It has strong **nonlinear mapping** ability and good fault tolerance ability and can pass through the adaptive learning process. BP neural network memorizes the content in the weights of the network and applies the learning results to new values, which is especially suitable for solving problems with complex internal mechanisms. Therefore, on the basis of analyzing the time series of gold prices, we established a BP neural network model to predict gold futures prices.

5.2 The Establishment of BP Neural Network Model

5.2.1 The Working Principle of BP Neural Network

Generally speaking, BP neural network includes three or more **layers** of input layer, hidden layer and output layer.

Each layer of the structure has multiple **neurons**, and the neurons in different layers are connected by weights. The information between neurons in different layers cannot flow freely, but must be transmitted to the neurons in the next layer.

The input layer transmits the received external input information to the hidden layer; the hidden layer transmits the information to the output layer through the weight and activation function between neurons; the output layer outputs the final result.

The specific three-layer neural network transmission process is shown in Figure 2.

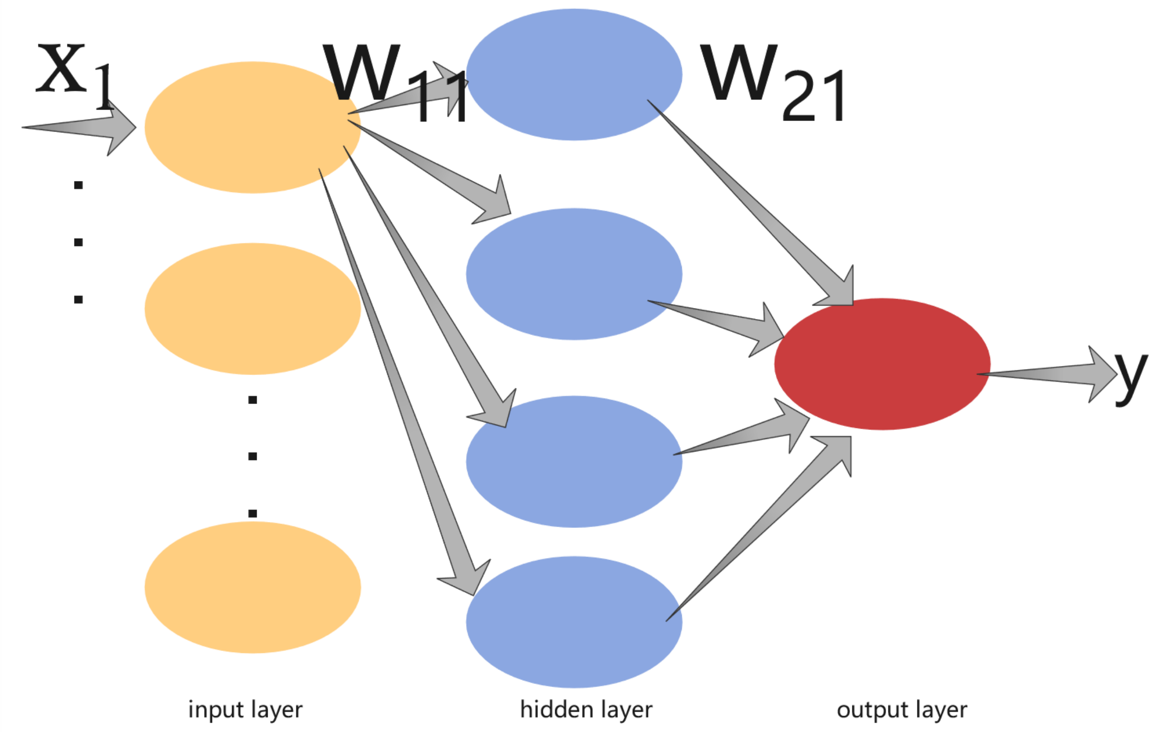

Figure 2: principles of bp neural network

In the figure, x is the input value of the neural network, y is the output value of the neural network, w is the weight from the input layer to the hidden layer and the weight from the hidden layer to the output layer, b is the bias from the input layer to the hidden layer, h is the bias from the hidden layer to the output layer.

5.2.2 Algorithms Used for BP Neural Network Operation

The algorithm of the BP neural network is divided into the following parts.

The first is the **forward** propagation of information, from the input layer to the hidden layer and finally to the output layer.

The second is the **backward** propagation of information. When the error between the output result and the real value exceeds the given error interval, the weights and thresholds of the model are adjusted by backward propagation, so that the error function presents a gradient descent trend.

Finally, after repeated **corrections**, the error is controlled within the set range, and obtain the weight corresponds to the minimum mean square error.

The specific neural network learning process is shown in Figure 3.

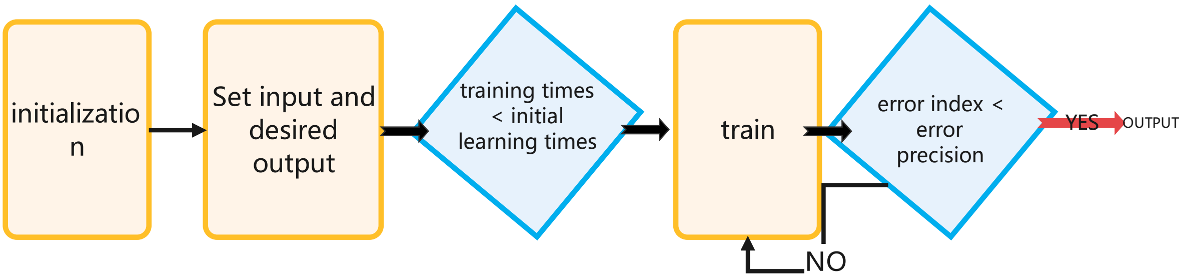

Figure 3: neural network training

Determination of activation function: Hyperbolic tangent sigmoid transfer function and Linear transfer function. Transfer functions with n representing the input signal and an as the output

5.2.3 BP Neural Network Model for Time Series Forecasting

In a time series model, the value of the next moment depends on the value in the historical time period, and it is worth thinking about how long the historical time period is to be the best model. The optimal autoregressive **order** (ie, the length of the time period) can be determined by comparing the errors of the BP time series neural network models trained with time periods of different lengths.

Considering that if the order is too long, the prediction accuracy will decrease, and if the order is too short, it is not suitable for the characteristics of long-term investment in gold. Combining two factors, the final decision is to choose a model with an order of 20.

The error values of different order models are shown in Table 1.

Table 2:error values of different order

Order	SSE	MAE	MSE	RMSE	MAPE	R
5	0.012356	0.011536	0.00024912	0.015783	1.6242%	0.80194
20	0.014072	0.013195	0.00028144	0.016776	1.8611%	0.78124
50	0.014486	0.01341	0.00028972	0.017021	1.8898%	0.76791

In the BP neural network time series model based on historical values, the predicted value at a certain moment depends on the historical values in all previous periods. It is explained here that in the process of training the neural network, the input x and output y data of the training set should be actual values. When the BP network model is obtained for testing, the input value in the model is still the actual historical value.

This is because, from the perspective of building a time series model, an assumption is made that future values depend on historical values; from the perspective of neural network identification, only when the input feature value given to the neural network is real, the output predicted value will be consistent with the actual situation. If you need to predict future values, you can only use this method to predict future trends. However, when predicting values in the future, the historical values in the previous period can only be derived from the predicted values, which will generate cumulative errors. The accumulated errors lead to predictions The values are erratic and appear as an image of an oscillating trend line.

5.3 Results and Analysis

5.3.1 Visually Analyze Results Based on Figures

From Figure 4 and Figure 5, the accuracy of the prediction results of the bp neural network can be intuitively analyzed.

The prediction error e_{bi} is defined as follows

$$e_{gi}=p_{gi}-fp_{gi} \tag{4}$$

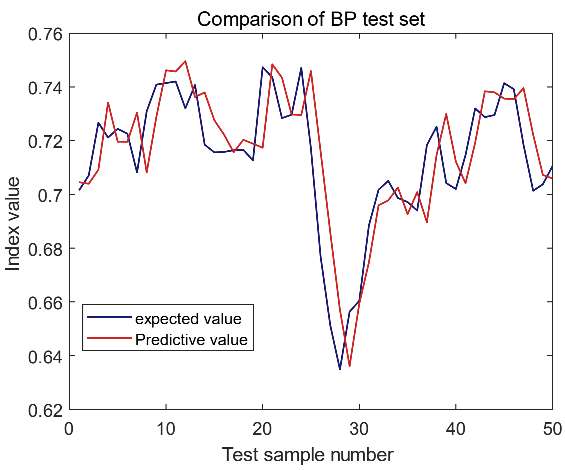

Figure 4: comparison of bp test set

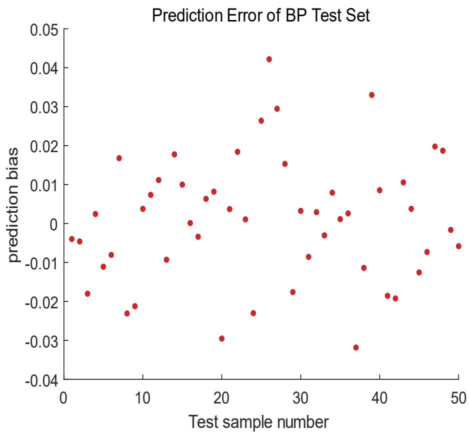

Figure 5: prediction error of bp test set

As can be seen from Figure 4, the predicted value (red line) is delayed compared with the actual value (blue line).

This is an unavoidable result because we only use the given data, i.e. historical values, to build a time series forecasting model for forecasting financial markets.

Because of the elements in the financial market, such as the dollar and other currencies, stocks, oil, as well as gold and bitcoin mentioned in this article, their price changes, the influencing factors are very complex and multi-faceted.

Starting from the historical price alone, it is only possible to predict the trend of a period of time in the future without special changes in the short term. For the prediction of sudden points in the image, a single time series prediction model is powerless.

As can be seen from Figure 5, the prediction effect of about half of the points is quite good, and only a very small number of points with large errors exceed two percent error.

Next, we use several statistical calculation methods to determine the degree of error.

5.3.2 Statistical Calculation Methods to Determine the Error

Note: In the following formula, in order to ensure universality, use y_i to represent the true value, and $\hat{y}_i$ to represent the predicted value

SSE（Sum of Squares due to Error）

This statistical parameter calculates the sum of squares of the errors between the fitted data and the corresponding points of the original data, the calculation method is listed in formula (5)

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \tag{5}$$

The closer the SSE is to 0, the better the model selection and fitting, and the more successful the data prediction.

The following **MSE** and **RMSE** are similar to SSE, so the judgment method is the same, the calculation methods are listed in Equation (6) and Equation (7) respectively.

$$MSE = \frac{SSE}{n} = \frac{1}{n}\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \tag{6}$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \tag{7}$$

MSE is used by many algorithms because it is fast to compute and easier to operate than RMSE. But it does not take into account the original error value (because the error is squared), which may cause the indicator to fail to correlate to the size range of the original error value.

MAE（Mean Absolute Error）

In the above formula, in order to avoid the positive and negative offset of the error, the square of the difference is calculated. There is also a formula that can do the same thing, which is to calculate the absolute value of the difference:

The formula is as follows in Equation (8):

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i| \tag{8}$$

The disadvantage of MAE is that it does not take into account the average of the actual values.

MAPE (Mean Absolute Percentage Error) is one of the most commonly used metrics to evaluate forecast accuracy. MAPE is the sum of each absolute error divided by the actual value. In fact, it is the average of the error percentages.

Its calculation formula is listed in formula (9):

$$MAPE = \frac{1}{n}\sum \frac{|\hat{y}_i - y_i|}{y_i} \tag{9}$$

MAPE divides each error value by the actual value, so there is a skew: if the actual value at a certain moment is very low, and the error is large, it will have a large impact on the value of MAPE. As a result, optimizations to MAPE can sometimes lead to strange predictions, most likely lowering the predicted value than the actual value.

The **R** - coefficient of determination is defined as the ratio of SSR (Sum of squares of the regression) and SST (Total sum of squares), which is used to characterize the quality of a fit through changes in data. The normal value of R ranges from zero to one, and the closer it is to one, the stronger the explanatory power, and the better the model fits the data.

Its calculation method is given in formula (10)

$$R^2=\frac{SSR}{SST}=\frac{SST-SSE}{SST}=1-\frac{SSE}{SST}=1-\frac{\sum_{i=1}^{n}\ (y_i-\hat{y}_i)^2}{\sum_{i=1}^{n}\ (y_i-\bar{y}_i)^2} \tag{10}$$

The error values of the model are as follows in Table 2:

Table 3:error values of this bp neural network

Order	SSE	MAE	MSE	RMSE	MAPE	R
20	0.014072	0.013195	0.00028144	0.016776	1.8611%	0.78124

From the statistical analysis of the error of the predicted data, the error of this model is within the acceptable range, and it can basically meet the fitting requirements.

6 Bitcoin Price Prediction Model Based on Grey System

6.1 Reasons to Choose Grey Systems Theory for Prediction

Due to the volatile nature of cryptocurrencies, and based on previous research we can know that Bitcoin price depends on many uncertain factors, predicting the price is difficult.

According to the research of Jalali (2020), the accuracy of predicting Bitcoin price through neural network models is about 50%, and the use of grey system theory can greatly improve the **accuracy**. Using grey system theory, a first-order grey model (GM (1,1)) was used to simulate the trend of time series using first-order differential equations. He stated in the paper that the results show that the GM (1,1) model can accurately predict the price of Bitcoin, and by choosing the appropriate time frame and managing investment assets, a confidence level of about 98% maximum **profit** can be obtained.

The purpose of establishing this model is to use the grey system theory to predict the price of Bitcoin and its changes.

6.2 Grey System Theory and GM (1,1) Model

6.2.1 Introduction of Grey System Theory and GM (1,1) Model

Grey system theory focuses on **small sample size and little information**, and is classified according to the "color" of the system. Grey indicates partially known information

The GM (n,m) model is a grey prediction model, where n represents the order of the differential equations used in the model, and m represents the number of variables. The GM (1,1) model is a classic grey forecasting model.

The main reasons for using the GM (1,1) model are its **simplicity** of modeling, **implementation** of the model, and **low demand** for data volume. In this system, four observation points are required to check uncertain data and reduce the error rate.

The GM (1,1) model is a first-order grey model for forecasting time series. In this model, a system is described by a first-order differential equation, and the model is updated whenever new data becomes available. To comply with the randomness of the data, we use the cumulative generation operator (**AGO**).

The differential equation GM (1,1) computes the correlation values for the previous steps of the prediction system. Use this predicted value and the Inverse Accumulation Generation Operator (I-AGO) to get the predicted value of the data.

6.2.2 The GM (1,1) Algorithm

1. Suppose $X^0 = \{x^0(k)\}, \{k: 1,2, \dots, n: n \geq 4\}$ is the non-negative sequence of the original data, and n is the size of the sample data, the first-order sequence of its cumulative generator operators is equal to $X^1 = \{x^1(k)\}, \{k: 1,2, \dots, n: n \geq 4\}$ where

$$x^1(k) = \sum_{i=1}^{k} x^0(i), k = 1,2,3, \dots, n \tag{11}$$

2. The average generated value of consecutive adjacent data is obtained by the following formula

$$Z^1 = \{z^1(k)\}, \{k = 1,2,3, \dots, n\} \tag{12}$$

where

$$z^1(k) = \frac{1}{2}x^1(k-1) + \frac{1}{2}x^1(k) \tag{13}$$

3. The "whitening equation" for the GM (1,1) mode is as follows

$$\frac{\mathrm{d}x^1(t)}{\mathrm{d}t} + ax^1(t) = b \tag{14}$$

With the discretization of Equation (14), the differential equations are "greyed " as follows:

$$x^0(k) + az^1(k) = b \tag{15}$$

4. Then, we compute the values of a and b using ordinary least squares estimation

$$\hat{a} = (B^T B)^{-1} B^T Y_N \tag{16}$$

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} \tag{17}$$

$$B = \begin{bmatrix} -z^1(2) & 1 \\ \vdots & \vdots \\ -z^1(n) & 1 \end{bmatrix} \tag{18}$$

$$Y_N = \begin{bmatrix} x^0(2) \\ \vdots \\ x^0(n) \end{bmatrix} \tag{19}$$

5.We next calculate the time response of the GM (1,1) equation based on the values of a and b

$$\hat{x}^1(k+1) = \left[x^0(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a}. \tag{20}$$

6. Finally, obtain the reconstructed value of the original data using the following equation

$$\hat{x}^0(k+1) = \hat{x}^1(k+1) - \hat{x}^1(k) \tag{21}$$

6.3 Results and Analysis

6.3.1 Visually Analyze Results Based on Figures

Figure 6 draws the predicted value and the actual value in the same coordinate system, and Figure 7 uses a scatter plot to represent the deviation between the predicted value and the actual value

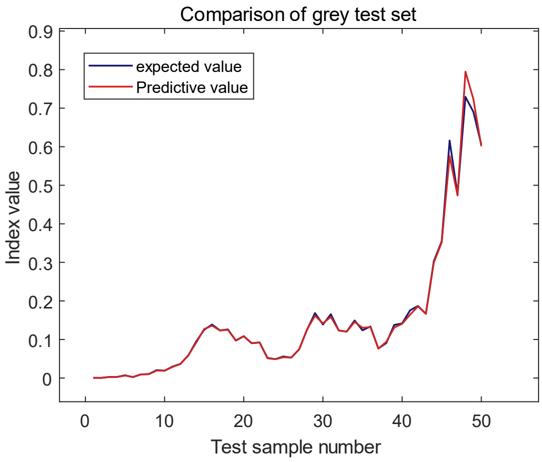

Figure 6: comparison of grey test set

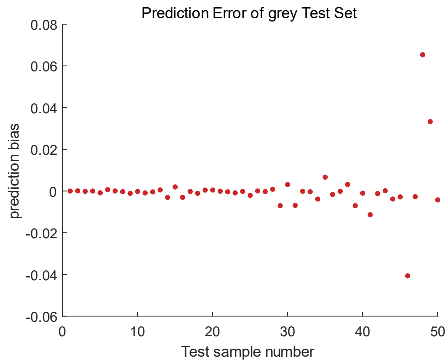

Figure 7: prediction error of grey test set

Intuitively, the predicted value of the gray forecast is **relatively close** to the real value, and the matching degree with the up and down trend is very high. Naturally, part of the reason is that the predicted time series is short.

Next, we use the posterior difference test method to quantitatively explain the error of the gray prediction model.

6.3.2 Analyzing Errors with Posterior Difference Test

The steps to calculate the posterior difference are as follows:

1. Calculate residuals on predicted values

$$e(k) = x^{(0)}(k) - \hat{x}^{(0)}(k), k = 1,2, \dots, n \tag{22}$$

2. Let the variances of the original sequence be $x^{(0)}$ and the residual sequence E be S_1^2 and S_2^2 respectively, then we have

$$S_1^2 = \frac{1}{n}\sum_{k=1}^{n} \left[x^{(0)}(k) - \bar{x}\right]^2 \tag{23}$$

$$S_2^2 = \frac{1}{n}\sum_{k=1}^{n} [e(k) - \bar{e}]^2 \tag{24}$$

where

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k), \bar{e} = \frac{1}{n}\sum_{k=1}^{n} e(k) \tag{25}$$

3. the posterior difference are

$$C = S_2/S_1 \tag{26}$$

The distribution of the posterior difference is shown in Figure 8. The lower the C value, the more accurate the prediction. Generally, the prediction is more accurate when C<0.65.

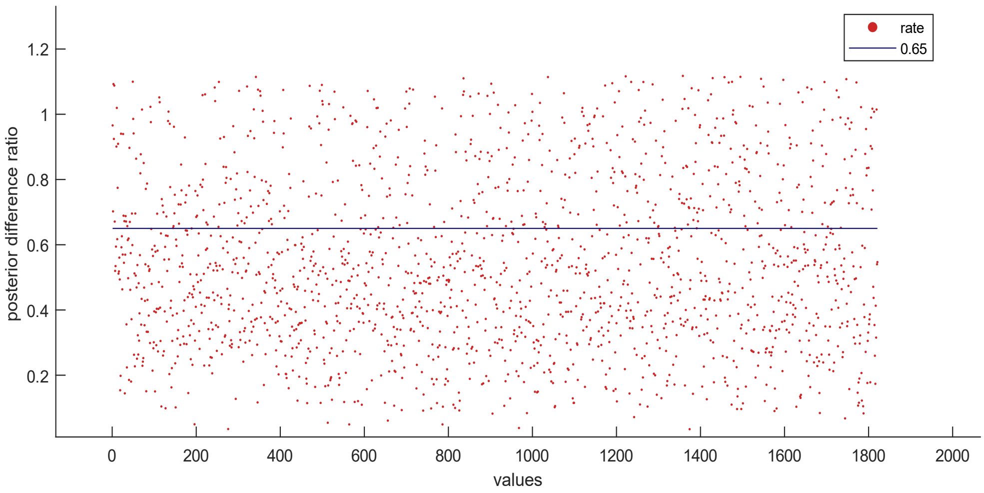

Figure 8: the C values of the grey prediction model

We can observe that although most of the points are distributed below the line of the exact C value, there are still many predicted values that are not accurate because there is too little data available for prediction in the given data.

There are about 70% of the points that can reach the qualified standard. Moreover, we observed that the unqualified points are evenly distributed on the ordinate, so the points with too large prediction error are not in large proportion among all the points.

7 Trading Decision Model Based on the Exhaustive Algorithm

7.1 Risk Measurement Method Based on VaR Value

7.1.1 Introduction to Value at Risk Theory

VaR is a **risk measurement** method that uses statistical methods to study the tail loss in the return series in the financial time series, and it is also the most important method to measure market risk in recent years.

One of the most prominent advantages of VaR is that it can express market risk as a single number at a certain level of confidence.

The VaR value represents the maximum possible loss of a financial asset under a certain confidence level $1-\alpha$ in a specific period of time in the future. The formula is:

$$\text{prob}\ (r < -VaR) = \alpha \tag{27}$$

where

r is the yield rate of Δt: $t \sim t+1$, ,$r > 0$ means profit, $r < 0$ means loss, VaR takes the form of yield and is a positive number.

7.1.2 Variance-covariance Methods to Calculate VaR Values

Variance-covariance methods, also known as Analytic Methods, estimate the market risk of a portfolio **through correlation and historical volatility**,

The key to the analytical method lies in the estimation method of the combined value function and the distribution form that the market factor obeys, and the parameter measurement method is used to obtain VaR.

Its core is to estimate the variance-covariance matrix of asset returns.

The basic steps are: first, use historical data to find the standard deviation and covariance of the returns of the asset portfolio; second, assume that the returns of the asset portfolio are normally distributed, and calculate the distribution that reflects the degree of deviation of the distribution from the mean under a certain confidence interval. critical value; third, establish the relationship between risk loss and asset portfolio to introduce VaR value.

Due to space reasons, the specific method of solving the VaR value will not be discussed in depth here. The value of VaR can be directly solved through the data through the function **portvrisk** of the financial toolbox library of **Matlab**.

7.2 Trading Decision Model Based on Programming Exhaustive Algorithm

7.2.1 Wrapped Investment Method Function

1. The function to **calculate the principal** of the day from the principal of the previous day and the price of gold and Bitcoin, encapsulated into function (1):

Take the previous day's gold and bitcoin holdings and multiply by the percentage change in gold and bitcoin, respectively.

2. The method of **investing in gold**, encapsulated into function (2):

If we hold cash < 200 US dollars, we will invest all in gold; if we hold cash >= 200 US dollars and <= 400 US dollars, we will invest 200 US dollars; if we hold cash > 400 US dollars, we will invest half of the cash we hold

3. The method of **investing in Bitcoin**, encapsulated into function (3):

If we hold cash < 200 US dollars, we will fully invest in Bitcoin; if we hold cash >= 200 US dollars and <= 400 US dollars, we will invest 200 US dollars; if we hold cash > 400 US dollars, we will invest half of the cash we hold

4. The method of **selling gold**, encapsulated as a function (4):

If we hold gold < 200 US dollars, we will not sell it; if we hold gold >= 200 US dollars and <= 400 US dollars, we will sell until remaining 200 US dollars of gold; if we hold cash > 400 US dollars, we will sell half of the gold held.

5. The method of **selling bitcoin**, encapsulated as a function (5):

If we hold bitcoin < 200 US dollars, we will not sell it; if we hold bitcoin >= 200 US dollars and <= 400 US dollars, we will until remaining 200 US dollars of gold; if we hold cash > 400 US dollars, we will sell half of the bitcoin held.

The above algorithms are shown in Figure 9

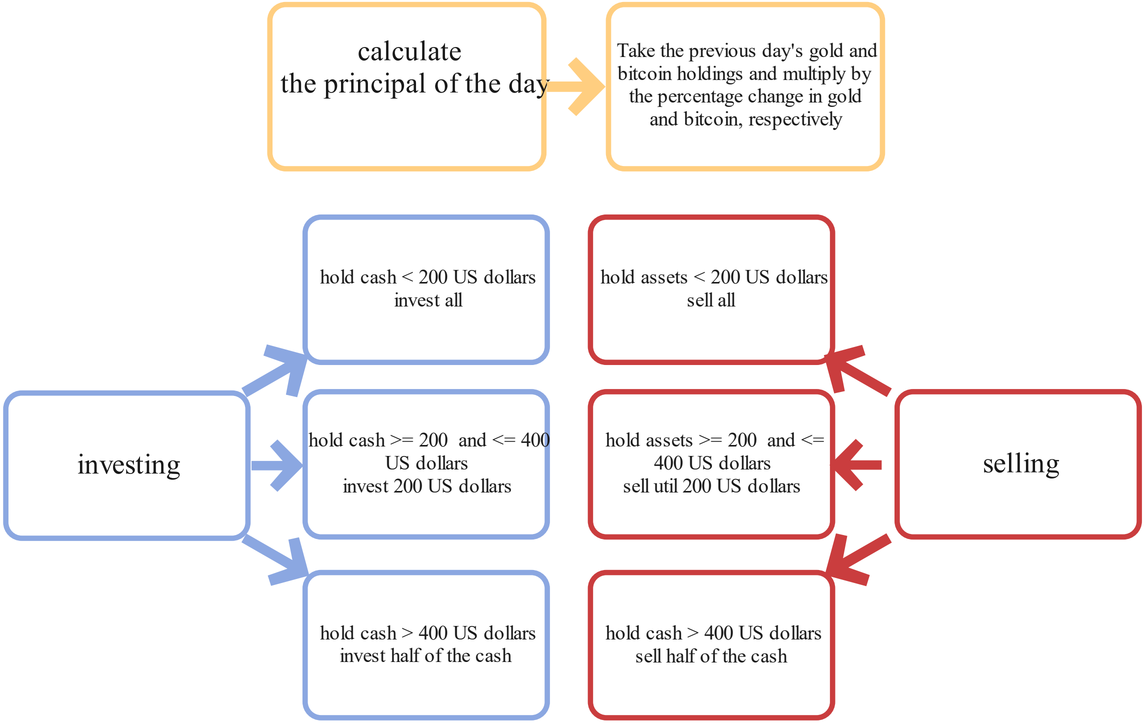

Figure 9: wrapped functions

7.2.2 Investment Algorithm

In this subsection, the bold words represent variables, and the input value can be freely set.

1. Strategies for long-term investment in gold by VaR risk levels:

It is evaluated every **gDays** trading day. If VaR obtained by the portvrisk function > **gVaRup**, use function (2) to invest in gold; if VaR obtained by the portvrisk function < **gVaRlow**, use function (4) to sell gold

2. Strategies for investing in or selling gold by its forecast value:

Whenever the decline of gold on a certain day exceeds **gDecrease**, if it is predicted that the decline of the next day is less than **gDecrease** or the increase of the next day, use function (2) to invest in gold

3. Strategies to invest or sell Bitcoin by Bitcoin forecast value:

Whenever the predicted increase exceeds **bIncrease**, use function (3) to invest in bitcoin immediately; whenever the predicted decline exceeds **bDecrease**, use function (5) to sell bitcoin immediately;

7.3 Assets after Five Years and Analysis

We enter different parameter values for calculation and find that the benefits are consistent with the intuitive feeling.

When the investment strategy is **aggressive**, with low expectations for buying and high expectations for selling, a substantial portion of the value can yield five-year returns of up to

120%.

But at the same time, there is also a great possibility that we will only get a return rate of less than **20%** or even a loss. The reason for analyzing the loss is mainly because many times when the price of Bitcoin fell sharply, it was purchased with wrong prediction values. The high yield is due to the fact that more gold was held when the price of bitcoin fell sharply, easing losses.

When the investment strategy is **moderate** or **ordinary**, it is easy to get a yield of more than **60%**, because the gold and bitcoin prices have generally shown an upward trend in the past 5 years,

Since the investment strategy is not aggressive, it is not easy to encounter a situation where the price of Bitcoin plummets.

8 Reasons Why This Investment Model Provides the Best Investment Method

In fact, this method is very close to the **exhaustive method**. Because the return on investment depends on only two factors, that is, the accuracy of the prediction model and the matching degree of the **input parameters**. The investment coefficient found through this model is absolutely "optimal".

At the same time, in order to avoid the suspicion of predicting the known data from the known data, we list the data of the first two and a half years separately, and take out the coefficient group of which the rate of return is greater than 20% to find the income of the next two and a half years. We found that more than 80% of the parameters gained more than 30% after two and a half years. This is enough to **justify** the model.

The reason why the rate of return in the last two and a half years is greater than that in the first two and a half years is because the growth rate of Bitcoin in the latter two and a half years is greatly accelerated. So if we make several successful speculations, we can get a high return.

9 Sensitivity Analysis when Changing Transaction Costs

When transaction fees are increased by **1%** each:

The trading strategies that were originally considered to be **aggressive** have **plummeted** from the original average return rate of about 68% to only about 26%. This is because aggressive trading strategies tend to trade more frequently and require high transaction fees. And **ordinary** trading strategies have dropped from an average return of about 58% to only about 40%. The trading strategies that are considered to be **moderate** have dropped from the original rate of return of about 56% to about 43%

When transaction fees are increased by 2% each:

Disregarding the aggressive trading strategies, the returns of the **ordinary** trading strategies decreased by about 22%, and the returns of the **moderate** trading strategies decreased by about 14%.Therefore, we can conclude that the more aggressive the trading strategy, the more easily the returns are affected by transaction costs.3

In Figure 10, it can be seen intuitively that with the change of transaction fees, the rate of return of different trading strategies changes.

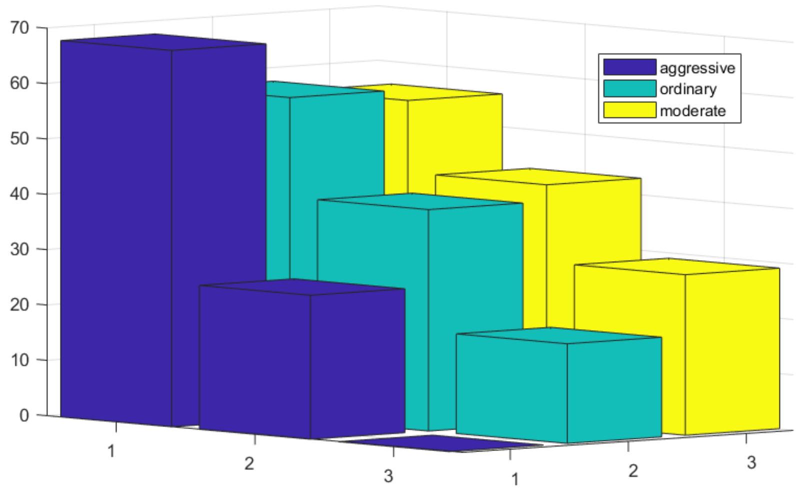

Figure 1 0 :return of different trading strategies changes

10 Strengths and Weaknesses

10.1 Strengths

The established model has the following strengths:

The model is scalable and incorporates financial market influences into an abstract and comprehensive framework. Whether it is the time series forecasting model we use or the exhaustive trading algorithm, it can be applied not only to the gold and bitcoin markets, but also to any financial market.

We did a good job of visualization, and gave graphic explanations in various places that were not easy to understand

Our model effectively achieves all objectives and is able to complete all the requirements given by the title

10.2 Weaknesses

Our model has the following limitations and related improvements:

It is difficult to accurately predict financial markets only through a single time series, and we believe that the accuracy can be greatly improved by adding multiple evaluation methods.

The time complexity of the exhaustive algorithm is very high. Due to time constraints, we can only take part of the value for calculation.

Memorandum

Dear reader:

We believe that even with such a wealth of financial and mathematical knowledge as you, there are still occasions when the inaccuracy of the forecasting model is not enough, or the error in the investment model is annoying.

Wouldn't it be a good thing if there were richer forecasting and investment models for reference? Here we innovatively propose several models that may not have been commonly used in the field of finance before. These include two time-series forecasting models for financial market prices and a model for portfolio investments.

We try to use the BP neural network model for medium and long-term financial market time series forecasting, and use the time series forecasting model based on the grey system for short-term forecasting. During our modeling process, they are generally reliable after inspection. Of course, there is still a lot of room for optimization.

In addition, we creatively apply the exhaustive method of computer programming to portfolio investment, and establish a model that combines VaR and predicted returns. As long as there is enough computing power. Only need to change the value of a few input parameters in the model, you can get the best decision time

Authors
February 22,2022

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Appendices

Appendix 1

Introduce: Determine whether to buy gold by VaR value

```
function VdealG(date,realprices,gVaRup,gVaRlow)
if date<=50
    return
end
past=realprices((date-50):1:date,2);
PortRisk=std(past);
PortReturn=0.01;
VaR = portvrisk(PortReturn,PortRisk);
if VaR>gVaRup
    buyG(date)
end
if VaR<gVaRlow
    sellG(date)
end
```

Appendix 2

Introduce: Implementation of Exhaustive Algorithm

```
function
gain=algorithm(anrate,realprices,gdecrease,bincrease,bdecrease,gVaRup,gVaRlow)
% function dealG(date,anrate,gdecrease)
% function dealB(date,anrate,bincrease,bdecrease)
% function VdealG(date,realprices,gVaRup,gVaRlow)
global money;
money(1,1)=1000;
for date=1:1826
    dealB(date,anrate,bincrease,bdecrease);
    dealG(date,anrate,gdecrease);
        if mod(date,50)==0
            VdealG(date,realprices,gVaRup,gVaRlow);
        end
end
```

```
gain=money(1826,1)+money(1826,2)+money(1826,3);
```