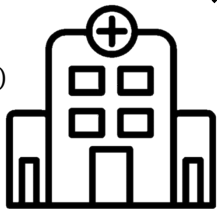


① Server divides all clients into  $m_1$  “public” clients and  $m_2$  “private” clients ( $m_1 < m_2$ ) according to their specified privacy budgets.

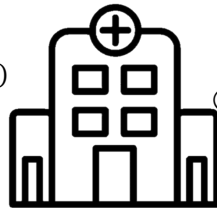
Client  $C_1^{(pub)}$   
[ $\epsilon = 10$ ]



“Public”  
clients

...

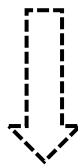
Client  $C_{m_1}^{(pub)}$   
[ $\epsilon = 8$ ]



③ Upload  $\Delta \tilde{\mathbf{x}}_{t_c}^{m_1}$   
⑦ Broadcast  $\mathbf{x}_{t_c}$



Server

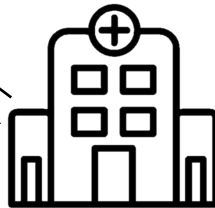


Client  $C_1^{(pri)}$   
[ $\epsilon = 1$ ]

“Private”  
clients

...

Client  $C_{m_2}^{(pri)}$   
[ $\epsilon = 0.8$ ]



### Local training

② If there are adequate privacy budgets to perform the next training round, repeat  $\tau$  iterations and compute the noisy weight updates  $\Delta \tilde{\mathbf{x}}_t^m$  via DPSGD

$$\Delta \tilde{\mathbf{x}}_{t_c}^m \leftarrow \tilde{\mathbf{x}}_{t_c+\tau}^m - \mathbf{x}_{t_c}$$

### Private updates projection

④ Server computes top-k eigenspace  $V_k$  of the “public” updates  $\Delta \tilde{\mathbf{x}}_{t_c}^{(pub)}$ .  
⑤ Server projects and reconstructs the “private” updates  $\Delta \tilde{\mathbf{x}}_{t_c}^{(pri)}$  using  $V_k$ .  
⑥ Server averages the projected “private” updates and non-projected “public” updates  
 $\mathbf{x}_{t_c+\tau} \leftarrow \mathbf{x}_{t_c} + \text{AVG} \left( \Delta \tilde{\mathbf{x}}_{t_c}^{(pub)} + V_k V_k^T \Delta \tilde{\mathbf{x}}_{t_c}^{(pri)} \right)$

