Problem 1

a)

$(x,y)=(8,7)$ is matched to the point $\begin{bmatrix} 1 & 0 & 0 & -12 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 8 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \\ 1 \end{bmatrix}$ in world coordinate, and the same applies to $(u,v)=(2,7)$ which is $\begin{pmatrix} 14 & 7 & 1 \end{pmatrix}$ in world coordinate.

Therefore we could have line

$$OP_1 : \frac{x+12}{8} = \frac{y}{7} = \frac{z}{1}$$

and

$$OP_2 : \frac{x-12}{2} = \frac{y}{7} = \frac{z}{1}$$

and the intersection of the two line is the 3D location of that point, which is $\begin{pmatrix} 20 & 28 & 4 \end{pmatrix}$

b)

We have $t=\begin{pmatrix} -24 & 0 & 0 \end{pmatrix}$ $R=I$,

$E=[t]R$

Therefore the essential matrix is

$$E=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -24 \\ 0 & 24 & 0 \end{bmatrix} R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -24 \\ 0 & 24 & 0 \end{bmatrix}$$

c)

Since the focal point is at $y=0$, and all the points on the line are at $y=0$, we can consider only the *xz*-plane.

For the porjection of point x in camera1 we have

$$X_L = \frac{f(x+12)}{z}$$

and the projection point x in camera2 which is also u is

$$X_R = \frac{f(x-12)}{z} = u = \frac{x-12}{2-x}$$

$$x = \frac{12+2u}{1+u}$$

Therefore the disparity d is

$$d = \frac{f(x+12)}{z} - \frac{f(x-12)}{z} = \frac{24}{z} = \frac{24}{2-x} = \frac{24}{2-\dfrac{12+2u}{1+u}} = -\frac{12}{5}(1+u)$$

Problem 2

a) Pure Horizontal Translation: $t=[t_x,0,0]^T$, $R=I$

i. For essential matrix E we have

$$E=[t]R=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

and

$$E^T=[t]R=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix}$$

Since $E^T e_A=0$ and $Ee_B=0$, we have the epipoles $e_A=[1,0,0]^T$, $e_B=[1,0,0]^T$

ii. Set $P=[x_A,y_A,1]^T$ and $P'=[x_B,y_B,1]^T$, we have

$$P^T EP'=[x_A,y_A,1]\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}\begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix}=t_x(y_B-y_A)=0$$

Therefore the epipolar line $l_B: y_B=y_A$

iii. First construct 3 mutually orthogonal unit vectors e_1,e_2,e_3, set the first vector the direction of translation $e_1=\frac{t}{|t|}=[1,0,0]^T$, and set the second vector the normalized cross product of e_1 with the direction vector of the optical axis $e_2=\frac{1}{\sqrt{t_x^{\ 2}+t_y^{\ 2}}}[-t_y,t_x,0]^T=[0,1,0]^T$, then set the third vector the normalized cross product of the first two vector $e_3=e_1\times e_2=[0,0,1]$. Thus we have the rectification matrix $R_{rect}=\begin{bmatrix} e_1^T \\ e_2^T \\ e_3^T \end{bmatrix}=I$, therefor the solution to rectifying transforms is $H_A=R_{rect}=I$ and $H_B=R\cdot R_{rect}=I$.

b) Pure Translation Orthogonal to the Optical Axis: $t=[t_x,t_y,0]^T$, $R=I$

i. For essential matrix we have

$$E=[t]R=\begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

Since $E^T e_A = 0$ and $Ee_B = 0$, we have the epipoles $e_A = [t_x, t_y, 0]^T$ and $e_B = [t_x, t_y, 0]^T$

ii. Set $P = [x_A, y_A, 1]^T$ and $P' = [x_B, y_B, 1]^T$, we have

$$P^T EP' = [x_A, y_A, 1]\begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}\begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix} = t_x(y_B - y_A) + t_y(x_A - x_B) = 0$$

Therefore the epipolar line $l_B : t_x(y_B - y_A) + t_y(x_A - x_B) = 0$

iii. Set the first vector $e_1 = \frac{t}{|t|} = [\frac{t_x^2}{\sqrt{t_x^2+t_y^2}}, \frac{t_y^2}{\sqrt{t_x^2+t_y^2}}, 0]^T$,

the second vector $e_2 = \frac{1}{\sqrt{t_x{}^2 + t_y{}^2}}[-t_y, t_x, 0]^T = [\frac{-t_y^2}{\sqrt{t_x^2+t_y^2}}, \frac{t_x^2}{\sqrt{t_x^2+t_y^2}}, 0]^T$,

and the third vector $e_3 = e_1 \times e_2 = [0,0,1]$

Thus we have the rectification matrix $R_{rect} = \begin{bmatrix} e_1^T \\ e_2^T \\ e_3^T \end{bmatrix} = \begin{bmatrix} \frac{t_x^2}{\sqrt{t_x^2+t_y^2}} & \frac{t_y^2}{\sqrt{t_x^2+t_y^2}} & 0 \\ \frac{-t_y^2}{\sqrt{t_x^2+t_y^2}} & \frac{t_x^2}{\sqrt{t_x^2+t_y^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

therefor the solution to rectifying transforms is

$$H_A = H_B = R_{rect} = \begin{bmatrix} \frac{t_x^2}{\sqrt{t_x^2+t_y^2}} & \frac{t_y^2}{\sqrt{t_x^2+t_y^2}} & 0 \\ \frac{-t_y^2}{\sqrt{t_x^2+t_y^2}} & \frac{t_x^2}{\sqrt{t_x^2+t_y^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c) Pure Translation Along the Optical Axis $t = [0,0,t_z]^T$, $R = I$

i. For essential matrix we have

$$E=[t]R=\begin{bmatrix} 0 & -t_z & 0 \\ t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since $E^T e_A = 0$ and $Ee_B = 0$, we have the epipoles $e_A = [0,0,1]^T$ and $e_B = [0,0,1]^T$

ii. Set $P = [x_A, y_A, 1]^T$ and $P' = [x_B, y_B, 1]^T$, we have

$$P^T EP' = [x_A, y_A, 1]\begin{bmatrix} 0 & -t_z & 0 \\ -t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix} = t_z(y_A x_B - y_B x_A) = 0$$

Therefore the epipolar line $l_B : (y_A x_B - y_B x_A) = 0$

iii. Set the first vector $e_1 = \frac{t}{|t|} = [0,0,1]^T$,

the second vector $e_2 = \frac{1}{\sqrt{0^2+0^2}}[-t_y, t_x, 0]^T$,

We found that e_2 doesn't exist, therefore we are not able to find its rectifying transform matrices, the epipolar rectification is impossible.

d) Pure Rotation $t = [0,0,0]^T$, R is an arbitrary rotation matrix

i. For essential matrix we have

$E = [t]R = 0$

Since $E^T e_A = 0$ and $Ee_B = 0$, the epipoles e_A and e_B could be any existing matrix.

ii. Set $P = [x_A, y_A, 1]^T$ and $P' = [x_B, y_B, 1]^T$, we have

$P^T 0 P' = 0$

Therefore l_B does not exist.

iii. For $e_1 = \frac{t}{|t|} = \frac{0}{|0|}$, e_1 doesn't exist, therefore we are not able to find its rectifying transform matrices, the epipolar rectification is impossible.