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ECE 369 Homework 1  
Proposition and Predicate Logic

### Exercise 1.1

#### Problems:

- 3. (a) true
- 3. (b) false
- 3. (e) true
- 3. (f) false

- 8. (a)  $\neg PF \cup PF$
- 8. (c)  $PF \cap \neg PS$
- 8. (f)  $\neg PF \cap \neg FD$

- 13. (a)  $H \rightarrow K$
- 13. (c)  $K \rightarrow H$
- 13. (d)  $K \leftrightarrow A$

### Exercise 1.2

#### Problems

- 42.
  - 1.  $C \rightarrow \neg F$  hyp
  - 2.  $F \rightarrow \neg S$  hyp
  - 3.  $S \rightarrow \neg F$  hyp
  - 4.  $\neg F \rightarrow S$  hyp
  - 5.  $C$  hyp
  - 6.  $\neg F$  1, 3, mp
  - 7.  $S$  4, 6, mp
  
- 47.
  - 1.  $(J \cup L) \rightarrow C$  hyp
  - 2.  $\neg T$  hyp
  - 3.  $C \rightarrow T$  hyp
  - 4.  $(J \cup L) \rightarrow T$  1, 3, hs
  - 5.  $\neg (J \cup L)$  4, 2, mt
  - 6.  $\neg J \cap \neg L$  5, De Morgan's Law
  - 7.  $\neg J$  6, sim

### Exercise 1.3

#### Problems

2. (e) true, if  $y = 0$ ;

2. (f) true,

2. (h) false, if  $x = 0$ ;

11. (a)  $(\exists x)[P(x) \rightarrow (\forall y)(T(y) \rightarrow F(x, y))]$

(b)  $(\forall x)[P(x) \rightarrow (\exists y)(T(y) \rightarrow F(x, y))]$

(c)  $\neg\{(\forall x)[P(x) \rightarrow (\exists y)(T(y) \rightarrow F(x, y))]\}$

16. (a)  $(\forall x)[B(x) \rightarrow (\forall y)(F(y) \rightarrow L(x, y))]$

16. (i)  $(\forall x)[F(x) \rightarrow (\exists y)(B(y) \rightarrow F(x, y))]$

16. (l)  $(\forall x)[F(x) \rightarrow (\exists y)(B(y) \rightarrow F(x, y))]$

### Exercise 1.4

F: Flowers

P: Flowers are purple

R: Flowers are red

PA: Pansies

4. 1.  $(\exists x)[F(x) \rightarrow R(x)]$  hyp

2.  $(\exists x)[F(x) \rightarrow P(x)]$  hyp

3.  $(\forall x)[PA(x) \rightarrow F(x)]$  hyp

4.  $PA(x) \rightarrow F(x)$  3, ui

5.  $[F(a) \rightarrow R(a)]$  1, ei

6.  $[F(a) \rightarrow P(a)]$  2, ei

7.  $PA(a) \rightarrow R(a)$  4, 5, hs

8.  $PA(a) \rightarrow P(a)$  4, 6, hs

9.  $(\exists x)[PA(x) \rightarrow P(x)]$  7, eg

10.  $(\exists x)[PA(x) \rightarrow P(x)]$  8, eg

Some pansies are Purple.

Some pansies are red.

PI: Flowers are pink  
 TH: Flowers have thorns  
 B: Bad smell.  
 W: Weed

5. 1.  $(\exists x)[F(x) \cap TH(x)]$  hyp
  2.  $(\forall x)[TH(x) \rightarrow B(x)]$  hyp
  3.  $(\forall x)[B(x) \rightarrow W(x)]$  hyp
  4.  $[F(a) \cap TH(a)]$  1, ei
  5.  $F(a), TH(a)$  4, sim
  6.  $TH(x) \rightarrow B(x)$  2, ui
  7.  $B(x) \rightarrow W(x)$  3, ui
  8.  $B(a)$  5, 6, mp
  9.  $W(a)$  7, 8, mp
  10.  $(\exists x)[W(x)]$  9, eg
- Some flowers smell bad.

9. (a) Let  $Q(x, y)$ : x talks to y  
 $(\forall y)(\exists x)Q(x, y)$  :

Everybody is talked by someone, but someone can be different person.

$(\exists x)(\forall y)Q(x, y)$ :

Someone talks to everyone. Someone should be the same person.

So  $(\forall y)(\exists x)Q(x, y)$  cannot imply  $(\exists x)(\forall y)Q(x, y)$ .

(b) The step4 to step5 is wrong.

4.  $(\forall y)Q(a, y)$  3, ug

5.  $(\forall y)(\exists x)Q(x, y)$  4, eg

The above is the right deduction.

33.    1.  $(\exists x)(M(x) \rightarrow (\forall y)(R(x, y)))$       hyp  
       2.  $(\forall x)(\forall y)(R(x, y) \rightarrow T(x, y))$       hyp  
       3.  $(\exists x)M(x)$       hyp  
       4.  $(M(a) \rightarrow (\forall y)(R(a, y)))$       1, ei  
       5.  $(M(a) \rightarrow (R(a, y)))$       4, ui  
       6.  $R(x, y) \rightarrow T(x, y)$       2, ui  
       7.  $M(a, y) \rightarrow T(a, y)$       5, 6, hs  
       8.  $(\exists x)(M(x) \rightarrow (T(x, y)))$       6, 7, eg  
       9.  $(\exists x)(M(x) \rightarrow (\forall y)(T(x, y)))$       8, ug
37.    1.  $(\forall x)(F(x) \rightarrow (\exists y)(C(y) \rightarrow O(x, y)))$       hyp  
       2.  $(\forall x)(D(x) \rightarrow (\forall y)(C(y) \rightarrow \neg O(x, y)))$       hyp  
       4.  $F(x) \rightarrow (\exists y)(C(y) \rightarrow O(x, y))$       1, ui  
       5.  $F(x) \rightarrow (C(a) \rightarrow O(x, a))$       4, ei  
       6.  $D(x) \rightarrow (\forall y)(C(y) \rightarrow \neg O(x, y))$       2, ui  
       7.  $D(x) \rightarrow (C(y) \rightarrow \neg O(x, y))$       6, ei  
       8.  $(D(x) \rightarrow \neg O(x, y))$       7, hs  
       9.  $(O(x, y) \rightarrow \neg D(x))$       9, cont  
       10.  $(F(x) \rightarrow O(x, a))$       6, hs  
       11.  $(F(x) \rightarrow \neg D(x))$       9, 10, hs  
       12.  $(\forall x)(F(x) \rightarrow \neg D(x))$       11, ug

Problem I:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = ((A \cup B) \cap C)$$

Problem II:

- (i)  $(\forall x)(P(x) \rightarrow \neg I(x))$
- (ii)  $(\forall x)(I(x) \rightarrow V(x))$
- (iii)  $(\forall x)(V(x) \rightarrow \neg P(x))$

No.

Because not only ignorant people can be vain, suppose good person also can be vain. Then Professor can be good person. So professor can be vain.