Name: Tian Qiu 00265 35063
Instructor: Womble

1. $P = 0.17$ X are films directed by woman

(a) $P(X=3) = \binom{15}{3}(0.17)^3(1-0.17)^{15-3} = 0.2389$

(b) $P(X\geq 2) = 1 - P(X=0) - P(X=1) = 1-(1-0.17)^{15} - \binom{15}{1}0.17\cdot(0.83)^{15-1}$

$= 1 - 0.0611 - 0.187 = 0.7519$

(c). $np = 15\times 0.17 = 2.55$

(d). $\sqrt{np(1-p)} = 1.4548$

2. 2012 27

2013 50

(a). $P_k = \frac{\lambda^k e^{-\lambda}}{k!}$ $\lambda = 4$.

$P(k=3) = \frac{4^3 \cdot e^{-4}}{3!} = 0.1953$

(b) $E(\text{Month}) = 4 \cdot E(\text{week}) = 4\lambda = 16$.

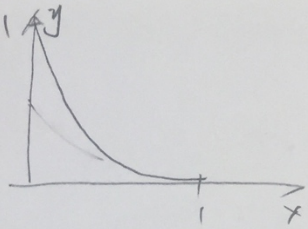
(c). $Var(4k) = 16\, Var(k) = 16 \cdot \lambda = 64$

$\sigma_{4k} = \sqrt{64} = 8$

(d). $P(4k=13) = C_4^3 \left[P(k=3)\right]^3 \cdot \left[P(k=4)\right]^1$

$= 4\times (0.1953)^3 \cdot \frac{4^4 \cdot e^{-4}}{4!} = 0.005821$

3. (a)



$$\int_0^1 2e^{-3x}\,dx$$

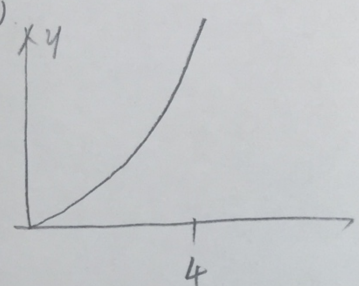
$$= 2\left[\frac{1}{-3}e^{-3x}\right]_0^1$$

$$= 2\left[\frac{1}{3}\left(e^{-3} - 0\right)\right]$$

$$= \frac{2}{3}\cdot e^{-3} \neq 1$$

$\therefore$ not legitimate pdf.

(b)



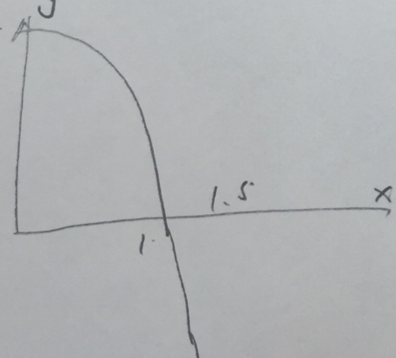
$$\int_0^4 \frac{3}{152}(x+2x^2)\,dx$$

$$= \frac{3}{152}\left[\frac{1}{2}x^2 + \frac{2}{3}x^3\right]_0^4$$

$$= \frac{3}{152}\left[8 + \frac{2}{3}\times 64\right] = 1.$$

$\therefore$ Yes.

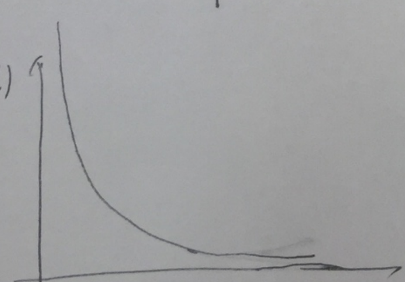
(c).



not

because $f(x)$ must be > 0.

(d)



$$\int_1^{\infty} x^{-2}\,dx$$

$$= \left[x^{-1}\right]_1^{\infty}$$

$$= 0 - (-1)$$

$$= 1$$

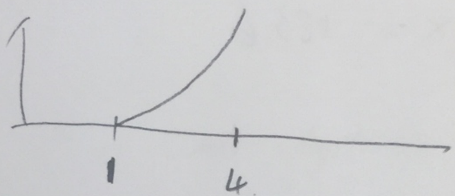
Yes.

4. (a) $\int_1^4 k(x^3-1)\,dx$

$= k\left[\frac{1}{4}x^4 - x\right]_1^4$

$= k\left[64 - 4 + \frac{1}{4} - 1\right] = 1 \Rightarrow k = 4/237$

(b)



5. (a) $P(1 \le x < 3) = \int_2^3 \frac{6-x}{6}\,dx = \left[x - \frac{x^2}{12}\right]_2^3 = \left[3 - \frac{9}{12} - 2 + \frac{4}{12}\right]$

$= 1 - \frac{5}{12} = \frac{7}{12}$

(b). $E(x) = \int_2^4 x \cdot \frac{6-x}{6}\,dx = \left[\frac{x^2}{2} - \frac{x^3}{18}\right]_2^4$

$= 8 - \frac{64}{18} - 2 + \frac{8}{18} = 6 - \frac{56}{18} = \frac{26}{9}$

(c). $F(x) = \int_{-\infty}^x f(t)\,dt = \int_{-\infty}^x 1 - \frac{t}{6}\,dt$

$= \int_2^x 1 - \frac{t}{6}\,dt = \left[t - \frac{t^2}{12}\right]_2^x = x - \frac{x^2}{12} - 2 + \frac{1}{3}$

$= \begin{cases} -\frac{x^2}{12} + x - \frac{5}{3} & 2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$

(d). $0.5 = F(x) = -\frac{x^2}{12} + x - \frac{5}{3}$

$x = 2.837$ ($x_2 > 4$ is not accepted).

(e) $Var(x) = \int_{-\infty}^{\infty} \left(x - \frac{26}{9}\right)^2 \cdot \frac{6-x}{6}\,dx = \frac{26}{81}$ $2 < x < 4$

(f) $\sigma_x = \sqrt{Var(x)} = .5665$

6. (a). $P(R \geq 550) = P(Z \geq \frac{550-500}{40}) = 1 - 0.8944 = 0.1056$

(b) $P(\frac{-62}{40} \leq Z \leq \frac{10}{40}) = (0.9394 - 0.5) + (0.5987 - 0.5) = .5381$

(c) $Z = 0.84 = \frac{x - 550}{40} \Rightarrow x = 583.6$

7. (a). $u = 51.75$
$\sigma = 14.37$

$P(Z < \frac{51.75 - 65}{14.37}) = 0.6808$

(b). $Z = 1.645 = \frac{x - u}{\sigma} \Rightarrow x = 75.38$

(c). $Z = 0.76 = \frac{x - u}{\sigma} \Rightarrow x_2 = 62.6712$

$x_1 = 51.75 - (x_2 - u) = 40.828$

$\therefore \quad 40.828 < x < 62.6712$

8. (a). $np \geq 5$ & $n \cdot (1-p) \geq 5$ is normal.

$np = 30 \cdot 0.63 = 18.9 > 5$ $\therefore$ normal to binomial
$nq = 30 \cdot 0.47 = 14.1 > 5$ is appropriate.

(b). $P(X \leq 18) = P(X \leq 18.5) = 0.4398$

$\sigma^2 = n \cdot p \cdot (1-p)$
$= 30 \times 0.63 \cdot 0.37$
$= 6.993$
$\sigma = 2.64$