STAT 350 – Spring 2009
Exam 1 SOLUTION

Your Name: _____ Your Seat: _____

Section Time (circle): 10:30 11:30 4:30

Note:

- You are responsible for upholding the Honor Code of Purdue University. This includes protecting your work from other students.
- Show your work on all questions. Unsupported work will not receive full credit. Credit will not be given for dumb luck. Showing work includes defining any random variable or event you use in the solution. Showing work also includes identifying any named distribution you are using in the solution and the values of all relevant parameters.
- Make it clear what your final answer is to each question. It may help to circle your answer.
- Decimal answers should be exact or to at least four significant digits (exceptions as discussed in class).
- Standard Normal (Z) values/probabilities must be taken from the table provided.
- You are allowed the following aids: a one-page (8.5×11 inch) cheat sheet, a scientific calculator, and pencils.
- Turn off and put away your cell phone before the exam begins!

Question	Points Possible	Points Missed
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

Exam Score: _____/ 100

1. (10 points; 5 points each part) Use the following set of number to answer the questions below. Assume you will be making a histogram of this data using the following cut-points: 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100.

3.695	14.007	17.703	18.947	20.453	20.692	20.723	21.507
26.671	28.106	28.805	31.591	33.202	33.939	35.026	39.994
43.833	43.981	44.943	45.430	48.031	49.455	49.755	52.470
52.710	55.004	55.620	55.669	55.723	56.312	56.748	58.890
59.652	61.486	63.623	65.098	65.722	67.247	67.647	70.169
70.249	70.682	71.429	72.494	72.660	73.517	75.063	76.971
78.219	78.477	78.900	79.272	80.005	81.201	81.700	82.501
82.719	83.089	83.852	84.843	85.069	85.271	86.262	87.021
87.207	88.380	89.003	89.890	90.252	90.469	91.064	91.169
92.490	93.680	93.881	95.002	95.864	97.761	97.811	97.952

a. If you are making a frequency histogram, what would be the height of the bar for the 30 to 40 bin?

5

There are 5 observations (highlighted) with values between 30 and 40

b. If you are making a density histogram, what would be the height of the bar for the 30 to 40 bin?

Relative Frequency = (# of observation in that class) / (total # of observations)

Density = (relative frequency) / (class width) = (5/80)/10 = 0.00625

2. (10 points; 5 points each part) Base your answers to the questions below on the following data set:

21	33	38	44	47	54	54	55	60	61	66	67	68	69	70	71	72	76	93

a. Find the median of this data set.

61

b. Find the lower quartile of this data set.

The lower quartile is the median of the smallest 10 observations (highlighted)
LQ is halfway between 47 and 54: (47 + 54) / 2 = 50.5

3. (10 points; 5 points each part) You have a sample of 35 observations from a population, you find that $\sum_{i=1}^{35} x_i = 2288$ and $\sum_{i=1}^{35} x_i^2 = 174{,}526$.

   a. Find $\bar{x}$

   2288 / 35 = 65.37143

   b. Find s_x

$$s_x^2 = \frac{SS_{xx}}{n-1} = \frac{174526 - \frac{(2288)^2}{35}}{35-1} = \frac{24956.17143}{34} = 734.005042$$

$$s_x = \sqrt{s_x^2} = 27.09252742$$

4. (10 points) X is a random variable with the following PDF. Find the mean of X.

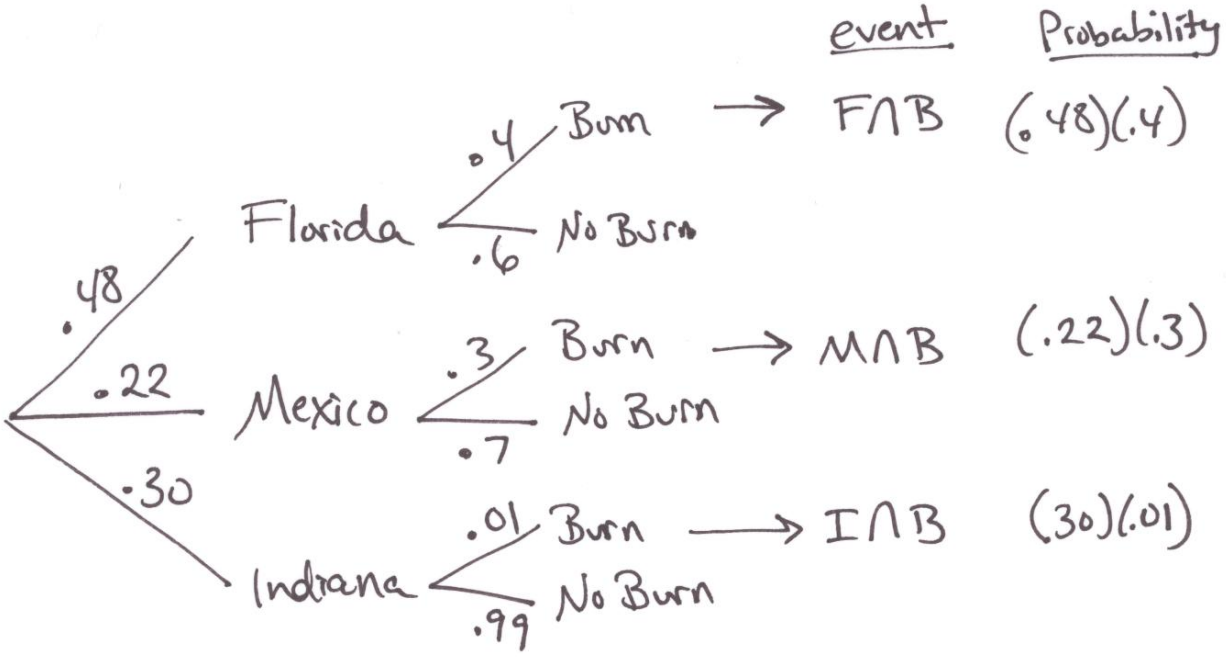
$$f(x) = \begin{cases} 0.34 - 0.04x & \text{if } 1 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{1}^{6} x(0.34 - 0.04x)\,dx = 37/12 = 3.08333$$

5. (10 points) For Spring Break, 48% of Purdue undergraduates will go to Florida, 22% will go to Mexico and the rest will stay in Indiana. 40% of the undergraduates that go to Florida will get sunburns, 30% of the undergraduates that go to Mexico will get sunburns, and only 1% of the undergraduates that stay in Indiana will get sunburns. What percent of all Purdue undergraduates will get sunburns over Spring Break?

P(Burn)
= P(Florida)P(Burn | Florida) + P(Mexico)P(Burn | Mexico) + P(Indiana)P(Burn| Indiana)
= (0.48)(0.40) + (0.22)(0.30) + (0.30)(0.01) = 0.261

26.1%



6. (10 points) You randomly select 27 tulip bulbs from a large discount bin at your local nursery. The nursery manager tells you that 84% of the bulbs in the bin are for red tulips. If you plants these bulbs and all 27 bloom, what is the probability you will get at least 25 red tulips?

X = # of red tulips
X ~ Binomial (n = 27, pi = 0.84)

n(1 – pi) < 5, therefore the normal approximation to the binomial is NOT appropriate

$P(X \geq 25) = P(X = 25) + P(X = 26) + P(X = 27)$

$$\binom{27}{25}.84^{25}.16^{2} + \binom{27}{26}.84^{26}.16^{1} + \binom{27}{27}.84^{27}.16^{0}$$

= 0.1704066

7. (10 points; 5 points each part) In the Hoosier Lottery, if you picked 2 of the 6 winning numbers, you win a free "quick pick". The probability of winning a free quick pick is 0.137. Assume you play the Hoosier Lottery once a week for the next 15 weeks.

   a. What is the expected value of the proportion of weeks in which you win a free quick pick?

   $E(p) = \pi = 0.137$

   b. What is the standard deviation of the proportion of weeks in which you win a free quick pick?

   $$SD(p) = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.137(1-0.137)}{15}} = 0.088781$$

8. (10 points) The lifetime of a particular type of battery has a mean of 19.4 hours with a standard deviation of 4.1 hours. If you have 50 of these batteries, what is the probability that the average lifetime of those 50 batteries is between 18 and 19 hours?

   $E(X_1) = 19.4$
   $SD(X_1) = 4.1$
   $n = 50$

   $$\bar{X} \sim Normal\left(\mu = 19.4, \sigma = \frac{4.1}{\sqrt{50}}\right)$$

   $$P(18 < \bar{X} < 19) = P\left(\frac{18-19.4}{4.1/\sqrt{50}} < Z < \frac{19-19.4}{4.1/\sqrt{50}}\right) = P(-2.41 < Z < -0.69)$$

   $$= \Phi(-0.69) - \Phi(-2.41) = 0.2451 - 0.0080$$

   $= 0.2371$

9. (10 points) Body-mass-index (BMI) is a ratio of weight to height or length. In humans, high values of BMI indicate a person is over-weight (too heavy for their height). In wildlife, BMI (usually measured as a ratio of weight to length) is used as a measure of "condition" - individuals with high BMIs are considered healthy and those with low BMIs are considered to be in underweight and, therefore, in poor health (obesity doesn't happen in the wild). 35 black bears from the George Washington and Jefferson National Forest in Virginia were captured and the BMI was calculated for each bear. For the 35 bears sampled, the average BMI was 246 and the standard deviation was 41. Obtain an 80% confidence interval for the true mean BMI of the population of black bears living in the George Washington and Jefferson National Forest in Virginia.

$\alpha = 1 - 0.80 = 0.20$

$z_{\alpha/2} = \Phi^{-1}(1 - 0.20/2) = \Phi^{-1}(0.90) = 1.28$

80% CI: $246 \pm 1.28\frac{41}{\sqrt{35}} \rightarrow$ (237.12926, 254.8707)

10. (10 points) You wish to estimate the proportion of black bears in the National Forests that are underweight (have a BMI below a particular value). How many bears will you need to sample to ensure that your resulting 95% CI will be within 0.05 of the true value?

$\alpha = 1 - 0.95 = 0.05$

$z_{\alpha/2} = \Phi^{-1}(1 - 0.05/2) = \Phi^{-1}(0.975) = 1.96$

$n = \pi(1-\pi)\left[\frac{z_{\alpha/2}}{B}\right]^2 = 0.5(1-0.5)\left[\frac{1.96}{0.05}\right]^2 = 384.16$

384 or 385 are also a $\pi(1-\pi)\left[\frac{z_{\alpha/2}}{B}\right]^2$ acceptable answers