Instructor: Womble

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Tian Qiu. HW5 STAT350.

1. (a). $E(x) = \int_a^b x \cdot f(x)\, dx$

$$= \int_a^b \frac{x}{b-a}\, dx$$

$$= \left[\frac{x^2}{2(b-a)}\right]_a^b$$

$$= \left[\frac{b^2-a^2}{2(b-a)}\right] = \frac{a+b}{2}$$

(b). $Var(x) = \int_a^b (x - \frac{a+b}{2})^2 \frac{1}{b-a}\, dx$

$$= E((x - \frac{b+a}{2})^2)$$

$$= E(x^2) - (\frac{a+b}{2})^2$$

$$= \int_a^b \frac{x^2}{b-a}\, dx - \frac{(a+b)^2}{4}$$

$$= \left[\frac{x^3}{3(b-a)}\right]_a^b - \frac{a^2+2ab+b^2}{4}$$

$$= \frac{a^2+a \cdot b+b^2}{3} - \frac{a^2+2ab+b^2}{4}$$

$$= \frac{1}{12}(4a^2+4ab+4b^2-3a^2-6ab-3b^2)$$

$$= \frac{1}{12}(a^2-2ab+b^2) = \frac{(a-b)^2}{12}$$

2. a). X hours the candle burns.

$$P(X \geq 7) = \frac{10.5-7}{10.5-6.5} = \frac{3.5}{4} = \frac{7}{8}$$

b). $$P(X \leq 8) = \frac{8-6.5}{10.5-6.5} = \frac{1.5}{4} = \frac{3}{8}$$

c) $$P(7 \leq X \leq 10) = \frac{3}{4}$$

d). $$E(X) = \frac{a+b}{2} = \frac{17}{2} = 8.5$$

e). $$\sigma_x = \sqrt{\frac{(a-b)^2}{12}} = \sqrt{\frac{4^2}{12}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}.$$

3. a). Normal Distribution

b) Normal

c). It is non-normal distribution. because plot is not Linear.

d). Almost normal.

A Little bit concave plot.

4. a). $E(X) = 100{,}000 = \frac{1}{\lambda}$

$\lambda = 1/100{,}000$

b). $\sqrt{Var(X)} = \sigma_X = \sqrt{\lambda^{-2}} = 100{,}000$.

c). X is timing belt.

$P(X \geq 120{,}000) = 0.4207$

d) $P(75000 \leq X \leq 125000) = 0.1974$

e). $P(X \leq 70{,}000) = 0.3821$

5. (a) $E(X) = 64 \cdot 0.1 + 65 \cdot 0.7 + 66 \cdot 0.2 = 65.1$

(b)

64, 64	$(0.1)^2$	64
64. 65	0.07	64.5
64, 66	0.02	65
	0.07	64.5
	$(0.7)^2$	65
	0.14	65.5
	.02	65
	.14	65.5
	.04	66
	P.	$\bar{x}$

$E(\bar{X}) = 65.1$.

c). $E(\bar{X}) = E(X)$.

d). If confirmed textbook

$\mu_{\bar{x}} = \mu \Rightarrow E(\bar{X}) = E(X)$.

6. a) min = 8 in

$E(x) = 7.75$

$\sigma_x = 0.1$

$P(x \geq 8) = 0.0062$

b) $\sigma_{\bar{x}} = \sqrt{\frac{\sigma_x^2}{n}} = \sqrt{\frac{0.1^2}{35}} = 0.0169$

$E(\bar{x}) = 7.75$

$P(\bar{x} \geq 8) = 0$

c) because the number of samples is different. more samples leads to more contribution to expected value.

7. $\mu_x = 397.34$

$\sigma_x = 20$.

a) $\sigma_{\bar{x}} = \sqrt{\frac{\sigma_x^2}{n}} = \sqrt{\frac{400}{40}} = \sqrt{10}$.

$P(\bar{x} \leq 393) = 0.0849$

b) $P(395 \leq \bar{x} \leq 403) = 0.7336$

c)

Let's calculate confidence interval.

$(400 \pm (\sigma_{\bar{x}} \times 1.96)) = 400 \pm 6.198$

$\because 397.34 \in [393.8, 406.198]$.

We have 95% confidence to say this 397.34 in the interval $[393.8, 406.198]$

$\therefore$ We cannot say population mean CO_2 increased