Sample Final Exam

ECE 608
Name: ______________________________

Read all of the following information before starting the exam:

1. **NOTE:** Unanswered questions are worth 25% credit, rounded down. Writing any answer loses this "free" credit, which must be earned back by the quality of the answer. If you wish a question to be treated as unanswered, but you have written on it, clearly write "DO NOT GRADE" in the answer area. In a multi-part question, unanswered *parts* are worth 25%. (You may *not* define your own parts for this purpose!) Unanswered parts of a problem are combined before rounding down.
2. Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
3. No calculators, or materials other than pen/pencil and blank paper are allowed except those we distribute during the exam. (Exception: a formula sheet may be allowed if stated in advance by the instructor.)
4. Please keep your written answers brief; be clear and to the point. Points will be deducted for rambling and for incorrect or irrelevant statements. Where algorithms are requested, you may be penalized for inefficient algorithms, and an exponential algorithm may be considered entirely incorrect on the basis of inefficiency alone. Any pseudo-code or written text will be disregarded if correctness is not apparent.
5. This test has 4 problems including the trival yes/no course-evaluation question. Except where indicated, each problem is approximately equal in value. Multi-part problems divide the problem score approximately equally among the parts, except where indicated.
6. Good luck!

1. Did you fill out the course evaluation? (1 point)

2.

Given a graph $G = (V, E)$, suppose each edge $e \in E$ is labelled with a label $\sigma(u, v)$ from a finite set Σ of labels, and assigned a probability $p(u, v)$ so that the sum of the probabilities leaving a each vertex u is 1. Define the probability of a path as the product of the probabilities of its edges, and the label of a path to be the sequence of labels of its edges.

Describe the design and structure of an efficient algorithm that takes as input a start vertex $v_0 \in V$ and a sequence $s = \langle \sigma_1, \ldots, \sigma_k \rangle$ of labels from Σ. The algorithm should find and return the most probable path starting at v_0 with label s. Argue briefly that your algorithm is efficient and correct. You do *not* need to give pseudocode.

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3.

a. **(5 points)** What single-source shortest-path problems can be addressed by the Bellman-Ford algorithm but not by Dijkstra's algorithm?

b. Clearly and completely describe the Bellman-Ford algorithm. You may use pseudo-code, but it is not required. In return for accepting "Do not grade" points on this part of this problem, we will provide you a copy of the pseudo-code for Bellman-Ford algorithm for reference use in solving later parts of this problem.

(Problem 3 continued on next page)

c. Prove the correctness of the Bellman-Ford algorithm.

(Problem 3 continued on next page)

Problem 3 (continued)

d. Prove a tight asymptotic bound on the worst-case runtime of the Bellman-Ford algorithm.

4.

Consider a 3-CNF formula Φ containing m clauses. We say that Φ is K-satisfiable if there is an assignment to the variables of Φ that makes at least K of the clauses of Φ true. Note that K may be smaller than m, so this is a weaker notion than ordinary satisfiability. Consider the following language:

$$\text{MAXSAT} = \{(\Phi, K) \mid \Phi \text{ is a 3-SAT formula that is } K\text{-satisfiable}\}$$

Prove carefully that the language MAXSAT is NP-complete. You may use any NP-completeness result covered in class (CKTSAT, SAT, 3-SAT, CLIQUE) but no other.

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