Homework 12
Due: 2016/04/25 in Class at 8:30 a.m.

1 Little's Law, Utilization

a) If a factory maintains an average inventory of 300 jobs, and an average job takes six weeks to be completed, what is the production rate of the factory in units of jobs per year?
b) Consider an $M/M/3$ queueing model. The average number of customers in the system is 6.4, and the average number in the queue is 4.0. What is the average utilization of each server?

2 M/M/c Waiting time

a) We discussed the $M/M/1$ average queue length in the class. Find formulae for the average waiting time in system W and average waiting time in buffer W_q.
b) Compute W and W_q for $M/M/c$ queues, where $c = 2$ and $c = 3$.
c) Plot and compare the W and W_q for each of these cases.
d) Suppose you are given a dataset with inter-arrival and service times for some number of customers. Write computer code to compute the waiting time in the buffer for each customer, by using the so-called "Lindley recursion": the waiting time for customer n is

$$w_q(n) = \max\{0, w_q(n-1) + v_{n-1} - a_n\}, \tag{2.1}$$

where $w_q(n-1)$ is the waiting time for customer $n-1$, v_{n-1} is the service time of customer $n-1$ and a_n is the inter-arrival time between customers $n-1$ and n. Assume that customer 0 does not have to wait; i.e., $w_q(0) = 0$. Notice that the amount of time a customer spends in the buffer is unaffected by her service time (naturally).

e) Let's model the cash register at Greyhouse coffee using an $M/M/1$ queue. Using the dataset provided, and the code from d), estimate the average waiting time of a customer who arrived at Greyhouse. Also estimate the load on the system. How does this number compare to the theoretical waiting time at the same load factor? If the numbers are significantly off, what in your opinion is the reason for this?

3 Discouraged customers

Solve problem 7 in your textbook.

4 Runway Capacity

Solve problem 10 in your textbook.

5 A model for Starbucks

Consider the Starbucks in PMU. Suppose that both cash registers operate at all times. Assuming that the service times are exponentially distributed, suggest an appropriate queueing model for this system.
a) Set up the transition diagram and steady-state equation
b) Solve the steady equations for the distribution of the number of cusomters at Starbucks.
c) What is the average waiting time in the system for a typical customer?
d) Suppose Starbucks were to introduce a single automated register, where the average service time is half of what it takes for a human operator. Assuming that the service time is exponentially distributed, would the automated register reduce the average waiting time for a typical customer?