Tian Qiu
ECE 369 Homework 2
Mathematical Proofs, Program Proofs

Proof Techniques
Text Exercise 2.1
Problems 51, 55

1. For x = 3, $x^2 = 9$, the product of x and x square is 27 which obviously is not even.
2. For any integer n,
$$(n-1)^2 \geq 0$$
$$n^2 - 2n + 1 \geq 0$$
$$n^2 + 1 \geq 2n$$
$$n + \frac{1}{n} \geq 2$$
Therefore, for any integer n, $n + \frac{1}{n} \geq 2$ is always true.

Induction
Text Exercise 2.2
Problems 40(b), 53

1. Base case n = 1, $1 + \frac{1}{2} = 2 - \frac{1}{2} = \frac{3}{2}$

   Assume $P(K) = 1 + \frac{1}{2} + \frac{1}{4} + .. + \frac{1}{2^k} = 2 - \frac{1}{2^k}$ is true

   Then $P(K+1) = 1 + \frac{1}{2} + \frac{1}{4} + .. + \frac{1}{2^k} + \frac{1}{2^{k+1}} = 2 - \frac{1}{2^k} + \frac{1}{2^{k+1}} = 2 - \frac{2}{2^{k+1}} + \frac{1}{2^{k+1}} =$

   $2 - \frac{1}{2^{k+1}}$ which K+1 is correct.

   Hence $P(n) = 1 + \frac{1}{2} + \frac{1}{4} + .. + \frac{1}{2^k} = 2 - \frac{1}{2^n} < 2 \; for \; n \geq 1$
2. Base case n = 1, $(x-1)/(x-1) = 1$

   Assume $P(K) = \frac{x^k - 1}{x-1} = a$, where a is an integer

   Then

   $P(K+1) = \frac{x^{k+1}-1}{x-1} = \frac{x((x-1)*a+1)-1}{x-1} = \frac{((ax^2-ax)+x)-1}{x-1} = \frac{(a*x+1)(x-1)}{x-1} = ax + 1$ for x is not equal to 1, which K+1 is correct.

   Hence$x^n - 1 \; is \; divisible \; by \; x - 1 \; for \; x \neq 1$

Program Proofs
Text Exercise 2.3
Problem 15

1. Base case n = 1, $i_0 = 1, j_0 = a[1]$, n = 2, $i_1 = 2, j_1 = a[2]$

   Assume $Q(k): j_k = \max\{a[1], \ldots, a[k]\}$ is true

   Then $Q(k+1) := j_{k+1} = \max\{a[1], \ldots, a[k+1]\} = \max\{j_k, a[k+1]\}$ which means Q(k+1) is correct.

   Hence $Q\ is\ a\ loop\ invariant.$

   At loop termination $i = n,\ j_k = \max\{a[1], \ldots, a[n]\}$. The function is correct.

Additional Problems

A. $(10k+i)^2 = 100k^2 + 20ki + i^2$ whose end is determined only by i^2
   where $0 \le i \le 9$
   a. i=0, $(10k+i)^2$ end with 0
   b. i=1, $(10k+i)^2$ end with 1
   c. i=2, $(10k+i)^2$ end with 4
   d. i=3, $(10k+i)^2$ end with 9
   e. i=4, $(10k+i)^2$ end with 6
   f. i=5, $(10k+i)^2$ end with 5
   g. i=6, $(10k+i)^2$ end with 6
   h. i=7, $(10k+i)^2$ end with 9
   i. i=8, $(10k+i)^2$ end with 4
   j. i=9, $(10k+i)^2$ end with 1

Obviously from above, any square of positive integer end with 0,1,4,5,6, or 9.

B. Determine which Fibonacci numbers are even and use a form of mathematical induction to prove your hunch.

1. Base case F (0) = 0, F (3) = 2
   Assume for any positive integer k which can be divided by 3, $F(k)$ is even
   Then F(K+3) = F(K+2) + F(K+1) = F(K+1) +F(K) +F(K+1) = 2F(K+1) + F(K) which F(K) is even. So F(k+3) is even.
   Hence, for any positive integer k which can be divided by 3, $F(k)$ is even.