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1.

(a) T(n) = 9T(n/3) + n^2 = 1 for n > 2, otherwise T(n) = 1.

a = 9, b = 3
$log_b a = 2;$
As described in situation 2 T(n) = $\Theta(n^2 \log n)$

(b) a = 6, b = 2
$\epsilon = log_2 6 - 2.4$ > 0 is a constant
f(n) = n^2.4 = O($n^{log_2 6-\epsilon}$)
Hence, as described in situation 1, T(n) = $\Theta(n^{log_2 6})$

(c) a = 12, b = 4
f(n) = n^2 = $\Omega(n^{log_4 12+0.1})$
$\epsilon =$ 0.1 > 0 is a constant

Also, 12f($\frac{n}{4}$) $\leq cf(n)$

$$\frac{12\mathrm{n}^2}{16} \leq cf(n)$$
$$\frac{3f(n)}{4} \leq cf(n)$$

for constant c = .75 < 1

so, as described in situation 3, T(n) = $\Theta(n^2)$

2.

Base case:

$T(n) = 1 < 12 <= 12*n$, for $n < 4$ and n is positive integer.

Assume:

For $4 <= m < n$, $T(m) = T(\lfloor 2m/3 \rfloor) + T(\lfloor m/4 \rfloor) + m < 12m$ is right

To Prove:

$T(n) = T(\lfloor 2n/3 \rfloor) + T(\lfloor n/4 \rfloor) + n <= 12n$

Prove:

Left Hand Side = $2n * 4 + n *3 + n = 12n <= 12n$

Proved.

3.

0		cn		
1	$c(\lfloor 3n/5 \rfloor)$		$c(\lfloor 2n/5 \rfloor)$	
2	$c(3\lfloor 3n/5 \rfloor/5)$	$c(2\lfloor 3n/5 \rfloor/5)$	$c(3\lfloor 2n/5 \rfloor/5)$	$c(2\lfloor 3n/5 \rfloor/5)$
		……		
i				

if level i is the longest simple path from the root to a leaf.

Because 3/5 path will be the last path to meet $n < 2$ which means $O(1)$ requirement.

Then, $n \rightarrow (3/5)n \rightarrow n(3/5)^2 \rightarrow \ldots \rightarrow 1$

Since, $n(3/5)^k = 1$ when $k = log_{5/3}n$.

Height of the tree is k.

We expect the solution to the recurrence to be at most the number of levels times the cost of each level.

So where c is a constant, $T(n) = O(cnlog_{5/3}n) = O(nlogn)$