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**ECE369 hw4**

**Section4.1**

**17. Find the reflexive, symmetric, and transitive closure of each of the relations in Exercise 10.**

1. Reflexive closure of ρ = ρ

Symmetric closure of ρ = {(0,0),(1,1),(2,2),(4,4),(6,6),(0,1),(1,0),(1,2),(2,1),(2,4),(4,2),(4,6),(6,4)}

Transitive closure of ρ = {(0,0),(1,1),(2,2),(4,4),(6,6),(0,1,(1,2),(0,2),(2,4),(0,4),(4,6),(2,6),(0,6))

1. ρ = {(0,1), (1,0), (2,4), (4,2), (4,6), (6,4)}

Reflexive closure of ρ = {(0,0),(1,1),(2,2),(4,4),(6,6),(0,1),(1,0),(2,4),(4,2),(4,6),(6,4)}

Symmetric closure of ρ = ρ

Transitive closure of ρ = {(0,1),(1,0),(2,4),(4,2),(4,6),(6,4),(0,0),(1,1),(2,2),(4,4),(6,6)}

1. Reflexive closure of ρ = {(0,1),(1,2),(0,2),(2,0),(2,1),(1,0),(0,0),(1,1),(2,2),(4,4),(6,6)}

Symmetric closure of ρ = ρ

Transitive closure of ρ = ρ

1. Reflexive closure of ρ = ρ

Symmetric closure of ρ = ρ

Transitive closure of ρ = ρ

1. Reflexive closure of ρ = {(0,0),(1,1),(2,2),(4,4),(6,6)}

Symmetric closure of ρ = ρ

Transitive closure of ρ = ρ

**19. Two additional properties of a binary relation ρ are defined and answer the questions.**

1. **Prove that if ρ is an asymmetric relation on a set S, then ρ is irreflexive.**

Suppose ρ is asymmetric relation.

Then for any x ∈ S, (x, x) ∉ ρ

⇒ ∀x ∈ S, (x, x) ∉ ρ

Therefore, ρ is not irreflexive.

1. **Prove that if ρ is an irreflexive and transitive relation on a set S, then ρ is asymmetric.**

Suppose ρ is irreflexive and transitive relation

Let (x, y) ∈ ρ, then we need to show that (y, x) ∉ ρ

Suppose if (y, x) ∈ ρ

Then (x, y), (y, x) ∈ ρ and ρ is transitive

However, (x, y), (y, x) ∈ ρ is a contradiction for ρ is irreflexive

Therefore, (x, y) ∈ ρ ⇒ (y, x) ∉ ρ ρ is asymmetric。

**Section 4.4**

1. **Let P be the power set of {a, b, c}. A function f: P → Z follows: For A in P, f(A) = the number of elements in A. Is f one-to-one? Prove or disprove. Is f onto? Prove or disprove.**

P = {φ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

f(A) = number of elements in A

f is a function

f is not one-to-one function, proof:

f({a, b}) = 2

f({b, c}) = 2

f({a, b}) = f({b, c})

{a, b} ≠ {b, c}

f is not onto function, proof:

4 ∈ Z, but 4 has no pre-image in P.

1. **Let S = {P, Q, R} and T = {k, l, m, n}**
2. **Find the number of functions from S to T.**

The number of functions from S to T is 43 = 64.

1. **Find the number of injective functions from S to T.**

The number of injective functions from S to T is 4×3×2 = 24.

**Problem A.**

**You want to compute the big-O estimate for the running time of your algorithm. Will it be O(n log(n!)) or will it be O(n2 log n). Prove your answer.**

O(n log(n!)) = O(n2 log n)

Proof:

In order to compare O(n log(n!)) and O(n2 log n), we need to compare O(log(n!)) and O(n log n)

1. log(n!) = log(1) + log(2) + … + log(n-1) + log(n)

≤ log(n) + log(n) + … + log(n) + log(n)

= n log(n)

= O (n log n)

1. log(n!) = log(1) + …+ log(n/2) + … + log(n)

≥ 0 + … + log(n/2) + … + log(n/2)

= (n/2) log(n/2)

= O((n/2) log(n/2))

= O (n log n)

Combine 1 and 2 we get, O (n log n) ≤ O(log(n!)) ≤ O (n log n)

So, O(log(n!)) = (n log n)

Therefore,

O(n log(n!)) = O(n2 log n)