ACE Coursework 1 (20 marks)

Question 1 (4 marks)

Let P be the program:

$$
\begin{array}{l}
y = x + z; \\
if\ (y \geq 0)\{ \\
\qquad y = a \times y; \\
\}\ else\{ \\
\qquad y = -a \times y; \\
\}
\end{array}
$$

Show that $\vdash_{par} \{\top\}\ P\ \{y = a \times |x + z|\}$ is valid.

Question 2 (7 marks)

Let P be the program:

$$
\begin{array}{l}
max = i; \\
j = i + 1; \\
while\ (y < arr.length)\{ \\
\quad if(arr[j] > arr[max])\{ \\
\quad\quad max = j; \\
\quad \} \\
\quad j = j + 1; \\
\}
\end{array}
$$

Show that $\vdash_{tot} \{i < arr.length\}\ P\ \{\forall k \in [i, arr.length) :\ arr[max] \geq arr[k]\}$ is valid.

Question 3 (9 marks)

```
1   public static void sort(int[] arr) {
2        int len = arr.length;
3        for(int i = 0; i < len-1; i++){
4             for(int j = len-1; j > i; j--){
5                  if( arr[j] < arr[j-1]){
6                      int temp = arr[j];
7                       arr[j] = arr[j-1];
8                       arr[j-1] = temp;
9                  }//end if
10              } // end inner for loop
11       }//end outer for loop
12  }
```

1. The partial correctness of the Java program above can be proved by proving the loop invariant of the inner for loop and the loop invariant of the outer for loop. What is the loop invariant of the inner for loop? What is the loop invariant of the outer for loop? You may write your answer using either logical expressions or their equivalent English expressions. [4 marks]
2. Prove the loop invariant of the inner for loop by mathematical induction. [5 marks]