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Answers for Coursework 1

Question 1

$\vdash_{par}$ {T}   *Implied*

$y = x + z;$

$\{y \geq 0 \to y = |x + z| \wedge y < 0 \to y = -|x + z|\}$ *Assignment*

$if(y \geq 0)\{$

$\{|x + z| = y\}$ *Implied*

$\{a \times |x + z|\} = a \times y\}$ *Assignment*

  $y = a \times y;$

$\}\ else\{$

$\{|x + z| = -y\}$ *Implied*

$\{a \times |x + z|\} = -a \times y\}$ *Assignment*

  $y = -a \times y;$

$\}$

$\{y = a \times |x + z|\}$ *Implilied*

Question 2

$\vdash_{tol} \{i < arr.length\}$

$\{\forall k \in [i, i + 1): arr[i] \geq arr[k]\}$ *Implied*

$\boldsymbol{max = i}$;

$\{\forall k \in [i, i + 1) : arr[max] \geq arr[k]$ *Assignment*

$\boldsymbol{j = i + 1}$;

$\{\forall k \in [i, j) : arr[max] \geq arr[k]\}$ *Assignment*

$\boldsymbol{while(j < arr.length)}\{$

$\{\forall k \in [i, j): arr[max] \geq arr[k] \ \wedge\ j < arr.length \ \wedge\ 0 < arr.length - j = E_0\}$ *Invariant*

$\{(arr[j] > arr[max]) \to (\forall k \in [i, j + 1): arr[j] \geq arr[k] \ \wedge\ 0 < arr.length - (j + 1) < E_0) \wedge (arr[j] \leq arr[max]) \to (\forall k \in [i, j + 1): arr[max] \geq arr[k] \ \wedge\ 0 < arr.length - (j + 1) < E_0)\}$ *Implied*

  $\boldsymbol{if(arr[j] > arr[max])}\{$

$\{\forall k \in [i, j + 1) : arr[j] \geq arr[k] \wedge 0 < arr.length - (j + 1) < E_0\}\}$ *Assignment*

    $\boldsymbol{max = j}$;

  $\}$

$\{\forall k \in [i, j + 1) : arr[max] \geq arr[k] \wedge 0 < arr.length - (j + 1) < E_0\}$ *Assignment*

  $\boldsymbol{j = j + 1}$;

$\{\forall k \in [i, j): arr[max] \geq arr[k] \ \wedge 0 < arr.length - j < E_0\}$ *Assignment*

$\}$

$\{\forall k \in [i, j): arr[max] \geq arr[k] \ \wedge 0 < arr.length - j < E_0\}$ *Total − while*

$\{\forall k \in [i, arr.length) : arr[max] \geq arr[k]\}$ *Implied*

Question 3

1. Loop invariants
   a. Loop invariant for the inner loop: $\{\forall k \in [j, len - 1] : arr[j] \leq arr[k]\}$
   b. Loop invariant for the inner loop: $\{\forall k \in [0, i) : arr[k - 1] \leq arr[k]\}$

2. Answer:
   Convert the for-loop into a while-loop:

```
for(int j = len − 1; j > i; j − −){
        if(arr[j] < arr[j − 1]){
                int temp = arr[j];
                arr[j] = arr[j − 1];
                arr[j − 1] = temp;
        }
```

}

```
int j = len − 1;
while(j > i){

    if(arr[j] < arr[j − 1]){
        int temp = arr[j];

        arr[j] = arr[j − 1];

        arr[j − 1] = temp;

    }

    j = j − 1;
```

$\{\forall k \in [j, len-1] : arr[j] \leq arr[k]\}$

}

Assumptions:
When $j = len - 1$, $\{\forall k \in [j, len-1] : arr[j] \leq arr[k]\}$ is true.
Assume that when $j = len - n (n \in Z^+)$, $\{\forall k \in [j, len-1] : arr[j] \leq arr[k]\}$ is true,
When $j = len - (n+1)$, now need to prove that $\{\forall k \in [j, len-1] : arr[j] \leq arr[k]\}$ is true,

Proof: $To\ prove\ that\ \{\forall k \in [len-(n+1), len-1] : arr[len-(n+1)] \leq arr[k]\}$
$since\ \{\forall k \in [len-n, len-1] : arr[len-n] \leq arr[k]\},$
$Let\ small_0 = arr[len-n],\ small_1 = arr[len-(n+1)]$

$Go\ into\ the\ loop:$
$if\ arr[len-(n+1)] \leq arr[len-n]\ (which\ means\ that\ small_0 \geq small_1), then\ skip\ the\ if-statement, and\ after\ one\ loop: arr[len-(n+1)] < arr[len-n];$

$if\ arr[len-(n+1)] > arr[len-n], (which\ means\ that\ small_0 < small_1) then\ go\ into\ the\ if-statement,$
$temp = arr[len-n] = small_0;$
$arr[len-n] = arr[len-(n+1)] = small_1;$
$arr[len-(n+1)] = temp = small_0;$
$after\ one\ loop: arr[len-(n+1)] = small_0 < arr[len-n] = small_1;$

$After\ the\ loop, in\ either\ condition, arr[len-(n+1)] < arr[len-n];$
$Since\ \{\forall k \in [len-n, len-1] : arr[len-(n+1)] = small_0 \leq arr[k]\} \wedge arr[len-(n+1)] < arr[len-n],$
$thus, \{\forall k \in [len-(n+1), len-1] : arr[len-(n+1)] \leq arr[k]\}, proved.$