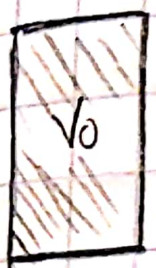
Taller 02

1.



u_1

$\cdot\ u_1 = V_0$

$u_2 = \frac{1}{3} u_1$

$u_3 = \frac{1}{3} u_2$

$\rightarrow \quad u_n = \frac{1}{3} u_{n-1}, \quad n \geq 1$

$\boxed{u_n = k^{n-1} u_1}, \quad k = \frac{1}{3}$

Para $u_n = (1/1000\,000)\, V_0$

$$\frac{\cancel{V_0}}{1\cdot 10^6} = \frac{1}{3}^{n-1} \cancel{V_0}$$

$$\log(1/10^6) = (n-1)\log(1/3)$$

$$n = \frac{\log(1) - \log(10^6)}{\log(1) - \log(3)} + 1$$

$$\boxed{n = 13.57 \approx 14}$$

Despues de 14 acciones habra menos de $V_0(10^{-6})$

2.

U : Población
n : año
r : Tasa

$1000(r) = 25$
$r = 1/40$

$\longrightarrow$ $U_n = U_{n-1}(1+r)$

$$\boxed{U_n = (1.025)^{n-1} U_1}, \quad K = (1+r)$$

Para $n = 15$, $U_1 = 200 \cdot 10^6$

- $U_{15} = 1.025^{14} \cdot 200 \cdot 10^6$

$$= \boxed{282.59 \cdot 10^6 \text{ personas}}$$

Para $U_1 = 200 \cdot 10^6$, $U_n = 750 \cdot 10^6$

- $750 \cdot \cancel{10^6} = (1.025)^{n-1}\, 200 \cdot \cancel{10^6}$

$\log(750/200) = (n-1)\log(1.025)$

$$n = \frac{\log(750) - \log(200)}{\log(1.025)} + 1$$

$n = 54.53$
$\approx \boxed{55}$

Despues de 55 años habra mas de $750 \cdot 10^6$ personas

3.

- $u_n = 4u_{n-1} - 1$, $n \geq 2$
  $k = 4$, $c = -1$

De modo que $u_n = ku_{n-1} + c$, entonces:

$$u_n = k(ku_{n-2} + c) + c = k[k(ku_{n-3} + c) + c] + c \cdots$$

$$= k^{n-1}u_1 + k^{n-2}c + \cdots + kc + c$$
$$= k^{n-1}u_1 + c[k^{n-2} + k^{n-3} + \cdots 1]$$

Veamos ahora que $P_n : \sum_{i=2}^{n} k^{n-i} = \frac{(k^{n-1} - 1)}{k-1}$, $n \geq 2$, $k \neq 1$:

1. Caso base $n=2$

$$\sum^{2} k^{n-i} = k^{2-2} = k^0 = 1$$, Adicionalmente

$$\frac{k^{2-1} - 1}{k-1} = \frac{k-1}{k-1} = 1$$

Donde $\boxed{1 = 1}$, por lo que P_2 se cumple.

2. Caso Inductivo.

Sea $n \in \mathbb{Z}$ cualquiera, tal que $n \geq 2$ y P_n se cumple. Operando a partir de lo anterior; tomando cualquier $k \neq 1$:

$$P_{n+1} = k^{n-1} + [k^{n-2} + k^{n-3} + \cdots + 1]$$
$$= k^{n-1} + P_n$$, Donde $P_n = \frac{k^{n-1} - 1}{k-1}$
$$= k^{n-1} + (k^{n-1} - 1)/(k-1)$$
$$= \frac{k^{(n+1)-1} - k^{n-1} + k^{n-1} - 1}{k-1}$$

$$= \frac{k^{(n+1)-1} - 1}{k-1}$$

De modo que P_{n+1} se cumple dado P_n.

Q.E.D

Así pues,

$$U_n = k^{n-1} U_1 + c\frac{k^{n-1}-1}{k-1}$$; Remplazando

$$\boxed{U_n = 4^{n-1} U_1 - \frac{1}{3}(4^{n-1}-1)}$$

- $U_n = 3U_{n-1} + 2$, $n \geq 2$

$k = 3$, $c = 2$

Analogamente,

$$\boxed{U_n = 3^{n-1} U_1 + 2(3^{n-1}-1)}$$

4.

- $U_n = -4U_{n-1} - 3$, $n \geq 1$

Siguiendo la demostración realizada en el punto 3.

$$\boxed{U_n = -4^{n-1} U_1 + \frac{3}{5}(-4^{n-1}-1)}$$

- $U_n = -2U_{n-1} + 13$, $n \geq 1$

Similarmente.

$$\boxed{U_n = -2^{n-1} U_1 - \frac{13}{3}(-2^{n-1}-1)}$$

5. $\cdot\ u_n = -2u_{n-1} + 6\ ,\ u_1 = 3$

$\cdot\ U_n = 3U_{n-1} + 5\ ,\ n \geq 1$

$U_0 = 1 \longrightarrow$ Lo que implica que existe un termino adicional para llegar a el n-esimo termino, con respecto a la formula previamente obtenida.

$$U_n = 3^{n-1+1}U_0 + \frac{5}{2}(3^{n-1+1} - 1)$$

$$= 3^n U_0 + \frac{5}{2}(3^n - 1)$$

a.

$$\boxed{= \frac{1}{2}[\, 7(3^n) - 5 \,]}$$

b.

$$\boxed{U_n = -2[\, 3(2^{n-2}) - 1 + (-2^{n-1}) \,]}$$

6.

$u_1 = 7$ $\rightarrow$ $u_2 - u_1 = 10$ $\rightarrow$ $u_1 = 7$ $\rightarrow$ $u_n = u_{n-1} + 10(2^{n-2})$

$u_2 = 17$ $\quad u_3 - u_2 = 20$ $\quad u_2 = u_1 + 2^0(10)$

$u_3 = 37$ $\quad u_4 - u_3 = 40$ $\quad u_3 = u_2 + 2^1(10)$

$u_4 = 77$ $\quad u_4 = u_3 + 2^2(10)$

$$\boxed{\begin{array}{l} u_n = u_{n-1} + 5(2^{n-1}) \\ n \geq 2 \end{array}}$$

Para Hallar la solución general:

$u_n = u_{n-1} + 5(2^{n-1})$

$= (u_{n-2} + 5^{n-2}) + 5 2^{n-1}$

$= [(u_{n-3} + 5 2^{n-3}) + 5 2^{n-2}] + 5 2^{n-1}$

$= u_1 + 5[2^1 + 2^2 + \cdots + 2^{n-1}]$

$\checkmark \sum_{i=1}^{n-1} 2^i = \sum_{i=0}^{n-1} 2^i - 1 = 2^n - 1 - 1$

$= 2^n - 2$

$u_n = u_1 + 5(2^n - 2)$

$= 7 + (5)(2)(2^{n-1} - 1)$

$$\boxed{\begin{array}{l} u_n = 7 - 10[1 - 2^{n-1}] \\ n \geq 2 \end{array}}$$

7.

$U_0 = 400$ M
$t = 3$ años
$i = 0.21$
x: pago anual

Para culminar el pago en 3 años, se tiene.
$U_3 = 0$.

Donde, $U_n = U_{n-1}(1+i) - x$ (interes compuesto)

Resolviendo, para $k = 1.21$ y $C = -x$

$$U_n = (1.21)^n 400 - x\frac{(1.21^n - 1)}{0.21}, \quad \text{con } n = 3$$

$$U_3 = 0 = (1.21)^3\, 400 - x(1.21^3 - 1)/0.21$$

$$x = \frac{(1.21)^3 \cdot 400}{(1.21^3 - 1)} \cdot 0.21$$

$$\boxed{x = 142.87 \cdot 10^6}$$

Pagando de a 142.37 millones anuales se salda la deuda en 3 años.

8. La tasa de producción crece todos los meses, en un 1% :

$r_1 = 200(1.01)$

$r_2 = [200(1.01)]1.01$ $\longrightarrow$ $r_n = 200(1.01)n$

$r_3 = [200(1.01)(1.01)]1.01$, $= 202n$

Donde,

$202n - 1600 = 0$

$n = 1600/202$

$= 7.92 \approx 8$

Es decir, que solo desde el mes 8, la producción supera las ordenes de 1600 T esperadas.

Toda la producción previa a este punto se hace 0, pues el producido es menor que el maximo de lo que se puede vender.

Asi pues, para el café acumulado:

$\underline{U_n = (202n - 1600) + U_{n-1}}$, $n \geq 8$

$U_7 = 0$ $(U_{0 \leq n \leq 7} = 0)$

Para $n = 12$

$U_{12} = 202[6(13) - 28] - 5(1600)$

$= 10100 - 8000$

$\boxed{U_{12} = 2100\ T}$

Para $n = 24$

$U_{24} = 202[12(25) - 28] - 17(1600)$

$\boxed{U_{24} = 27744\ T}$

$U_n = U_{n-2} + [202(n-1) - 1600] + [202n - 1600]$

$= U_7 - (n-7)1600 + 202[8+9+\cdots+n]$

$\sum_{i=8}^{n} i = \sum_{i=1}^{n} i - \sum_{i=1}^{7} i$

$= \frac{1}{2}n(n+1) - 28$

$U_n = U_7 - (n-7)1600 + 202[\frac{1}{2}n(n+1) - 28]$

$U_n = 202[\frac{1}{2}n(n+1) - 28] - 1600(n-7)$

4.

$U_0 = 2000$, $U_n = U_{n-1}(1.05) + 100$

$$U_n = (1.05)^n \, 2000 + \frac{100}{0.05}(1.05^n - 1)$$

$$= 2000\left[(1.05)^n + 1.05^n - 1 \right]$$

$$= 2000\left[2(1.05)^n - 1 \right]$$

para $n = 10$

$$u_{10} = 2000 [2(1.03)^{10} - 1]$$

$$= 2000 [2.258]$$

$$= 4515.58 \approx 4515 \text{ arboles}$$ (No existen arboles decimales)

De modo que, para la productividad [P],

$$P = \frac{(u_{10} - u_0)}{u_0} \cdot 100$$

$$= \boxed{125.75\,\%}$$

Hubo una mejora del 125.75% en la plantación.

10.

$u_n = 3u_{n-1} + n$, $u_1 = 5 \longrightarrow u_2 = 17$

$\rightarrow u_n - 3u_{n-1} = n$, Ecuación no homogenea de la forma $u_n = a + bn$.

Solución homogenea:

$u_n - 3u_{n-1} = 0$

$m = \frac{3 \pm 3}{2}$ | $m_1 = 0$, $m_2 = 3$ | $U_n = A\,3^n$

Solución particular:

$(a+bn) - 3[a + b(n-1)] = n$

$a - 3a + bn - 3bn + 3b = n$

$3b - 2(a + bn) = n$

$(3b - 2a) + (-2bn) = n$

✓ $-2b = 1$

$b = -1/2$

✓ $3b - 2a = 0$

$2a = -3/2$

$a = -3/4$

Así pues,

$U_n = A(3^n) - 3/4 - \frac{1}{2}n \longrightarrow$ Para U_1

$5 = A(3) - 3/4 - 1/2$

$A = \frac{1}{3} \cdot \frac{25}{4} = \frac{25}{12}$

$$\boxed{U_n = \frac{25}{12}(3^n) - \frac{3}{4} - \frac{1}{2}n}$$

11.

- $u_n = u_{n-1} + 2^n$, no homogenea de la forma k^n
$u_n - u_{n-1} = 2^n$ donde $u_n = a k^n$

1. $u_n - u_{n-1} = 0$
$m^2 - m = 0$
$m(m-1) = 0$
$m_1 = 0, m_2 = 1$

| $u_n = A(1)^n$
$= A$

2. $a2^n - a2^{n-1} = 2^n$
$a2^n [1 - 2^{-1}] = \cancel{2^n}$
$a = 2$

| $2(2^n) - 2(2^{n-1}) = 2^n$
$2^n [2 - 2^0] = 2^n$
$2^n [1] = 2^n$
$2^n = 2^n$ ✓

$\rightarrow$ $\boxed{u_n = 2^{n+1} + A}$

- $u_n = 2u_{n-1} + n$, no homogenea de la forma n
$u_n - 2u_{n-1} = n$ donde $u_n = a + bn$

1. $u_n - 2u_{n-1} = 0$
$m(m-2) = 0$
$m_1 = 0, m_2 = 2$

| $u_n = A(2^n)$

2. $(a + bn) - 2(a + b(n-1)) = n$
$a - 2a + bn - 2bn + 2b = n$
$(2b - a) + (-bn) = n$
✓ $b = -1$ ✓ $2b - a = 0$
$a = -2$

| $u_n = -2 - n$

$\rightarrow$ $\boxed{u_n = A(2^n) - 2 - n}$

12.

$U_n = k U_{n-1} + 5$

$u_1 = 4$

$u_2 = 17$

· Hallar k

$U_2 = k U_1 + 5$

$17 = k(4) + 5$

$k = 12/4$

$\boxed{k = 3}$

· Hallar U_6

$$\rightarrow U_n = (3)^{n-1} \cdot U_1 + 5 \frac{[3^{n-1} - 1]}{2}$$

$$U_6 = 3^5 (4) + \frac{5}{2} (3^5 - 1)$$

$$= 243(4) + \frac{5}{2}(242)$$

$$= \boxed{1577}$$

13.

$$u_n = \frac{u_{n-1}}{u_{n-2}} \qquad n \geq 2 \quad , \quad u_1 = 1/6$$

0. Condición inicial, $u_1 = [6]^{-1}$ $\bullet$

1. $u_2 = \dfrac{u_1}{u_0} = \dfrac{1}{6u_0} = [6u_0]^{-1}$ $\star$

2. $u_3 = \dfrac{u_2}{u_1} = \dfrac{6}{6u_0} = [u_0]^{-1}$ $\blacktriangle$

3. $u_4 = \dfrac{u_3}{u_2} = \dfrac{6u_0}{u_0} = [6]$

4. $u_5 = \dfrac{u_4}{u_3} = [6u_0]$

5. $u_6 = u_5/u_4 = \dfrac{6u_0}{6} = [u_0]$

6. $u_7 = u_6/u_5 = \dfrac{u_0}{6u_0} = [6]^{-1}$ $\bullet$

7. $u_8 = u_7/u_6 = [6u_0]^{-1}$ $\star$

8. $u_9 = u_8/u_7 = 6/6u_0 = [u_0]^{-1}$ $\blacktriangle$

$\vdots$

Para $k \in \mathbb{Z}^+$

$$\boxed{u_n \begin{cases} [6]^{-1}, & n = 1 + 6k \\ [6u_0]^{-1}, & n = 2 + 6k \\ [u_0]^{-1}, & n = 3 + 6k \\ [6], & n = 4 + 6k \\ [6u_0], & n = 5 + 6k \\ [u_0], & n = 6 + 6k \end{cases}}$$

Para algun u_0

14. $\frac{u_n}{u_{n+1}}$

$$u_n = u_{n-1} + 2u_{n-2}$$

para $u_n - u_{n-1} - 2u_{n-2} = 0$

$m = 1 \pm \sqrt{1+8} / 2$ | $m_1 = 2$

$m = (1 \pm 3)/2$ | $m_2 = -1$

$$U_n = A(2)^n + B(-1)^n$$

De modo que:

$$\frac{u_n}{u_{n+1}} = \frac{A(2)^n + B(-1)^n}{A(2)^{n+1} + B(-1)^{n+1}} \div \frac{(-1)^n}{(-1)^n} =$$

$$= \frac{A(-2)^n + B(1)^n}{2A(-2)^n + (-1)B(1)^n}$$

$$= \frac{A(-2)^n + B}{2A(-2)^n + B}, \quad \text{con } n \in \mathbb{Z}^+ . \to \infty$$

1. $$\lim_{n\to\infty} \frac{A\frac{(-2)^n}{(-2)^n} + \frac{B}{(-2)^n}}{2A\frac{(-2)^n}{(-2)^n} - \frac{B}{(-2)^n}},$$

n: par $\to \lim_{n\to\infty^-} \frac{B}{(-2)^n} = 0$

n: impar $\to \lim_{n\to\infty^+} \frac{B}{(-2)^n} = 0$

$$= \frac{A+0}{2A-0} = \frac{A}{2A} = \frac{1}{2}$$

$$\Rightarrow \boxed{\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \frac{1}{2}}$$

13.

$-3, \; 21, \; 3, \; 124, \; 147, \; \ldots$

① ② ③ ④ ⑤ ⓝ

$u_1 = -3$

$u_2 = (-1)^2(-3)^2 + 12$ → $(-1)^2(-3)^2 + (-2)^2(3)$

$u_3 = (-1)^3(-3)^3 + -24$ → $(-1)^3(-3)^3 + (-2)^3(3)$

$u_4 = (-1)^4(-3)^4 + 48$ → $(-1)^4(-3)^4 + (-2)^4(3)$

$u_5 = (-1)^5(-3)^5 - 96$ → $(-1)^5(-3)^5 + (-2)^5(3)$

$$u_n = (-1)^n u_1^n + 3(-2)^n$$

16.

$u_n - 6u_{n-1} + 8u_{n-2} = 0 \quad , \quad n \geq 3$

$u_1 = 10 \quad , \quad u_2 = 28$

$m = 6 \pm \sqrt{4} \;/ 2 \quad \Big| \quad m_1 = 4, \quad m_2 = 2$

$u_n = A(4)^n + B(2)^n$, Evaluando

$u_1 = 10$

$10 = 4 \cdot 4 + B \cdot 2$

$B = 5 - 2A$

$B = 5 - 2$

$B = 3$

$u_2 = 28$

$28 = 16A + 4B = 16A + 20 - 8A$

$= 8A + 20$

$28 - 20 = 8A$

$A = 1$

$$\boxed{u_n = 4^n + 3(2^n)}$$

Para u_6

$u_6 = 4^6 + 3(2^6) = 4096 + 192$

$= \boxed{4288}$

17.

$u_n + 2u_{n+1} + u_{n+2} = 0$, $n \geq 1$

$u_1 = -1$, $u_2 = -2$

$k = n+2$

$k-1 = n+1$

$k-2 = n$

$\rightarrow$ $u_k + 2u_{k-1} + u_{k-2} = 0$ $\boxed{x^2 + 2x + 1 = (x+1)^2}$

$m = -2 \pm \sqrt{4-4} / 2$ | $m_1 = -1$

$m = -2 \pm 0 / 2$ | $m_2 = -1$

$u_k = A(-1)^k + Bk(-1)^k$

$\hookrightarrow u_{n+2} = A(-1)^{n+2} + B(n+2)(-1)^{n+2}$

$u_n = A(-1)^n + Bn(-1)^n$, Despejando

$u_1 = -1$, $u_2 = -2$

$-1 = -A - B$, $-2 = A + 2B = A + 2 - 2A$

$B = 1 - A$

$-4 = -A$

$\downarrow$

$A = 4$

$B = 1 - 4$

$B = -3$

$\Rightarrow$ $\boxed{u_n = 4(-1^n) - 3n(-1^n)}$

13.

$$u_n - 5u_{n-1} + 6u_{n-2} = f(n)$$

✓ Homogeneous $[f(n)=0]$

$m^2 - 5m + 6 = 0$ | $m_1 = 2$

$(m-2)(m-3) = 0$ | $m_2 = 3$

$$* \; U_n = A2^n + B3^n \equiv U_{H.O}$$

• $f(n) = 2$, $U_n = a + bn$

$(a+bn) - 5[a + b(n-1)] + 6[a + b(n-2)] = 2$

$a - 5a + 6a + bn - 5bn + 6bn + 5b - 12b = 2$

$2a + 2bn - 7b = 2$

$2(a+bn) - 7b = 2$

$\Rightarrow -7b = 0$, $a + bn = 1$

$b = 0$, $a = 1$

$$\boxed{U_n = U_{H.O} + 1}$$

• $f(n) = n$, $u_n = a + bn$

$2bn + 2a - 7b = n$

$\Rightarrow 2b = 1$, $2a - 7b = 0$

$b = 1/2$, $4a - 7 = 0$

$a = 7/4$

$$\boxed{u_n = u_{H.O} + \frac{7}{4} + \frac{1}{2}n}$$

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- $f(n) = 5^n$, $u_n = a5^n$

$(a5^n) - 5(a5^{n-1}) + 6(a5^{n-2}) = 5^n$

$5^n(a)[1 - 1 + 6/25] = 5^n$

$a \cdot \frac{6}{25} = 1$

$a = 25/6$

$$\boxed{u_n = u_{H.O} + \frac{25}{6} 5^n}$$

- $f(n) = 1 + n^2$, $u_n = a + bn + cn^2$

$(a + bn + cn^2) - 5[a + b(n-1) + c(n-1)^2] + 6[a + b(n-2) + c(n-2)^2] = (1 + n^2)$

$(2a + 2bn - 7b) + cn^2 - 5c(n-1)^2 + 6c(n-2)^2 = (1 + n^2)$

$(\quad) + cn^2 - 5cn^2 + 10cn - 5c + 6cn^2 - 24cn + 24c = (1 + n^2)$

$(\quad) + 2cn^2 - 14cn + 19c = (1 + n^2)$

$(2cn^2 + 19c) + (2a + 2bn - 7b - 14cn) = (1 + n^2)$

$2c[n^2 + 1] + 17c + (2a + 2bn - 7b - 14cn) = [n^2 + 1]$

$2c[n^2 + 1] + n[2b - 14c] + (17c + 2a - 7b) = [n^2 + 1]$

$\Rightarrow 2c = 1$, $2b - 14c = 0$, $17c + 2a - 7b = 0$

$c = 1/2$, $2b = 7$, $17 + 4a - (7)(7) = 0$

$b = 7/2$, $4a = 32$

$a = 8$

$$\boxed{u_n = u_{H.O} + 8 + \frac{7}{2}n + \frac{1}{2}n^2}$$

4.

$u_n - 3u_{n-1} + 4u_{n-2} = 0$,

$\lambda^k - 3\lambda^{k-1} + 4\lambda^{k-2} = 0$

$\lambda^k [1 - 3/\lambda + 4/\lambda^2] = 0$

$\Rightarrow \lambda^2 - 3\lambda + 4 = 0$

$U_n = |\lambda|^k$ [Forma sol.]

Usando los auto valores del polinomio característico:

$\rightarrow \lambda^2 - 3\lambda + 4 = 0$, Donde $\lambda = \alpha + i\beta$

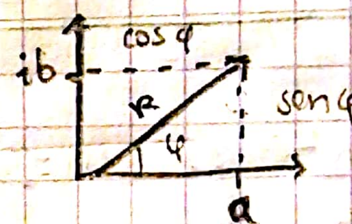
$\lambda = 3 \pm i\sqrt{7} / 2$

$\lambda_1 = \frac{1}{2}(3 + i\sqrt{7})$

$\lambda_2 = \frac{1}{2}(3 - i\sqrt{7})$

$\rightarrow \alpha = \frac{3}{2}, \quad \beta = \frac{1}{2}\sqrt{7}$

$R = |\lambda| = \sqrt{\alpha^2 + \beta^2} = 16/4 = 2$



$\{Re\} = \cos\varphi = \frac{\alpha}{|\lambda|} = \frac{3}{4}$

$\{Im\} = \operatorname{sen}\varphi = \frac{\beta}{|\lambda|} = \frac{1}{4}\sqrt{7}$

$\tan\varphi = \frac{\sqrt{7}}{4} \cdot \frac{4}{3} = \sqrt{7}/3$

$\varphi = \tan^{-1}(\sqrt{7}/3) = 0.72 \text{ rad}$

Con $\lambda = \alpha + i\beta = |\lambda| e^{i\varphi} = |\lambda| [\cos\varphi + i \operatorname{sen}\varphi]$

$\Rightarrow$ Para ecuaciones de 2º orden con raíces complejas:

* $e^{\alpha x}[A\cos(\beta x) + B\sin(\beta x)]$

Análogamente

$U_n = |\lambda|^n [A\cos(\varphi n) + B \operatorname{sen}(\varphi n)]$
$= 2^n [A\cos(0.72n) + B \operatorname{sen}(0.72n)]$, General

Donde $u_0 = 0$, $u_1 = 20$;

Despejando.

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$u_0 = 0$, $u_1 = 20$

$0 = A\cos(0) + B\,\text{sen}(0)$

$A = 0$

$20 = 2B\,\text{sen}(0.72) + 0$

$10 = B \cdot \frac{1}{4}\sqrt{7}$, $\text{sen}\,\varphi = \frac{1}{4}\sqrt{7}$

$B = \frac{1}{\sqrt{7}} 40$

$\Rightarrow u_n = 2^n \left[\frac{1}{\sqrt{7}} 40\, \text{sen}(\varphi n) \right]$, $\varphi = \tan^{-1}(\sqrt{7}/3)$

$n \geq 2$

10.

Fibonacci

20.

Fibonacci

$$u_n = u_{n-1} + u_{n-2}$$
$$u_n - u_{n-1} - u_{n-2} = 0$$
$$u_0 = 0, \quad u_1 = 1$$

$$g(x) = 1 + x + u_2 x^2 + u_3 x^3 + \dots$$
$$= 1 + x + (u_0 + u_1)x^2 + (u_2 + u_1)x^3 + \dots$$

$$g(x) = 1 + x + x^2[u_1 + u_2 x + u_3 x^2 + \dots] + x^2[u_0 + u_1 x + u_2 x^2 + \dots]$$
$$= 1 + x + x^2\left[\frac{g(x) - u_0}{x}\right] + x^2 g(x)$$
$$= 1 + x - u_0 x + g(x)x + g(x)x^2$$

$$g(x)[1 - x - x^2] = 1 + (x - x)$$

$$\boxed{g(x) = \frac{1}{(1 - x - x^2)}}$$

Para hallar la solución, se puede aplicar el metodo por autovalores, Donde $\boxed{u_n = \lambda^n}$

$$u_n - u_{n-1} - u_{n-2} = 0$$
$$\lambda^n - \lambda^{n-1} - \lambda^{n-2} = 0$$
$$(\lambda^2) \cdot \cancel{\lambda^n}[1 - 1/\lambda - 1/\lambda^2] = 0 \cdot (\lambda^2)$$
$$\lambda^2 - \lambda - 1 = 0$$
$$\lambda = 1 \pm \sqrt{1+4}\,/2$$
$$\lambda = 1 \pm \sqrt{5}/2$$

$$\lambda_1 = \tfrac{1}{2}(1 + \sqrt{5})$$
$$\lambda_2 = \tfrac{1}{2}(1 - \sqrt{5})$$

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$U_n = A\left[\frac{1}{2}(1+\sqrt{5})\right]^n + B\left[\frac{1}{2}(1-\sqrt{5})\right]^n$, general

Despejando.

$U_0 = 0$, $U_1 = 1$

$0 = A + B$

$B = -A$

$\downarrow$

$B = -\frac{1}{\sqrt{5}}$

$1 = \frac{A}{2}(1+\sqrt{5}) - \frac{A}{2}(1-\sqrt{5})$

$1 = \frac{A}{2}[2\sqrt{5}]$

$1 = \sqrt{5}A$

$A = \frac{1}{\sqrt{5}}$

$$\boxed{U_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]}$$

21. $U_n - 2U_{n-1} = 3^n$, $n \geq 1$
$U_0 = 1$

Aplicando el metodo por autovalores:

1. Homogenea, $U_n = \lambda^n$

$\lambda^n - 2\lambda^{n-1} = 0$
$\lambda^n [1 - 2/\lambda] = 0$
$\lambda - 2 = 0$

$\lambda = 2$
$U_n = A2^n$, formula completa

2. Particular, $U_n = B3^n$

$B3^n - 2B3^{n-1} = 3^n$
$B3^n [1 - 2/3] = 3^n$
$B = 1/(1/3)$
$B = 3$

$U_n = 3 \cdot 3^n$
$= 3^{n+1}$

3. General

$$U_n = A2^n + 3^{n+1}$$

Donde $u_0 = 1$

$$1 = A + 3$$

$$A = -2$$

$$\Rightarrow \boxed{U_n = 3^{n+1} - 2^{n+1}}$$