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Quick and Precise Clustering of Arbitrarily Shaped Flat Patterns Based on Stringy Effect

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Abstract

Grouping a given number of arbitrarily shaped flat patterns to form a cluster which occupies minimal-area convex enclosure is very useful in solving cutting stock problem. This study is aimed at improving the effectiveness of conventional clustering processes by incorporating a new technique for the determination of optimal conditions for the sliding process. The new technique is referred to as 'stringy effect' which is based on minimizing the distance between centroids of the patterns during clustering. The efficiency of the proposed method is shown with the help of some typical multiple flat patterns.

Keywords : Cutting Stock Problem, Clustering, Flat Pattern

1. Introduction

The cutting stock problem is of interest to many industries like garment, paper, ship building, aircraft and sheet metal industries for minimizing the use of raw materials required in the development of various products. Adamowicz [1] attempted a heuristic approach to solve the irregular cutting stock problem by dividing it into two sub-problems, called clustering and nesting. Clustering is to specify a collection of patterns that fit well together before nesting on a given stock sheet. When the size of the stock sheet is similar to that of the cluster, i.e. for small scale cutting stock problem, it is only necessary to fit the selected cluster on a given rectangular or irregular stock sheet without any self-duplication. In this case, the ability to generate clusters with the properties of minimal-area convex enclosure are highly appreciated. When the problem becomes large scale, i.e. the size of the stock sheet is much greater than that of the cluster, it is necessary to duplicate the cluster on the given stock sheet. In this case, the evaluation method by considering the area of convex enclosure of the cluster is widely adopted to determine the best cluster among the available candidates. Basically, no matter what kind of problem, the determination of the cluster with minimal-area convex enclosure is essential.

An important constraint that is often imposed is the orientation of patterns. For handling such situations, Adamowicz [1] suggested a geometrical method of generating a no-fit-polygon (NFP) which is used to guide the relative movement between two patterns with the consideration of overlapping between patterns. This method is also called as sliding technique in which a "movable" pattern slides along a "stationary" pattern. A typical sliding step is illustrated in Fig. 1(a) and (b). While sliding, the collision-free sliding distance that the movable pattern can slide without any interference is determined by projection. The clusters generated by this technique generally do not provide solutions near global optimization. By introducing the concept of breakpoint, Adamowicz and Albano [2] proposed an "extended search" which can finally lead to the generation of cluster with the properties of a minimal-area non-inclined rectangle enclosure. In general, the detection of minimal-area non-inclined rectangle enclosure is achieved by aligning patterns with a vertical or horizontal datum, as shown in Fig. 1(c). However, this approach is only suitable for finding the minimal-area non-inclined rectangle enclosure but not for obtaining the minimal-area convex polygon. To find the minimal-area convex enclosure, Grinde and Cavalier [3] attempted another geometrical method to generate breakpoints. Generally, this method is related to the study of the change in convex hull with each sliding step, as shown in Fig. 1(d) for one such step. Although it can guarantee the generation of minimal-area convex enclosure of two patterns, it is quite computation intensive due to the detection of joining edges and the analysis of the change in the convex hull while sliding.

It may be concluded that while the current approaches are mathematically and geometrically elegant, they are computationally intensive. It is also important to know whether these approaches work well for the clustering problem with multiple patterns instead of only two patterns. This study is focused in addressing these two problems by introducing some new approaches to the sliding techniques which have yielded significant results. These new approaches are introduced in the following sections.

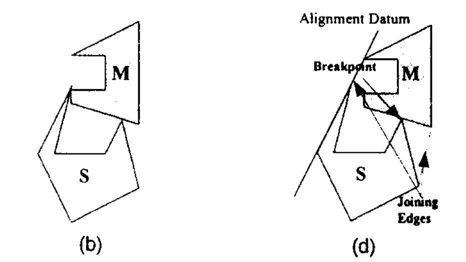


Fig. 1 (a) Detection of the collision-free sliding distance; (b) New cluster obtained as a result of sliding; (c) Adamowicz's approach; (d) Grinde's approach.

2. Representation of Pattern Geometry

Flat patterns with entities of lines and arcs only are considered presently. Polygonal representation methods are proposed to represent both concave and convex arcs as sets of straight lines. The determination of the number of dividing points is achieved through an algorithm [4] which relates the area of the original pattern to that of the approximate pattern, based on the accuracy requirements of the user. On the other hand, clearance or offset generation is an essential step that contributes towards the success of CAD/CAM technology. An algorithm to generate the required offset, called "Three Point Island Tracing (TPIT)" technique, is incorporated, and the details of this technique are presented in Reference [5].

3. Stringy Clustering Algorithm

The criterion to evaluate the best cluster among the generated nodes is by adopting a stringy effect that relates the feasibility of a cluster to its corresponding energy level, i.e., the distance between the center of gravity of patterns. Besides determining the best cluster, this principle can help in optimizing and detecting the possible relative motion between two patterns during the clustering process.

(a) Problem definition

How to collect N pieces of flat patterns together to form a cluster which occupies minimal-area convex enclosure is complicated (NP-Complete) but very often encountered in solving cutting stock problems. However, several types of enclosures may be defined based on the problem on hand (see Fig. 2). The area of convex enclosure is related to the convex hull of the union of patterns which can be imagined as a large rubber band surrounding the set of all vertices. The ratio of the total area of patterns to the area of enclosure, term as **Enclosure Efficiency Ratio (EER)**, reflects the compactness of the enclosure. Mathematically, a prefect enclosure is obtained when EER equals to 1; with lower values of EER indicating poor clusters.

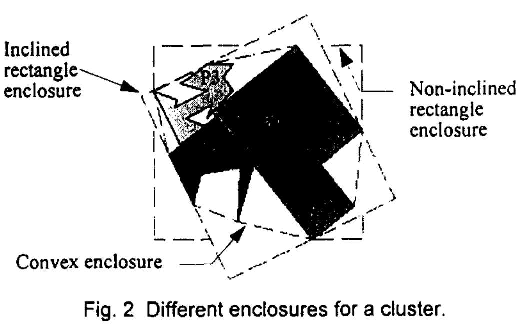


Fig. 2 Different enclosures for a cluster.

(b) Overview of the clustering system

The clustering system can be divided into 3 main modules. The first one is the preprocessing module which loads the flat patterns that are reference sets containing geometry and the optimization type (convex enclosure). A decision on the clustering sequence (CS) is essential when the number of patterns are greater than two. Nee [6] suggested a clustering sequence following the descending order of area of patterns, which is adopted in the present work. A processing module then applies the sliding technique to collect the patterns one by one together to form a cluster. A stringy effect is proposed and used in this work to detect any breakpoint in each intermediate clustering step. This concept also serves as an evaluation technique for the selection of best cluster. In the last clustering step, Grinde's approach is used to generate the minimal-area convex enclosure. The output module will display the best cluster and all computational results will be stored as specific system files for future use.

(c) Stringy effect of rigid body

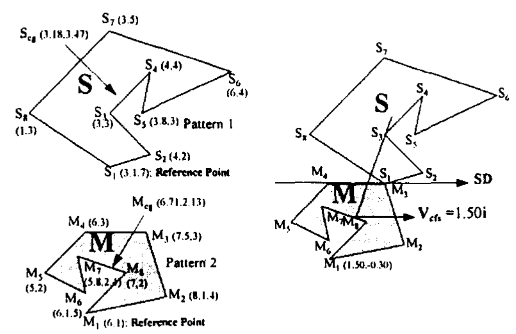
Following Newton's law, the attractive force (F) between two particles is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance (D_{cg}) between them. It is possible to map the above phenomenon on the clustering problem with two arbitrary flat patterns which have the same thickness. Suppose A_s and A_m denote the area of these two patterns and G is a constant, we have :

$$F = G \cdot A_s \cdot A_m / D_{cg}^2 \tag{1}$$

In this case, the line of force can be regarded as a string that connects the centroids of these two patterns when the movable pattern slides around the stationary pattern. From one iteration to the next, an attempt is made to position the patterns in contact with each other and with an intention to trace the location which provides the smallest value of D_{cg} between patterns, i.e. the attractive force (F) can be kept maximum while the stability of the system can be increased. This approach is called as "Stringy Effect".

(d) Proposed technique for sliding

To illustrate the working principle of the sliding algorithm with stringy effect, two typical patterns, shown in Fig. 3(a), with both concave and convex features are selected. The pattern that has greater area is designated as "Stationary Pattern" and the other as "Movable Pattern" and the vertices are arranged in a counter-clockwise direction and described by the ordered lists of vertices $\{S_i, i=0, .., n_s\}$ and $\{M_i, i=0, .., n_m\}$ respectively. In order to analyze the relative movement between patterns, the bottom-left vertices S_{ref} in the stationary pattern and M_{ref} in the movable pattern are defined as the reference points. The former will be kept constant throughout the sliding process while the latter will be continuously changing in each iteration. At the beginning, the centroid (S_{cg}) and the reference point (S_{ref}) of the stationary pattern are located at (3.18,3.47) and (3.00,1.70), respectively. Before coinciding the top-right vertex (7.50,3.00) of the movable pattern with the bottom-left vertex (3.00,1.70) of the stationary one, the centroid (M_{cg}) and the reference point (M_{ref}) of the movable pattern are (6.71,2.13) and (6.00,1.00), respectively. After repositioning, the centroid of the movable pattern is translated to (2.21,0.83) and the reference point to (1.50,-0.30), as shown in Fig. 3(b).



(a) (b)

Fig. 3 (a) Selected flat patterns for the sliding process; (b) Initial stage of the sliding process.

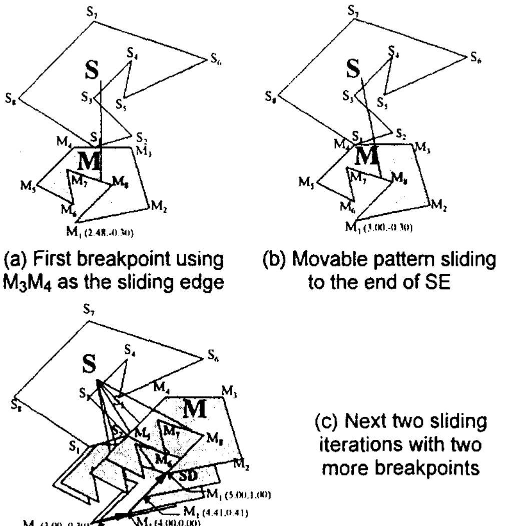
From one iteration to the next, the movable pattern slides around the stationary pattern following counterclockwise direction and maintaining connectivity between them. The collision-free sliding distance (D_{cfs}) that the movable pattern can slide without any interference is determined by a Branch-and-Bound and Freeman Code (BBFC) projecting technique. The details of this technique to calculate D_{cfs} are given in Reference [7]. Considering the sliding step as shown in Fig. 3(b), there is no feasible projecting region geometrically, i.e. no collision region, and hence the movable pattern slides from the start of the sliding edge M_4 (1.50,1.70) to the end of the sliding edge

M_3 (3.00,1.70) with D_{cfs} equal to edge M_3M_4 (1.50 units). A vector $\mathbf{V_{cfs}}$ with unit vector, magnitude and direction equal to $\mathbf{u_{cfs}}$, D_{cfs} and **SD** respectively is defined here. Geometrically, vector $\mathbf{V_{cfs}}$ starts from point M_{cg} and extends to M_{cfs} which is the terminal point of the collision-free sliding with respect to point M_{cg}. In the first sliding step, M_{cfs} and $\mathbf{V_{cfs}}$ are calculated as (3.71,0.83) and 1.5**i**, respectively. The following theorem is introduced for demonstrating the methodology to detect and locate the breakpoint in a given **SD**.

Theorem : Given a sliding direction (**SD**), the location of a breakpoint M_{os} which leads to shortest C.G. distance between patterns is obtained by projecting the centroid S_{cg} on the sliding edge (SE). At the breakpoint, the line of force between the patterns intersects SE perpendicularly.

Although the location of breakpoint can be successfully found, it is essential to test its position in order to assure that the breakpoint lies inside the sliding edge. There are two geometrical requirements, called the distance and directional constraints. The former limits the optimal sliding distance which must be smaller than that of the collision-free one to avoid interference. The latter avoids the movable pattern moving clockwise so that the system can be protected from sliding back. These constraints are summarized as :

1. Distance constraint : The optimal sliding distance must be smaller than the collision-free sliding distance, i.e. $D_{os} < D_{cfs}$;
2. Directional constraint : The optimal sliding direction must be same as that of the collision free sliding direction, i.e. $\mathbf{u_{cfs}} - \mathbf{u_{os}} = 0$ where $\mathbf{u_{os}}$ is the unit vector of the optimal sliding vector V_{os} (from M_{cg} to M_{os}) and its magnitude equals to D_{os}.



(a) First breakpoint using M_3M_4 as the sliding edge

(b) Movable pattern sliding to the end of SE

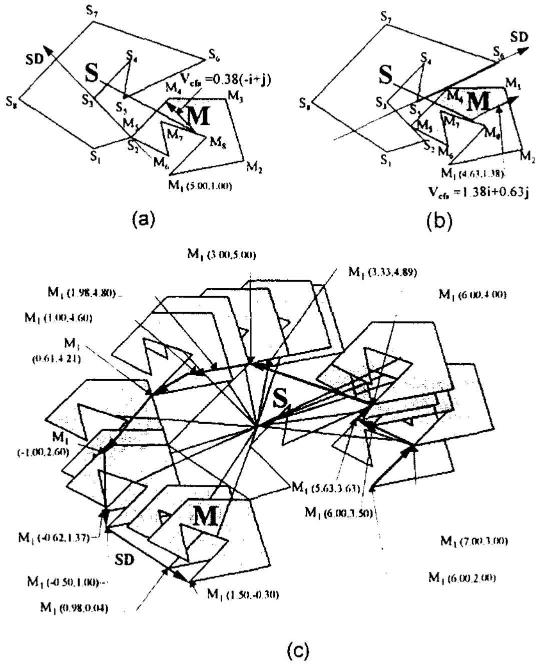
(c) Next two sliding iterations with two more breakpoints

Fig. 4 Generation of break points during sliding.

In the example, it is calculated that the location of breakpoint M_{os} and vector $\mathbf{V_{os}}$ are equal to (3.18,0.83) and 0.97**i**, respectively. In order to get the minimum C.G. distance along **SD**, the movable pattern is required to slide a distance of 0.97 unit, as shown in Fig. 4(a). This is the first breakpoint in the whole sliding process and the distance between centroids reaches a minimum in this **SD** and equals to 2.64 units. After reaching the breakpoint, the movable pattern can directly slide to the end of the sliding edge and so vertex M_4 is finally connected with vertex S_1, as shown in Fig. 4(b). By using the same technique, it was found that there are two more breakpoints when the movable pattern slides along edges S_1S_2 and M_4M_5 in Fig. 4(c). The figure also indicates the coordinates of M_{ref} in the consecutive sliding iterations.

When the sliding process is further continued as shown in Fig. 5(a), it is found that the next two iterations cannot meet the geometrical constraints. The movable pattern is connected with the stationary one at vertex S_2 and it will be sliding along **SD** defined by edge S_2S_3 on the stationary pattern. By projecting vertex (M_4) on the stationary pattern, distance D_{cfs} and vector $\mathbf{V_{cfs}}$ are calculated as 0.53 units and 0.38(-**i**+**j**), respectively. At this time, the location of breakpoint M_{os} and vector $\mathbf{V_{os}}$ are calculated as (3.78,4.06) and 1.93(-**i**+**j**). If the movable pattern slides to a distance equal to D_{os} (1.93 units), interference between patterns will happen as a result of distance constraint. Geometrically, distance D_{cfs} is in fact the optimal one in such sliding direction, as shown in Fig. 5(b).

Figure 5(b) shows a situation of not meeting the directional constraint. The movable pattern tries to slide along **SD** defined by edge S_5S_6. It is calculated that the value of D_{cfs} and vector $\mathbf{V_{cfs}}$ are equal to 1.51 units and 1.38I+0.63j, respectively. The coordinates of breakpoint and vector $\mathbf{V_{os}}$ are equal to (3.92,1.86) and -1.42i-0.65j,. **SD** is directed from the bottom-left to the top-right. In contrast, vector $\mathbf{V_{os}}$ is directed from the top-right corner to the bottom-left position, and hence does not satisfy the directional constraint. No matter it can satisfy the distance constraint or not, the movable pattern cannot slide to the breakpoint. Fig. 5(c) gives a summary of the rest of the sliding steps.



(a) (b)

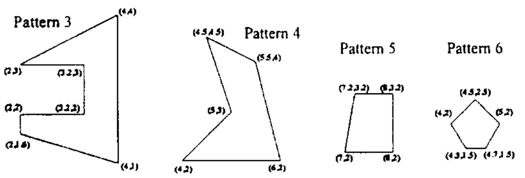
(c)

Fig. 5 (a) D_{os} greater than D_{cfs} – distance constraint; (b) Optimal sliding direction opposite to constraint the sliding direction (**SD**) – directional constraint; (c) Summary of further sliding steps.

For the chosen two patterns, it is found that the shortest C.G. distance is equal to 2.35 units in sliding iteration #8, which does not correspond to any breakpoint. Although the global minimum of C.G. distance is not obtained by the consideration of breakpoints in this case, it is possible to obtain the best solution in other cases. Upon finishing the sliding technique, i.e. the movable pattern returns to the initial position, the best cluster with shortest C.G. distance is generated and is then subjected to a grouping process [6] which extracts all internal and common boundaries of the cluster. The purpose of generating a "Grouped" cluster is to shorten the computational time in the subsequent operations by providing smaller number of entities. Finally, the centroid of the grouped cluster is calculated for use in the next clustering step.

4. Case Studies

A computer program has been developed in C++ object oriented language incorporating the new sliding technique described above based on stringy effect. This clustering system is a part of the main system **MANest** (**M**ulti-purpose **A**utomatic **Nest**ing) that addresses the cutting stock problems relevant to various industries. In this section, the capabilities of the clustering software is demonstrated step by step with the help of some typical flat patterns. Four additional flat patterns, as shown in Fig. 6(a)-(d), are obtained from standard CAD graphic files such as IGES or DXF to test the efficiency of the proposed clustering methods for multiple patterns. In order to check the efficiency of the algorithm for handling different situations during clustering, two of the patterns (Fig. 6(c) and (d)) with relatively small area are selected intentionally. In addition, additional pieces of patterns 2 and 6 are introduced so that a total of eight patterns will be collected together to form a cluster. The clustering sequence follows the descending order of area, that is "1-2-2-3-4-5-6-6".



(a) (b) (c) (d)

Fig. 6 Additional patterns introduced to generate a multi-pattern cluster.

The best clusters generated by the sliding technique with the consideration of breakpoints using different approaches are presented in Fig. 7(a)-(c), and Table 1 summarizes the computational results.

(a) (b) (c)

Fig. 7 Best clusters generated by (a) Stringy approach, (b) Grinde's approach and (c) Adamowicz's approach.

Table 1 Computational results of clustering obtained with different sliding techniques

Technique	EER	Computational time (sec)
Stringy	**0.66**	**65**
Grinde	0.64	88
Adamowicz	0.57	73

It is clear that the new sliding technique with stringy effect can provide the best results (smallest-area convex enclosure) when compared with the others. The result of Grinde's approach is very close to the stringy effect as far as EER is concerned. The total computational times vary significantly (23 seconds), with the new approach giving the fastest time. These differences are mainly due to the method of generating breakpoints and analyzing the properties of the resultant cluster during each sliding step. Grinde's approach attempts the generation of breakpoint and selection of best cluster by analyzing the change in convex hull. This operation is tedious and quite dependent on the number of vertices and basically demands a very long computational time when the number of vertices in the patterns increase. The convex enclosure generated by Admowicz's approach is 15% less efficient than that obtained by using stringy effect. The computational time for Admowicz's approach is also longer than that for stringy sliding technique, and the difference is due to greater demand on computing operations in the former case. In Admowicz's approach, the generation of breakpoints and the evaluation of best cluster are related to the tasks that calculate the extreme points (with smallest and largest values of x and y-coordinates). The number of operations are proportional to the number of vertices. In general, unlike the approaches of Grinde or Adamowicz, the stringy effect involves much simpler geometrical calculations with a given sliding direction and edge during the generation of breakpoints and the evaluation of best cluster.

5. Conclusions

A new strategy for quick and precise generation of cluster for flat patterns composed of line segments and arcs has been successfully developed in this work. The feasibility of the proposed algorithm has been verified by several sets of patterns in terms of the compactness of clusters and the computing speed. By using a stringy effect, the collection of patterns with the features of minimal or near minimal-area convex enclosure can be obtained rapidly and precisely.

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