

where p denotes the cushion pressure, h the thickness of the ice field, r the size of the air cushion, σ the tensile strength of the ice and β is related to the width of the ice wedge.

Take β = 0.5, r = 40 feet and σ = 150 pounds per square inch (psi).

Do not convert to the metric system, however, make sure the units are consistent before proceeding further with the problem.

You need to,

1. Use the newton raphson method to find out the value of pressure for h = 0.6 ft.

2. Find the optimal relaxation factor for this problem with the help of a suitable plot.

3. Tabulate the results of p for h = [0.6,1.2,1.8,2.4,3,3.6,4.2] assuming a suitable relaxation factor. This must be done in python.

Objective: To find the minimum pressure cushion corresponding to the height of the ice-field by solving the supplied equation inn the problem statement using the Newton-Raphson numerical method in a Python code.

Explanation and Procedure:

The Newton-Raphson method helps us in finding the roots of the equation `f(x)=0`. In other words, it is an iterative root-finding algorithm that successively produces better results (closer to the analytical solution) with each iteration. The most basic version starts with a real valued function “`f`”, its derivative “`f’`”, and an initial guess “` *x*0 `” for a root of “`f`”. If “`f`” satisfies certain assumptions and the initial guess is close, then,

`x\_1=x\_0-(f(x\_0)/(f’(x\_0))`

x1=x0−f(x0)f′(x0)

is a better approximation of the root than “`*x`*0.” Geometrically, “(*x*1, 0)” is the [x-intercept](https://en.wikipedia.org/wiki/X-intercept) of the [tangent](https://en.wikipedia.org/wiki/Tangent) of the [graph](https://en.wikipedia.org/wiki/Graph_of_a_function) of *f* at (*x*0, *f*(*x*0)): that is, the improved guess, *x*1, is the unique root of the [linear approximation](https://en.wikipedia.org/wiki/Linear_approximation) of *f* at the initial guess, *x*0. The process is repeated as

xn+1=xn−f(xn)f′(xn)

until a sufficiently precise value is reached.

is a better approximation of the root than “`x\_0`.” Geometrically, “`(x\_1, 0)`” is the x-intercept of the tangent of the graph of “`f`” at “`(x0, f(x0))`”: that is, the improved guess, “`x\_1`”, is the unique root of the linear approximation of “`f`” at the initial guess, “`x\_0`”. The process is repeated as

`x\_1=x\_0-(f(x\_0)/(f’(x\_0))`

Until a sufficiently precise value (based on the set tolerance) is reached.

The Newton-Raphson method helps us in finding the roots of the equation `f(x)=0`. In other words, it is an iterative root-finding algorithm that successively produces better results (closer to the analytical solution) with each iteration.

This method requires the assumption that the function is linear. Thus, the ‘x’-intercept of the tangent drawn at the guess point is the “root” obtained from this iteration. This will serve as the new “guess” value:

`x\_1=x\_0-(f(x\_0)/(f’(x\_0))`

This process is repeated iteratively as:

`x\_n=x\_(n-1)-(f(x\_(n-1))/(f’(x\_(n-1)))`

(Where “`n`” is the iteration.)

When done right, with each successive iteration, the ‘x’-intercepts (roots from each iteration) must keep getting closer to the analytical value (denoted by “`x^\*`”).

The role of the “Relaxation factor” (“`alpha`”):

The relaxation factor determines with what ‘speed’ each iteration proceeds:

`x\_n=x\_(n-1)-alpha\*(f(x\_(n-1))/(f’(x\_(n-1)))`

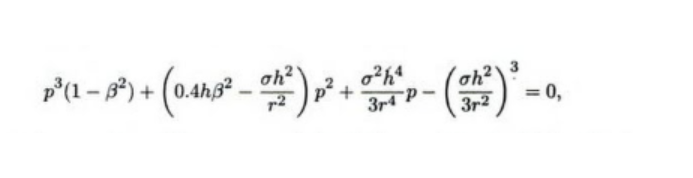
If :

`alpha>1` It is said to be over-relaxed; the iterations proceed quicker.

`alpha<1` It is said to be under-relaxed; the iterations proceed with a more conservative pace.

Thus, a general idea would be that larger the value of “`alpha`”, lesser the number of iterations would be needed. However, an improper value of can result in the method overshooting the solution and oscillating about it, thus dramatically increasing the number of iterations.

A very small value of “`alpha`” can also needlessly increase the number of iterations to achieve the set tolerance. However, small and conservative values for the relaxation factor increase in necessity as the complexity of the function increases.



The given equation explains the relationship between the height of the ice, “h” , and the corresponding minimum cushion pressure, “p”.

For a given value of “h” say (0.6ft.), the equation takes the form:

`”f(p)=0`”.

Thus, the corresponding minimum pressure is the root of the above equation. For finding the root, the Newton-Raphson method was employed.