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# Teaching Feynman's Quantum Physics at Secondary Schools Using Current Digital Technologies

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**Abstract.** We present our experience and preliminary results from teaching modern physics using smartphones at one of Slovak secondary schools. To be more specific, we developed teaching materials in the framework of flipped learning, applying Feynman's many-path quantum model together with current digital technologies - GeoGebra simulations ([www.geogebra.org](http://www.geogebra.org)) designed for smartphones. Using theoretical and mixed methods research, we report our current progress in answering the following research questions: What impact do smartphones have on the teaching and learning process? How simulations in the context of flipped learning contribute to the visualization, motivation and students' mental models in the modern physics world?

## INTRODUCTION

At present, we can see different approaches to teaching modern physics in abroad and also in Slovakia: application of historical aspects, interpretation of the historical evolution of basic ideas, the hybrid quantum-historical building of ideas and the single-model Feynman's quantum approach [1–3]. The latest, nonstandard approach was developed by Richard Feynman as a third alternative version of quantum mechanics also called as “the sum-over-paths theory”, “the many paths approach”, “space-time approach”, “path integral formulation”. The teaching of Feynman's quantum mechanics at the first years of university study and also at secondary schools was and is realized in the UK [4], Argentina [5], Australia [6] or US [7].

Mashhadi [8] on a sample of secondary school students and Ayene [9] on a sample of university students in the basic course of quantum physics showed that teaching through historical development had led students to a misleading, mixed classical-quantum conception of the quantum particles' behavior [8–11]. Secondary school students imagined the electron as a fast-moving small spherical object orbiting like planets around the atomic nucleus and photons as shining, vibrating spheres. Quantum behavior in connection with the behavior of electrons has been frequently absent in student ideas. Students who passed the basic course focusing on the principle of uncertainty and the wave-particle dualism demonstrated three mental models of the understanding of quantum behavior: (1) the behavior of photons as objects of classical physics; (2) the mixed model of behavior and (3) correct models about quantum behavior. The principle of uncertainty was conceived by students as the external experimental error, the error of the measurement methodology.

Employing ideas of Mashhadi and Ayene, in his quantum physics course for secondary schools, Malgieri [11] presented a unified view of quantum behavior via the single-model Feynman's quantum approach. Malgieri's results show that the introduction of the wave-particle dualism based on Feynman's unified model can lead to a single mental model of students dealing with the quantum behavior of elementary particles. As for the uncertainty principle, Malgieri did not notice any significant difference from its standard way of teaching.

In Slovakia, teaching Feynman quantum model at the secondary school [12], inspired by the UK secondary school course Advancing physics AS [4] and the course of EF Taylor from MIT for non-physicists called Demystifying quantum mechanics [13], brought some incorrect ideas related to the Feynman model itself, but it did not lead to misconceptions dealing with the hybrid classical-quantum behavior of electrons. However, the Slovak modern physics

courses were taught in physics seminars with students who were motivated and interested in physics and being prepared to study physics at universities.

Since 2009, we offer and teach a university course entitled "Modern physics from the viewpoint of a physics teacher", where the Feynman model is one of its cornerstones. This course takes place during one semester and thanks to flipped learning [14–16], it is possible to go through a wide range of activities from relativity to quantum mechanics. In addition, students, future (pre-service) physics teachers, experience the flipped learning on their own, with various home activities (individual learning space of flipped learning), from the active study of texts to the active watching of videos or solving simple basic problems using an immediate feedback software. At the typical following face-to-face sessions, students of the course also see and try different interactive methods [17] in small group learning modes (group learning space of flipped learning) such as Peer Instruction, Inquiry-based Education, Interactive Demonstrations or Workshops to master the abstract concepts, ideas and themes of modern physics.

Our fundamental tool of visualization, research and experimentation is a dynamic mathematical software called GeoGebra ([www.geogebra.org](http://www.geogebra.org)). It allowed us to create interactive simulations with measuring tool elements suitable for students' active learning.

Motivated by our experience [14–16] and Malgieri's work [1,11], we have decided to explore students' mental models in the standard third-grade class at one of the Slovak grammar schools. We made two fundamental changes in comparison with the previous instruction at Slovak secondary schools. First of all, we based our teaching materials using Feynman's many-path quantum model on current digital technologies - interactive simulations designed for smartphones. The second fundamental change was the flipped learning model as the framework for mentioned interactive teaching methods [16].

The aim of this paper is to report how students perceive modern physics simulations designed for smartphones as tools of virtual experimentation and inquiry in introducing the Feynman quantum model at the secondary school physics level. More precisely, we will try to report our current progress in answering the following research questions: What impact do smartphones have on the teaching and learning process? How simulations in the context of flipped learning contribute to the visualization, motivation and mental models of the modern physics world?

## **OUR PEDAGOGICAL RESEARCH AT SECONDARY SCHOOL**

### **Teaching Content. Teaching Sequence and Technology**

We divided our pedagogical research into a theoretical and empirical part. In the theoretical part, we applied the logical-didactic analysis (1) of the above-mentioned education research in teaching Feynman's approach and (2) educational standards for physics education, part of The National Educational Program for general upper secondary education with four- and five-year programs in Slovakia from September 2015 [18]. Based on the following synthesis of knowledge, together with today's principles and technologies of flipped learning, we have developed the learning content and structure of lessons. Total number of face-to-face lessons in our teaching sequence was 20.

One of the important problems behind the introduction of the Feynman quantum mechanics is to solve more complex quantum systems using only secondary school mathematics. The solution is Edwin Taylor's idea [13] to use interactive pedagogical software. Interactive simulations offer students the visual perception of system behavior belonging to an important aspect affecting the creation of students' mental models in quantum mechanics [10]. On top of that, simulations appear not only as a suitable alternative in the absence of real experiments, but digital technology was positively accepted as a strong motivational element of learning among students [12].

Up to now, we were using simulations based on the Java programming language [12,19]. But the use of Java is now limited only to computers and it is not compatible with today's digital technologies like smartphones or tablets. Therefore, we have chosen Geogebra software ([www.geogebra.org](http://www.geogebra.org)), which is at present one of the best modeling tools for math and physics education. We have created 23 interactive Geogebra simulations, which became virtual experimentation tools for exploring quantum phenomena. Simulations were designed specifically for use on smartphones and tablets. In several situations, we also use other technologies - Google sheets, Google documents and simulations from other sources - e.g. Advancing Physics [4] or PhET (<https://phet.colorado.edu/>).

Today digital technologies represent strong means for (non)immediate feedback and interaction with and between students and with teacher [14]. Ubiquitous digital technologies, with their widespread availability in school and home, also offer natural accessibility of learning materials for almost all students including absenting ones [20].

The current structure of our lessons with the implementation of digital technologies as a tool of motivation, visualization, exploration and feedback is the result of our three-year research-based testing of individual elements of

both flipped learning and the Feynman quantum model at the university level or during popularization activities for secondary school students at UPJŠ in Košice. The table 1 gives an overview of the created and used software associated with each topic as a result of our theoretical research.

**TABLE 1.** Teaching sequence of 20 lessons with technology used to enhance understanding

<b>Title of Chapter</b>	<b>Topic</b>	<b>Title of simulation, GeoGebra (GG) / Java program (Java) / PhET simulation (Phet) / Google sheets (GS)</b>
Photons - quanta of light (5 lessons)	Introduction to quantum physics	Scale of Universe (Adobe Flash)
	Quantumness	Photon energy (GG)
	Measurement of Planck's constant	Photometer (GG)
	Planck's curve	Photoelectric effect (Phet) Planck's curve-heat radiation (GG) Photons and their use (GS)
Photons, partial reflection (2 lessons)	Partial reflection on one surface	Rear view mirror - demonstration (GG) Rear view mirror - measurement (GG)
	Partial reflection on two surfaces	Partial reflection on layer - demonstration (GG) Partial reflection on layer - measurement (GG)
Probability (2 lessons)	Partial reflection- probability	Partial reflection on glass - experiment (GG)
		Partial reflection - measurement (GG)
		Coin tossing-Statistical View (GG)
		Cube rolling-Statistical View (GG)
Feynman model (4 lessons)	Double-slit experiment	Monte Carlo Simulation in Python
		ManyPaths (Java)
		Famous double slit experiment (GG)
		Photon and mirror-quantum model (GG)
		Photon and mirror-measurement (GG)
		Photon and the propagation of light-quantum model (GG)
		Photon and the propagation of light-measurement (GG)
		Photon and refraction-quantum model (GG)
Electrons (7 lessons)	Quantum principle of the least action	Photon and refraction- measurement (GG)
		ManyPaths (Java)
		Action table (GS)
	Model of the hydrogen atom	Hydrogen Atom-quantum Model (GG)
		SChE in action (GG)
		SChE in computer speech (GG)
	Schrödinger equation	De Broglie Wave (GG)
		Schrodinger equation (Java)
		Principle of Indistinguishability (GG)
		Laser (Phet)
	Pauli exclusion principle	Models of the hydrogen atom (Phet)
	Principle of laser	
	Models of atom	

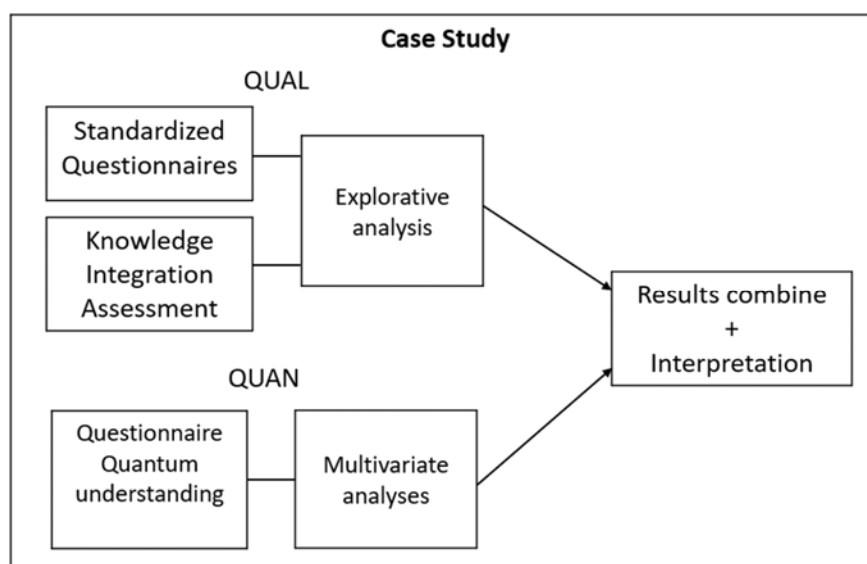
### Mixed Methods Research

As for empirical educational research, we decided to apply the mixed research. Basically, we can characterize a mixed research design as an approach in which the researcher collects and analyzes both qualitative and quantitative

data in accordance with research questions. Each design of research has specific methods of data collection and analysis to create a theory consistent with the paradigm of research [21].

In general, we choose the mixed research methodology in situations where one kind of data, quantitative or qualitative, is insufficient to describe the pedagogical phenomenon under investigation. Using a mixed method approach provides the researcher with the opportunity to capture both the trends and detail and add depth and context to quantitative results. Mixed research has the long-standing tradition in educational science and this study uses a mixed method approach, convergent design, incorporating qualitative and quantitative methods of data collection in an attempt to converge the results for enhanced understanding [21].

Qualitative data were collected in our research during group work activities from each topic. At the end of the research, we obtained the views and attitudes of students to teaching and digital technologies using questionnaires. Questionnaires (UEQ and CEQ) as a method of collecting qualitative data are transformed into quantitative data and evaluated by an appropriate statistical method. Quantitative data came from a quantum questionnaire described below. The main goal of our research is to map students' quantum mental models and understanding after teaching the Feynman model.



**FIGURE 1.** Converge Study, QUAL + QUAN design. Adapted from [21].

To respond the question, How simulations contribute to motivation to explore the quantum world?, we used a standardized, reliable and validated questionnaire called User Experience Questionnaire (UEQ) [22]. Up to these days, there are more than 20 language mutations of the original German version. For the purposes of our study, we translated the UEQ into the Slovak language.

UEQ is a quick way to evaluate the comprehensive impression of user satisfaction, feelings, total impression with a digital tool as a product - in our case GeoGebra simulations on smartphones and Java programs on a PC. It contains 26 items including six dimensions: Attractiveness, Perspicuity, Efficiency, Dependability, Stimulation and Novelty. Attractiveness means the overall impression of the tool, perspicuity expresses how easy is to get familiar with the tool and to learn how to use it, efficiency stands for easy to use, dependability expresses how the user feels in control of the interaction, stimulation means motivation and novelty is a synonym for creative. In the UEQ, student's responses are in the form of the semantic differential. It means to use a seven-point scale to express to what extent a person agrees with the given characteristic of the tool described by two opposite adjectives.

We planned to obtain also feedback from students about the quality of teaching and its conditions by another standardized questionnaire, entitled CEQ (Course Experience Questionnaire) [23]. We discovered the argument for choosing the CEQ in [23]. In our research, we used a 26-item version of the CEQ [24] and two open-ended questions for students' suggestions and ideas for future teaching. The individual questionnaire items have a 5-point Likert scale. The questionnaire was administered electronically as Google forms and preliminary processed by statistical tools directly implemented in Google forms.

Concerning developed mental models of students about the behavior of objects in the quantum world, we used conceptual test – quantum questionnaire for quantum conceptions mapping, developed for undergraduates in introductory quantum physics courses and for secondary school students [25]. This kind of conceptual test examines mental models of the whole class in the following areas: wave-particle dualism, uncertainty principle, Pauli exclusion principle and models of the atom. The content of the questionnaire is in accordance with the requirement for the Slovak school-leaving examination in physics (maturita in the Slovak language). For research purposes, we again prepared the Slovak version of the questionnaire.

The tool collects responses to group clusters and each cluster contains students with very similar mental models. Typically, the first cluster consisted of students with quantum thinking, the second one with conflicting quantum thinking and the third with conflicting mechanistic thinking. In our research, we used the quantum questionnaire twice as the pre-test and post-test showing what changes were brought after implementing the Feynman quantum model. The questions were again administered as a Google form. The detail statistical analysis of results will be processed in the open-source statistical software R using Jupyter technology [26].

## **Empirical Research Conditions**

The research was conducted at the grammar school Poštová 9 in Košice for three months in school year 2018-2019. The school chooses students on the basis of entrance exams and forms four classes of students. One class has a study profiling program with extended Mathematics, one with extended English language. The remaining two classes are educated according to the study program of the general grammar school. We conducted our pedagogical research in the third-grade class without profiling. In the subject physics, in the third year of study, according to the school curriculum, the teaching time is increased to two hours a week, with one divided hour.

Our research sample consisted of 33 students ( $N = 33$ ) of which 11 were boys and 22 were girls. The lessons took place alternately in a physics classroom equipped with a Wi-Fi internet connection and a data projector, and in two computer rooms equipped with desktops with the installed Java and connected to the internet. One of them was equipped with an interactive whiteboard, the other one with a ceramic board and a data projector with a projection on the classroom wall. Teaching in one of the computer rooms was always realized during divided lessons, where the class was split into two groups with 17 and 16 students respectively. When physics instruction was in a classical classroom, students used their own mobile devices (smartphones).

## **PRELIMINARY RESULTS FROM OUR RESEARCH**

In our second paper in this proceedings, entitled *Flipped Learning and Interactive Methods with Smartphones in Modern Physics at Secondary Schools* we present a typical, concrete example of an active learning activity dealing with the black-body radiation as an example of results of our theoretical research. This paper also shows a detailed description how we applied the flipped learning framework, what working time and workload of students were required during our teaching, where and when students used the simulations. This paper also demonstrates general strong benefit of flipped learning, creating extra time, which in a class allows instructors moderating discussions and debates about more difficult topics, identifying and resolving students' misconceptions, doing more complex hands-on activities or training key skills [16].

Empirical results from this paper also indicate that thanks to virtual simulations the interactive methods in the framework of flipped learning is really viable, even in classical classrooms with currently widely available digital technologies– smartphones and a Wi-Fi connection.

Since our didactic and statistical analysis of collected data from UEQ and CEQ together with the interpretation of results have just started, at this time we are able to report only some preliminary results. The quantum questionnaire requires the most sophisticated, cluster analysis, so we do not have any results from it now. Full results will be presented during the conference. In Figure 2 we can see preliminary results of UEQ questionnaire ( $N = 33$  students). They are displayed in the benchmark bar graph created by the authors of the questionnaire [22] which evaluates the used digital tool in all six dimensions of UEQ, particularly GeoGebra simulations on smartphones:

- excellent, it is in the 10% best product range
- good for product evaluation means that 10% of the best placed higher and 75% of the products rated worse than the product
- above average 25% better and 50% worse,
- below average is 50% better and 25% worse



- bad, 25% of the products are worse than the product itself

In our conditions, GeoGebra simulations have reached in dimensions attractiveness, perspicuity, and dependability stimulation rating above average. In the dimensions efficiency and novelty, we got the good rating.

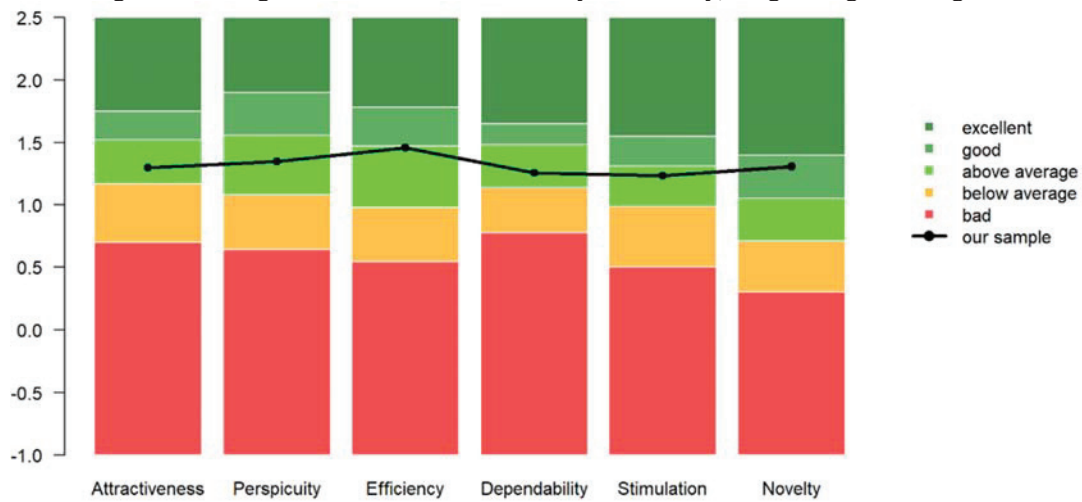


FIGURE 2. Results of the UEQ for GeoGebra simulations.

Students' views on teaching in the framework of flipped learning were identified using a standardized CEQ questionnaire. We have collected data from 31 students and 24 of them also wrote their opinions in open questions. As an illustration of CEQ results, we present Figure 3 summarizing results from two questions dealing with the role of digital technology in our course.

The use of digital technology helped me a lot to understand the subject matter we have discussed.

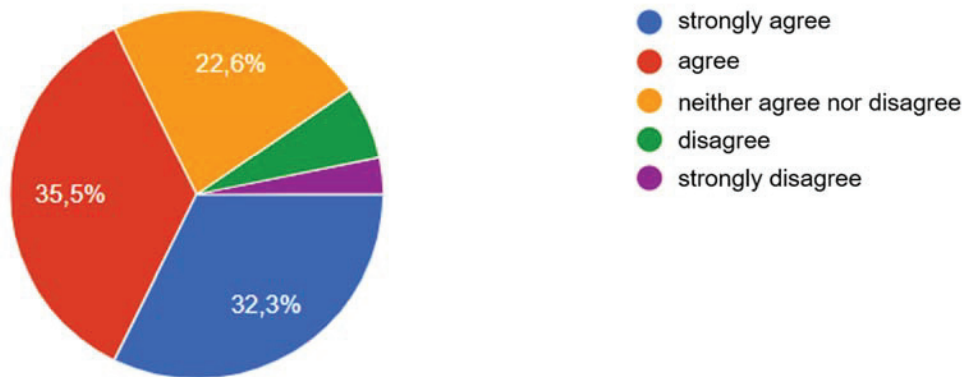


FIGURE 3. Results from the use of technology item - 31 students' responses in the CEQ.

Moreover, fourteen students consider the use of technology, GeoGebra simulations and Java programs, as the strongest aspect of instruction. Here are several students' answers to the question what were the strongest aspects of the course:

" Use of technology in teaching Physics. Practical tasks such as working on computers in GeoGebra ... "

" Simulations, they helped me to imagine very difficult things. Thank you! "

" Thanks to questionnaires and simulations, I have made more effort with this subject (Physics) than in other years."

"My work in physics was mostly about mechanical learning. This teaching method helps me to understand physics practically and increase my interest in physics. I like the simulations and variety of lessons, tasks and learning materials ... "

" I haven't known the simulations in GeoGebra ...before, so I'm glad I had the opportunity to learn how to work with these programs. "

## DISCUSSION AND CONCLUSIONS

We reported the results of theoretical and empirical research in teaching modern physics with quantum mechanics ideas at secondary school level. Our theoretical research led to creating the teaching sequence based on Feynman's "Explore All Paths" model for implementation at standard Slovak secondary (grammar) schools. The proposed structure of the teaching units was designed as much as possible in accordance with the physics subject curriculum from the educational standards and the requirements for the school-leaving examination.

Preliminary results from empirical research show that modern physics with quantum ideas in the flipped learning form is viable in Slovak education system conditions. The flipped learning really created more time for active learning and the use of interactive methods. As for technology, for effective flipped learning activities, we recommend realizing instruction in classes with internet access. Such conditions allow students to work in groups with their own mobile devices, providing tools for visualization, manipulation, and interactivity with learning content. Moreover, results indicate that simulations are inevitable, key element in understanding the abstract nonintuitive concepts of quantum mechanics through and effective teaching modern physics seems to be impossible without them.

GeoGebra simulations were also attractive and user-friendly for students and became a part of students' motivation. Our realized instruction showed that flipped learning offers much more space for the development of critical thinking, analytical skills, ability to work and discuss within the group.

More complete results with more valid interpretation will be reported after finishing our pedagogical and statistical analysis of all data obtained during the case study. Especially data from quantum questionnaire will give us very important information about mental models of students. However, our direct experience and observation from tasks, projects, and questions indicate that students should not have mixed model misconceptions as it is during standard quantum mechanical instruction.

As for generalizations, we have to say that our mixed research design is based on case study design, which will limit us in more general conclusions. However, since misconceptions are strongly invariant with respect school systems or size of classes, we think that our final results should show directions for further quantitative research confirming uncovered misconceptions.

According to our research and knowledge, our instruction is the first case study of flipped learning in Slovakia, adapted on the syllabi of modern physics in accordance with educational standards and requirements for school-leaving examination. To confirm the effectiveness of flipped learning not only in modern physics, we recommend further research aimed at quantifications of normalized gains used in reporting results of conceptual tests at secondary school level. Further research will be also beneficial for both flipped learning community and physics education community in Slovakia and abroad.

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