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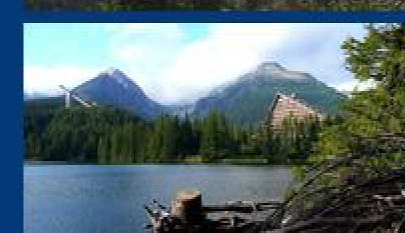

Geogebra - a complex digital tool for highly effective math and science teaching

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Abstract — **Dynamic geometrical systems bring new tools for visualization and exploring mathematical ideas, structures and relations in teaching of mathematics. Among this type of systems the Geogebra is coming to the fore these days, thanks to its extensive options. In the methodical textbooks for teachers created within Slovak national project "Modernization of education at primary and secondary schools" Geogebra has an important position among used mathematical projects. This article introduces the main and the newest possibilities of the software environment Geogebra which joins dynamical geometry, work with algebraic expressions, graphs, spreadsheet, and the CAS environment into an unique unit that can be usable in teaching from primary school to university.**

Keywords — **Digital literacy, Education, Mathematics, Dynamic geometry software.**

I. Introduction

Meaningful and effective use of digital technologies in teaching process can help with stimulation of active learning and evolving of pupils' positive attitude towards math and learning. Teaching supported by computer should be based not only on integration of digital technologies into traditional ways of teaching, it should also respect the newest trends in the area of the teaching theory and the newest results of the development of digital technology.

In the suggestions of the teaching process of the mathematical themes in the methodical textbooks for teachers created within Slovak national project "Modernization of education at primary and secondary schools" are used various types of mathematical systems. The most common systems are:

- dynamic geometrical systems (Geogebra, Cabri II, Cabri 3D, C.a.R),
- CAS-type programs (Wolframalpha, Mathematica, Derive, Maxima) [1],
- spreadsheets (MS Excel, Google documents).

Digital technologies bring various potential options of the improvement of the teaching process to the education. The most proclaimed are:

- visualization – to increase the plasticity you can use static and dynamic graphical representation of the data that allow you to create the adequate visual imagination in pupils' minds
- interactivity and dynamics - in educational applications you can implement an immediate response to external stimuli in the form of providing feedback or dynamic changes of objects dependent on the altered parameters. Aspects related to the provision of interactivity and feedback are further elaborate in [2]
- use of models and simulations of the processes – using digital technologies you can create various types of models for the representation of the objects and mathematical structures and also you can execute simulation of the real processes.

All of the above also offer dynamic geometrical systems and therefore also math teachers often use these systems to create digital teaching materials and contraptions.

II. Dynamic Geometrical Systems

Dynamic geometrical systems allow you to easily construct geometrical shapes and by manipulation with the objects examine and discover invariant characteristics of the geometrical shapes and their mutual relations. Connection of the algebraically specified rates, geometrical shapes and animations represents strong tool for experimental examination of geometrical models of real situations and solving of various types mathematical problems.

In general, the main reason for the integration of dynamic geometry systems into the teaching of mathematics is creation of an experimental, interactive and dynamic environment conducive to active learning. However if we compare tools and possibilities of mentioned systems in introduction, it seems that Geogebra plays a special role.

The last version of Geogebra [3,4] became a powerful integrated software environment which combines in its views (windows): algebra, graphics, spreadsheet, CAS and construction all benefits of dynamic geometrical systems, spreadsheets, graphic calculators, CAS systems. Since these views are mathematically connected and work in perfect harmony, it makes from Geogebra so strong, complex and unique program that now it has very important impacts on pedagogy in math and science teaching.

For example it is known [5, 6] that typically after any traditional instruction connections among ideas, concepts, their formal representations, and the real world are often lacking. Because students can change interactively properties of exploring objects like lines, points or velocity or position vs. time graphs in any mentioned view and see simultaneously what these changes cause in remaining ones, Geogebra enables students to see mathematical ideas as „living, breathing moving".

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So that in any school subject using math this immediate feedback gives students the opportunity to develop deeper understanding than the traditional method of seeing only static objects on a blackboard or on PC in the separated types of software.

It is also quite amazing that at present Geogebra has tools developed in the project GeogebraTouch which adjust the program for using at Interactive whiteboard with all power of interactive whiteboards.

Previous Geogebra versions and applets created in them were designed for desktop computers and laptops. So they are not able to work on intelligent smartphones or tablets like iPhone, Android, Windows phone or iPad.

However current Goegebra project GeoGebraMobile [7] will soon allow the use of GeoGebra applets in modern web browsers both on computers and new mobile phones. This important step will permit students and teachers to use GeoGebra's large pool of dynamic materials on virtually all devices with a web browser.

Finally we should mention that Geogebra project called GeogebraTube will provide a very simple, easy direct upload of any applet for sharing with students and rating and commenting in worldwide pedagogical community.

III. Integration of Geogebra into the methodical textbooks for math teachers

In the methodical textbook the main tools of the chosen mathematical programs usable for support of math teaching are demonstrated. In the next part of the book the choices of using mathematical programs in extending of dominant ICT competences of pupils in math teaching are presented. ICT competences extended in separated dimensions of digital literacy are described in [8].

In extending the ICT competence Organizing data and explore relations the dynamic geometric systems and spreadsheet are mostly used. Solving problems using these programs can be part of systems of education elaborated in the form of interactive educating activities. As an example we can use educating activity created in author's system ToolBook Instructor focused on discovering the attributes of tangent quadrangle [2].

In solving some problems various software systems are used in separated steps of the solution. As an example we can use a problem from the second chapter of methodical textbook [9]: Children were building block stairs (see figure 1). Figure out with how many blocks the stairs in few next steps are built.

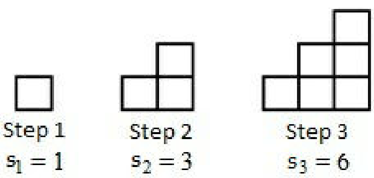

Figure 1. The first three steps of a building of stairs

For numerical and graphical representation of the data the spreadsheet MS Excel is used. For finding the number of the blocks in following steps of the building of the stairs it is easier to use recurrent relation:

$$s_n = s_{n-1} + n, \quad s_1 = 1.$$

Aforesaid relation indicates the fact that in nth step we add n new blocks to already built stairs. Recurrent relations are in the spreadsheet realized simply by defining formulas with links to vicinal cells. In the figure 2 there is a table created in described way along with graphic representation of the data.

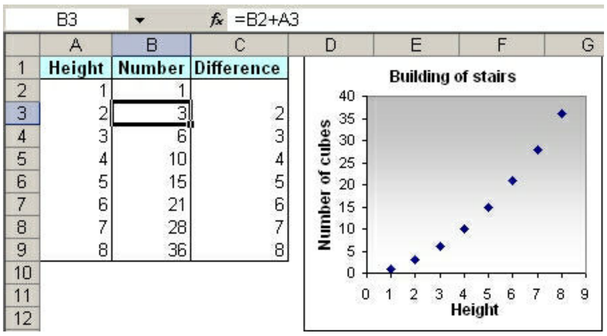

Figure 2. Numerical and graphical representation of data

From equally increasing difference between vicinal terms of the sequence we can say that the sequence indicates quadratic dependence. Analytical relation for calculating nth term of the sequence can be described in this form: $s_n = an^2 + bn + c, \quad a, b, c \in R.$

Deducing of analytical relation leads to the solution of system of linear equations with an unknowns a, b, c. For this purpose is possible to use CAS-type program. Mathematical program WolframAlpha, which is available online, will also formulate the analytic relation for calculating the nth term of sequence, after specifying the first few terms of sequence. This attribute is usable for checking of the results in the final step of solving the problem.

All the aforesaid programs are integrated into the program Geogebra, therefore all steps of solving the problem are realizable with this program. The sample of the possible solving of a problem using the program Geogebra is pictured in the figure 3.

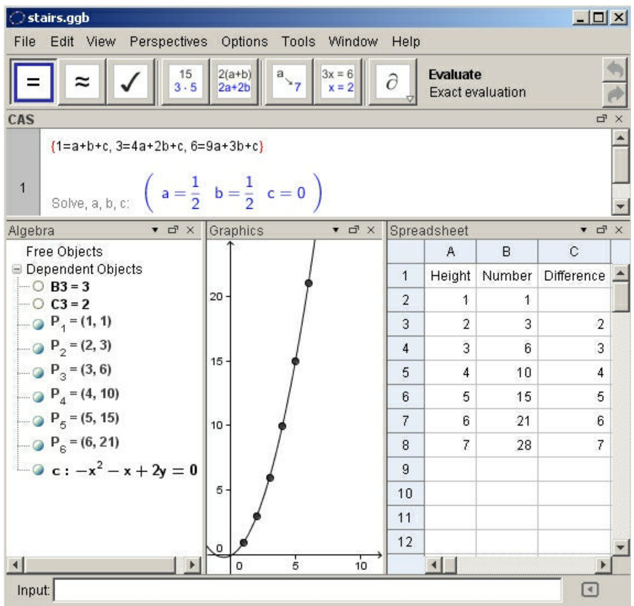

Figure 3. Solving of the problem in Geogebra environment

In the figure 3 there are pictured four main components of Geogebra, that are represented by Graphics, Algebra, Spreadsheet and CAS windows. Depending on active window the toolbar under the main menu adjusts. In the Spreadsheet window you can work with the data and use formulas and main functions in way similar to the program MS Excel. Data from the table are used for finding points using coordinates. Defined points are displayed in the Algebra window as depending objects. At the same time they are pictured in the Graphics window. The conic is brought forward through first five points, using the eponymous geometric order.

For defining coefficients a, b, c in analytical relation for calculating the nth term of the sequence we can use the first three terms of the examined sequence. Received system of linear equations is written and solved in the CAS window. The results imply that analytical relation for defining the terms of the sequence can be described in this form:

$$s_n = \frac{1}{2}n^2 + \frac{1}{2}n.$$

Analytical description of conic constructed in the Graphics window will be automatically listed among depending objects in the Algebra window. In this case the conic is parabola and is marked with c. For the set of all natural numbers is this description identical to the analytical relation for calculating the nth term of the sequence.

IV. Geogebra in the Teaching of Selected Mathematical Topics

Diversity of tools of the program Geogebra enables to use this system to solve different types of mathematical problems in the methodical textbook for mathematics teachers [9]. We can demonstrate these possibilities by means of the section about using mathematical programs to model real situations. Geogebra is used for a modelling of ladder fall along a perpendicular wall and for the investigation of a track of the center of ladder during its movement.

An interesting application of mathematical modeling is the solving of linear optimization problems. The people often solve decision-making problems in real life. The solving of these problems requires finding of optimal solutions taking into account certain restrictions that can be often represented with linear inequalities. The geometric model can be used to solve mathematical model of a real situation in simpler cases. The solution is based on geometric representation of solution of linear inequalities system and of an investigation of target function values in points of a plane determined by means of geometric model. Software CAS or Geogebra can be used for these purposes.

The main part of the methodical textbook is the fourth chapter which contains suggestions for lesson plans of selected mathematical topics. Several mathematical programs, applets on internet, and other educational software systems are applied to solving of the problems in the methodical textbook. Interactive notebooks prepared using system Mathematica have an important role for educational treatment of the subject Elementary functions. Similar interactive mathematical documents can be created using Geogebra. We see the greatest possible use of GeoGebra for computer assisted learning of the following topics of the methodical textbook:

- **planimetry** – investigation of relationships between elements of the right triangle, investigation of modifications of Pythagorean theorem and its generalization, investigation of relationships between angles in a circle, identification of the set of all points in the plane from which a given segment is seen under the same angle,
- **Congruent and similarity transformations in the plane** – exploring the properties of axial symmetry and their application in problem solving, composition of axial symmetries, creation and investigation of the model of pantograf – real mechanical tool for zooming in and out of plane figures,
- **Elementary functions –** investigation of functional dependencies through their graphical representations, investigation of properties of functions, applications of linear, quadratic and exponential functions to solving of real problems,
- **Derivation of functions** – understanding concept of derivation of the function at a point using geometric meaning, investigation of derivations of elementary functions using graphical representations, investigation of the properties of functions using derivation, use derivations to solve real problems focused on finding extremes of functions,
- **Probability** – modelling of random events and analysis of the received results, estimation of the probability using the relative frequency, graphical representations of random events by geometric probability,
- **Statistics** – statistical data processing and their graphical representation, investigation of the relationships between data (e.g. fig.4)

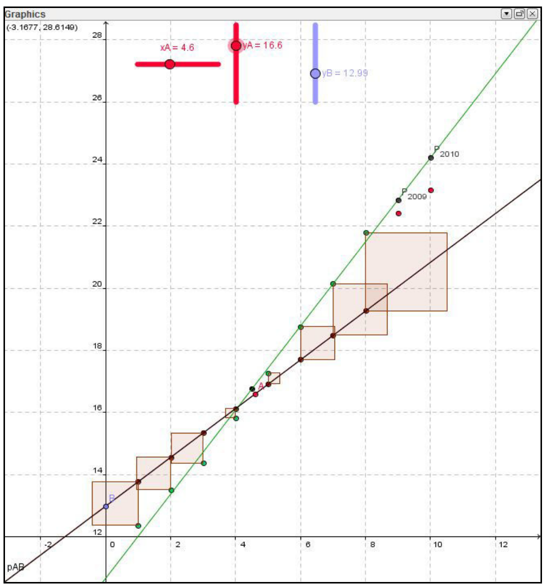

Fig. 4: Developing understanding of the least squares method in Statics.

The next section is focused on a detailed explanation of Geogebra application in estimating a theoretical value of probability using the method Monte Carlo and on a calculation of the probability of random events using geometric probability. We chose the following tasks from the methodical textbook: *Peter drew the points A, B, C, D with coordinates (0,0), (5,0), (5,5), (0,5) in the coordinate system. What is the probability that randomly selected point in the square ABCD has y-coordinate at least three times greater than x-coordinate?*

In the methodical textbook the estimation of the probability by means of modelling of random events is used at the beginning of problem solving. Spreadsheet Microsoft Excel is used to generate random points in the specified region of the plane, to evaluate the results, and to calculate the relative frequency. Geogebra can be also used to execute these operations. The parts of the solution are displayed in windows Graphics and Spreadsheet in the figure 5.

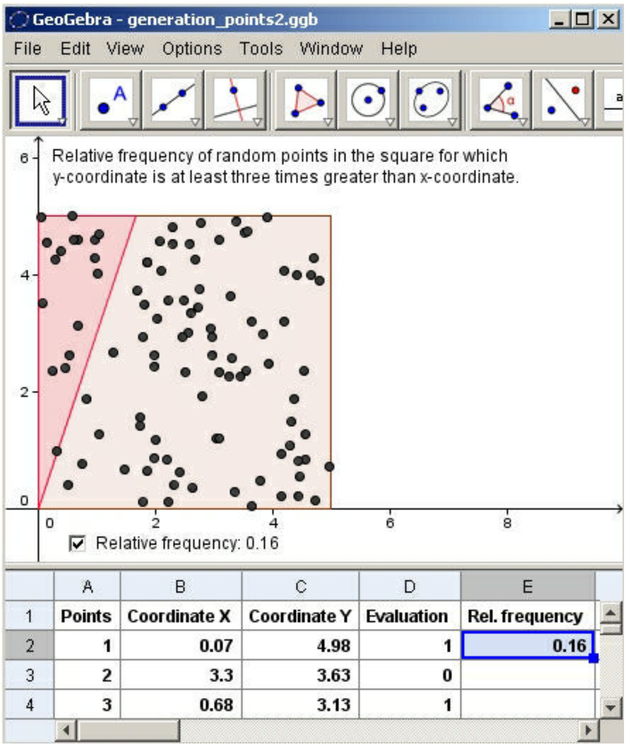

Figure 5. Realization of the method Monte Carlo using Geogebra

The coordinates from the interval <0, 5> of one hundred random points are randomly generated in the window Spreadsheet in the columns B and C using the function Random between which is placed in the linear expression. The column D contains evaluation of the condition, whether y-coordinate of a random point is at least three times greater than x-coordinate. The formula IF[C2>=3*B2,1,0] is used for this calculation. The relative frequency of favorable results of the random event is calculated in the cell E2 after counting of favorable cases. New one hundred random points can be generated through pressing button F9. The coordinates of random points in the columns B and C are used to define points in the coordinate system and to display the points in the graph.

After the estimation of the value of probability we approach to the accurate calculation of probability using the definition of geometric probability. We use areas of the figures as geometric measure in the definition of geometric probability. We draw the square ABCD with the length of side equal to 5 cm with the vertex A in the origin of the coordinate system according to assignment of the problem. The square ABCD creates the sample space of the random event. The triangle AED represents favorable results of the random event. The borders of the triangle AED are formed by the sides of the square ABCD and by the segment AE containing the points which y-coordinate is three times greater than x-coordinate. Described situation is shown in the figure 6.

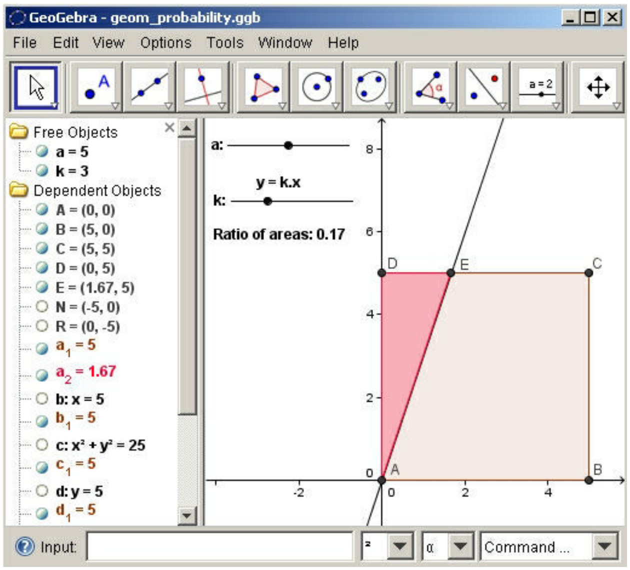

Figure 5. Use of the definition of the geometric probability

We can see in the figure 5 that a ratio of the areas of the triangle AED and the square ABCD equals 0.17. This value is just the probability of explored random event. The major advantage of prepared dynamic construction is the possibility to change the length *a* of the side of square ABCD and the slope *k* of line AE by means of the sliders. Students may satisfy that a change of the side length of square ABCD does not affect the value of the probability because created figures are similar with the same ratio of the similarity. The change of the slope *k* of line AE leads to showing of the results of analogical tasks which can be used to check students' solutions of the modifications of basic task.

V. Integration of Geogebra Teaching and Learning Materials for Science Subjects

Power and unique properties of Geogebra offer to use this software in any school subject applying mathematical ideas. For example in any area of physics like mechanics, electricity and magnetism, optics, relativity or quantum physics it supports very strongly all issues connected with one of four areas of digital and scientific literacy - creating and developing new ideas to understand the world around us by ICT and seeing how scientific method at this process works.

In physics especially understanding models and modelling by ICT were underestimated for a long time and now in our modernization project belongs to main key-skills which should be developed in student minds.

In this contribution we would like to demonstrate two Geogebra applications in visualizing and applying mathematical models used in physics.

First of them is connected with applying physical model simultaneously with its visualization in solving the following physical problem: *A tennis ball is dropped from Pisa Tower. When it reaches the ground and what is its speed?*

The expected solution of the problem using digital technologies consists in the following steps:

- Students have to find in reliable information sources needed information. In this case height of Pisa Tower which is 55.86 m together with good picture of the tower.
- Then Geogebra image tool allows students to put an image of Pisa Tower in graphics view.
- The following direct manipulation with coordinate system allows calibrating the system to be corresponding to real measures of tower.
- After that slider tool containing time as independent quantity is used for simulation purposes.
- Next point A is put in the system as a model of tennis ball with any fixed x-coordinate and y-coordinate given by free fall formula applied in this case: $y = 55.86\text{-}5t^2$.
- Finally activating animation property of the time slider point A starts to move as a real tennis ball, so a student can see what really happens. Simultaneously i tis possible to stop simulation when a ball hits the ground, so student can read time of fall. (see Fig.7)

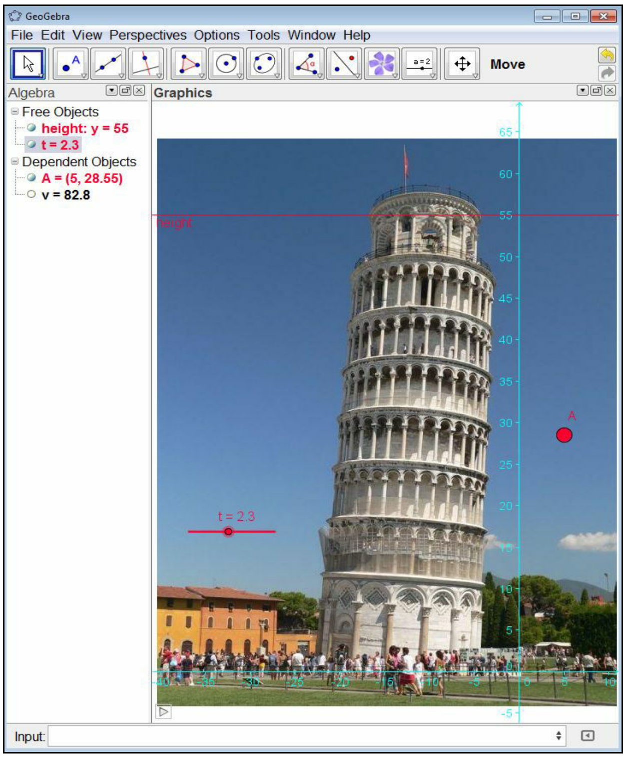

Fig.7 Virtual reality simulation in Geogebra

In connection with fall from Pisa Tower students can develop kinematical model y= $55.86\text{-}5t^2$ from more fundamental physical laws and knowledge.

Particularly Galileo's free-fall law, an immediate and direct conclusion of fundamental Newton's second law: *Each falling object with shape and density allowing neglecting air resistance falls with acceleration 9.81 m/s²*. It means that every seconds speed of the object increases its value by 9,81 m/s. The geometrical interpretation of the law is a half-line or ray running through points [0 s,0 m/s] and [1 s, 9.81 m/s].

Since the second fundamental kinematical knowledge says that the distance travelled by any object is represented by the area under a speed-time graph, powerful geometrical tools of Geogebra allows a very quick construction of corresponding area which can be interactively changed by simple dragging of a point lying on the speed-time graph. Exploring properties of the dynamically changing area, students are able to find required mathematical models describing changing speed and changing distance over time.

The second illustration shows a Geogebra application in electricity and magnetism. The fundamental law describing interaction between charges is Coulomb's law and fundamental concept describing electric field is vector of electric field or vector of electric force.

Its visualization is very important for students in correct understanding Coulomb's law as a physical model. Since physics education research shows that students have strong misconceptions due to lack of a clear and careful electric-field-vector operational definition in their minds, using this digital technology we give our student the opportunity to develop the required careful operational definition, where each step in Geogebra mimics perfectly every operation (action) needed in finding vectors of electric force or field.

Figure 8 is illustration of particular example where in Geogebra construction view student can see every operation corresponding to any displayed step in geometrical (graphics) window.

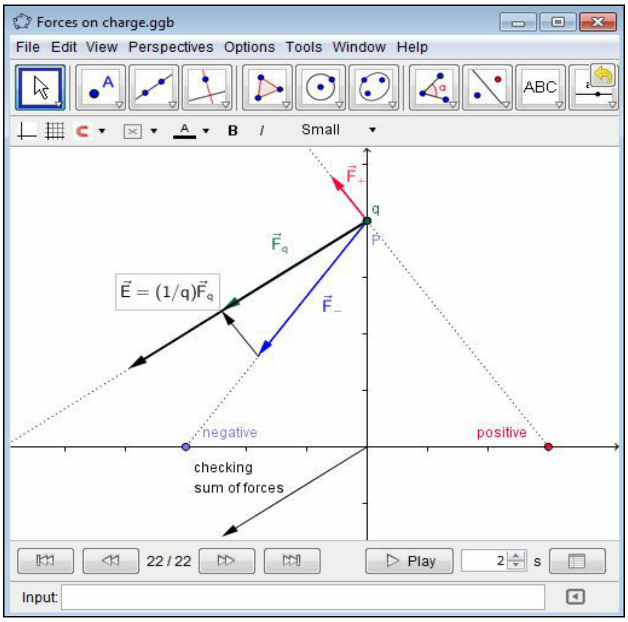

Fig.8 Geogebra Application in electricity and magnetism.

Our experience conclude than any phase of modeling as key science skill: visualizing, applying modifying or creating can be developed in Geogebra.

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