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Interactive Jupyter Notebooks with SageMath in Number Theory, Algebra, Calculus, and Numerical Methods

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Abstract— In 2014, physicists and data scientists F. Perez and B. Granger started an ambitious project Jupyter with goals of developing a powerful, universal and open virtual interactive scientific environment. Today, Jupyter is the most widely used technology for performing scientific calculations, open data analysis, processing, and reporting, but it has also become a very important educational tool. Our article summarizes pedagogical and technological aspects and experience from using Jupyter notebooks with SageMath in our research and teaching university courses dealing with main mathematical domains. The ideas will be introduced and reflected in the framework of STEM education with modern active learning approaches together with current visions of the EU strategy for advancing the economy and sustainable growth of the EU.

Keywords— blended learning, research, scientific work, CAS, SageMath, Jupyter

I. INTRODUCTION

Today science, technology, engineering, and math (STEM) are the primary driver for developed economies, their competitiveness, and the creation of most jobs. New, last-decade technologies such as mobile devices, cloud infrastructure, open data, artificial intelligence, and decentralized and social networks are rapidly changing all aspects of our lives. Our work, health, education, environment, and even roles as citizens become strongly dependent on STEM, digital and data literacy.

The European strategy for the advancement of the economy of the EU (<https://ec.europa.eu/>) called *Europe 2020: A strategy for smart, sustainable and inclusive growth* considered STEM education as playing a central role in contributing to the goal of fostering a dynamic and innovative knowledge-based economy. The current vision — *A sustainable Europe by 2030* confirms the role even more. Speaking about science in the EU, at the end of 2018, the European Open Science Cloud (EOSC, <https://eosc-portal.eu/>) was launched with the primary and highest ambition to change the way we do science in the EU — to open it to all.

The EOSC assumes that it will offer millions of European researchers, professionals, but also university students in STEAM (STEM with arts — the humanities and social sciences) a virtual environment with open and seamless services for storage, management, analysis and re-use of open research data, interoperable among all scientific domains or EU member states.

The e-infrastructure of EOSC was built via several Horizon 2020 projects. One of the significant projects, *OpenDreamKit* (2015-2019, <http://opendreamkit.org/>), was led by the 1st world mathematics ranking University of Paris-Saclay [1] in a consortium of the best European universities¹. The project's core strategy was to create and strengthen virtual research environments based on open Jupyter notebooks technology [2], [3]. More concretely, the project brought together well-established open mathematical digital tools such as SageMath, GAP, PARI and Singular. Now, these tools, actively used by research and education communities, can be handled and combined seamlessly in Jupyter notebooks.

In 2017 we started a very fruitful communication with members of OpenDreamKit, so we could also start to use these open-source tools in our research and teaching our mathematics courses dealing with number theory, algebra, calculus, and numerical methods. In the following sections, we summarize our research, pedagogical and technological ideas, and results from our experience using Jupyter notebooks with SageMath [4], [5].

II. JUPYTER WITH SAGEMATH AS A UNIVERSAL SCIENTIFIC COMPUTING ENVIRONMENT

A. Applications in Higher Education

Jupyter is an open, free web environment that runs on any free modern web browser. The product is an interactive Jupyter notebook that can be easily edited as e.g. a Word document. It enables users to insert texts, equations, graphs, e-voting, annotations, images, audio recordings, videos, and simulations. A crucial feature is the possibility of inserting computing or programming code (now from more than 100 computer languages, e.g., Python, Julia, R, or C++), which can also be performed in the given document or easily repeated.

The mentioned features can make these notebooks “living” teaching digital learning materials where students can interact as active modifiers and co-creators. Therefore, after creating Jupyter technology, Jupyter notebooks have been immediately used in education as tutorials, lab reports, assignments, worksheets, explanatory documents, or as online interactive e-textbooks not only in all STEM, but also in some humanities subjects and languages.

¹ Paris, Oxford, St. Andrews, Sheffield, Bremen, Erlangen, Gent, Zurich, Grenoble, and others

Today after five years long experience, we use Jupyter notebooks with SageMath as primary educational tools in our following bachelor math-oriented courses intended for data and computer scientists and natural scientists (physics with another science discipline):

- Mathematics I for Informaticians (2/2)
- Mathematics II for Informaticians (2/2)
- Numerical Methods (2/3)
- Fundamentals of Math for Physicists I (1/2)
- Fundamentals of Math for Physicists II (0/2)
- Methods of Physical Problems Solving (0/2)

Regarding mathematical content, the courses deal with all fundamental mathematical domains used as a language in STEM disciplines — number theory, algebra, calculus, and numerical methods. Particularly, you can see a syllabus from one of the courses in Tab. I.

From the technological viewpoint, we have decided to use Jupyter notebooks with SageMath (or shortly Sage) as a kernel — a computational engine inside Jupyter notebooks. Sage is a free Python-based open-source mathematics software [4], [5] allowing both Python coding and scientific computing as a computer algebra system. Sage combines the power of several existing open-source packages as a viable open alternative to Maple, Mathematica or MATLAB.

More concretely, Sage enables math work with any mathematical symbols, e.g. manipulation of mathematical expressions, solving and transforming equations, their systems (algebraic and differential); symbolic transformations and calculations, e.g. in algebra or calculus. Sage also allows very powerful 2D and 3D visualizations (via Python packages Matplotlib and Plotly) and advanced data and statistical analysis using R via Python package rpy2 [6].

TABLE I. TOPICS OF MATHEMATICS FOR INFORMATICIANS

Mathematics I (https://github.com/gajdosandrej/MTIa)
0. Introduction to SageMath
1. Integers and divisibility
2. Prime numbers and congruences
3. Applications of congruences and residue classes
4. Number systems and conversions
5. Matrices and determinants
6. Introduction to analytical geometry I
7. Applications of matrices and determinants
8. Introduction to analytical geometry II
9. Functions.
10. Elementary functions
Mathematics II (https://github.com/gajdosandrej/MTIb)
1. Limits of functions
2. Continuity of functions
3. Derivatives of functions
4. Applications of functions' derivatives
5. Numerical sequences and series
6. Indefinite integral
7. Definite integral I
8. Definite integral II
9. Functions of several variables I
10. Functions of several variables II
11. Functions of several variables III
12. Groups

We describe using R-based digital tools in our second conference paper *Open R and Python-based Digital Tools in Statistics, Random Processes, and Metrology*.

In Fig. 1, we demonstrate mentioned features of Sage in the Jupyter notebook from the course *Fundamentals of Math for Physicists I* dealing with the concept of real feel temperature described by a mathematical model — a function of several variables.

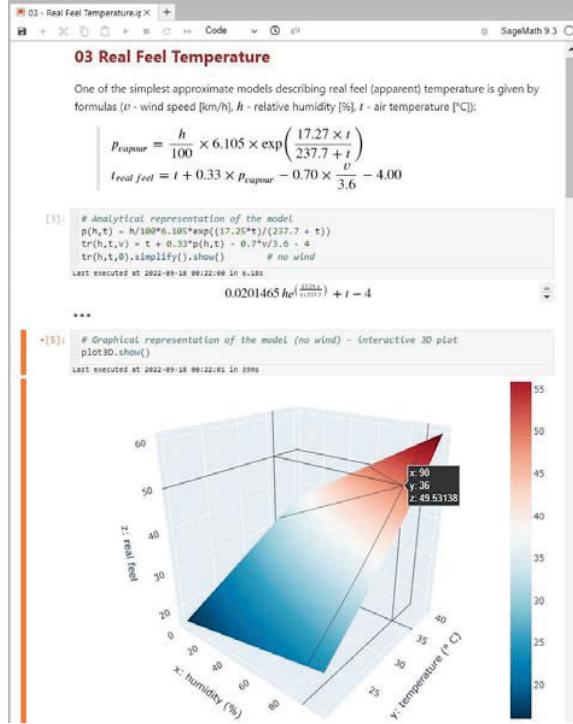


Figure 1. Excerpt from a Jupyter notebook with Sage, including a narrative text, formulas and 3D interactive graph

As for pedagogy, our approach is based on modern innovative educational methods based on principles verified and successfully used in higher education [7], [8]. We have decided on one of them, blended learning [9]–[11], which we started to apply more than ten years ago. Blended learning effectively employs digital technology with well-established pedagogical approaches enhancing active learning [12]. In Tab. II, we briefly describe all pedagogical approaches used in our courses in the frame of blended learning.

Now let us briefly explain an illustrative example how blended learning typically works in our courses. For each theme, five types of digital educational resources are prepared using Jupyter notebooks. Each topic is taught in two learning phases by flipped learning [13], [14].

The first phase - individual learning

Before a face-to-face group session with an instructor, students actively do a structured preparatory activity individually and online. The first part is an introductory Jupyter notebook with motivation and study instructions on what study materials should be learned.

Particularly, topic 2. in the course Mathematics I for Informaticians deals with primes and congruencies. Therefore, from the first Jupyter notebook students learn that prime numbers bring us to cryptography and encryption connected to personal privacy and online security of everyone in digital space. As for congruencies, students realize that congruences can be used to facilitate difficult calculations even beyond the available computing power of computers.

As for study materials, they actively learn from mandatory and supplementary materials. The mandatory study material is a Jupyter notebook with an educational text and interactive calculations (or simulations like in Fig. 2) demonstrating and visualizing new concepts and skills connected with prime numbers and congruencies. In order to better understand the concepts, students must also actively watch some segments from educational YouTube videos about divisibility.

The second part of the preparatory activity is a Jupyter notebook with simple tasks probing and verifying understanding of study materials. Solutions to these tasks directly in the notebook are sent to the instructor. Another way how to help student's in this first phase is to use social reader Perusall (see more in [15], [16]).

The second phase – group learning

Now, students come to the face-to-face group session with an instructor. They work in small groups with instructor assistance on various activities. For example, they answer conceptual questions or “two-minute” problems together (like in Fig.3), which probe the correct understanding of basic concepts (third educational resource). Here we apply the peer instruction method [17]–[19]. Or they solve more complex inquiry-based problems (fourth educational resource) by applying, exploring and developing concepts and knowledge gained from the first preparatory phase. These activities are designed by principles of inquiry-based science education [20].

STEM projects

The last important part of our blended learning are interdisciplinary projects in the spirit of STEM education [21]. Students in small groups work on these projects during the semester and in the end, they present the results and solutions of their investigations in the form of presentations. Our experience shows that STEM projects and students' solutions have good or excellent quality if we respect the basic steps of project based learning [22]. As examples of successful projects, we can mention *Hill's Ciphers*, *Detection and (self)correcting codes*, *GPS (Global Positioning System)*, *Image compression using singular number decomposition*, *Website rankings - the math of the Google search engine* or *RSA cipher*.

Finally, we would like to say that our teaching and preparing study materials would be very difficult without well-known distinguished study references, which promote active learning, conceptual understanding and the interactive use of digital technology. During the last years, we have been very satisfied with the following textbooks fulfilling these demands:

- SageMath and problem solving: Zimmerman et al. [4], Bard [23], Perry et al. [24]
- number theory: Stein [25], Crisman [26]
- algebra: Boyd & Vandenberghe [27], Klein [28]
- calculus: Hallet et al. [29], [30]
- numerical methods: Anastasiou & Mezei [31], Cheney & Kincaid [32], Sullivan [33]

TABLE II. ACTIVE LEARNING APPROACHES IN OUR COURSES

Pedagogical approaches
<i>Active learning (interactive approach)</i>
<ul style="list-style-type: none"> • Any pedagogical approach supporting linking knowledge and skills with understanding through the student's active approach, i.e. through his minds-on activity (always) and hands-on activity (mostly), while this active approach leads to immediate feedback from the student, even when discussing with peers teachers or teachers. • The traditional university instruction based on lectures and recitations supports active learning only in a very limited way
<i>Blended and flipped learning</i>
<ul style="list-style-type: none"> • a formal educational framework (meta-strategy) in which a student learns at least partly <i>through online learning</i>, with the student having control of time, place, method or pace of education; at least partly in <i>real physical space</i> (onsite) outside the home; while different methods of education along the educational path of each student are mutually interconnected to provide the student with an integrated learning experience. • Flipped learning — a special case of blended learning, in which the first contact with new concepts and knowledge is done by students alone in their individual learning space, online, asynchronously. Applying the knowledge and concepts via solving problems is realized with the teacher as a guide in interactive group face-to-face sessions, onsite, synchronously.
<i>Peer instruction (question-driven) instruction</i>
<ul style="list-style-type: none"> • An interactive teaching approach using audience response systems (e-voting) when solving problems or answering questions is managed via the <i>peer-instruction cycle</i>. • The peer-instruction cycle includes the following steps: <ul style="list-style-type: none"> (1) posing a question (problem) by the instructor (2) small-group work on solutions — peer instruction (3) collecting answers of students by e-voting (4) displaying the answers without showing the correct one (5) class-wide discussion (6) closure, e.g. summarizing the key points or giving an explanation • The typical class session is structured around three or four cycles per 45-min long time slot.
<i>Project based and/or team-based learning</i>
<ul style="list-style-type: none"> • An instructional approach using a project as learning tool for students to gain understanding as well as their mastery (knowledge and skills) of the curriculum. • Team-based learning, if the project provides opportunities of active learning in small group or team work • Typically, a project is an authentic, engaging and complex problem or challenge which is done for an extended period of time. • From the viewpoint STEM education, there is a certain level of project interdisciplinarity, when solving real-world problems requires knowledge and skills from different STEM disciplines.
<i>Inquiry-based science education</i>
<ul style="list-style-type: none"> • A form of science education that gives the student the opportunity to explore a subject or topic through minds-on and hands-on activities, investigation and posing of questions in same manner as researchers in the process of scientific inquiry. • To plan and realize scientific inquiry effectively, the typical IBSE activities should be designed with respect to a learning cycle. • Typically, it is the 5e learning cycle which includes five phases: <i>Engage, Explore, Explain, Elaborate and Evaluate</i>.

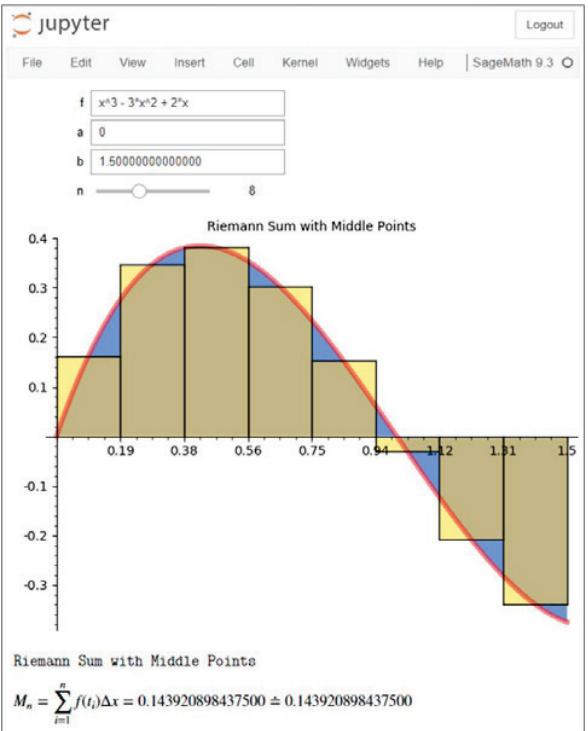
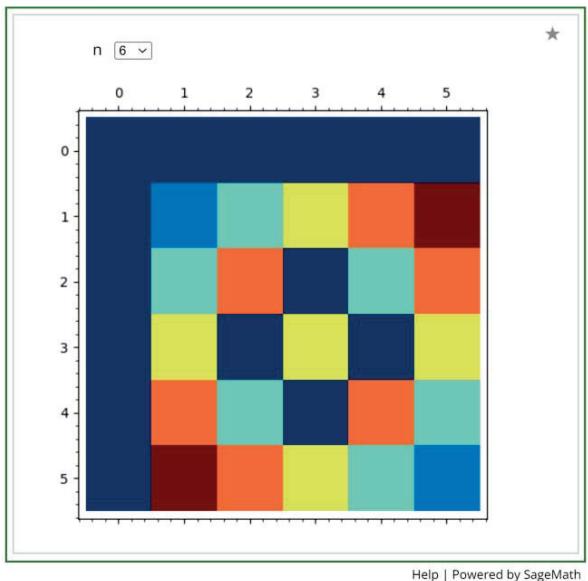
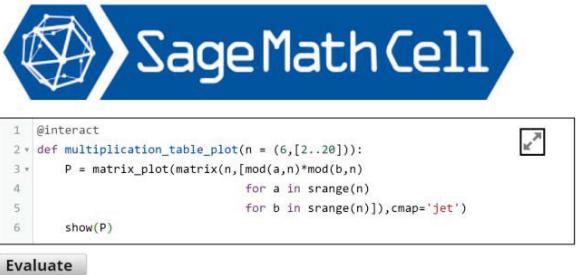


Figure 2. Interactive Jupyter simulations - SageMath interacts
(1) Visualization of multiplication in residue classes
(upper part)
(2) Numerical integration - a middle-point quadrature
(lower part)

When poll is active, respond at PollEv.com/andrejgajdos114

Elements of the set of residue classes with respect to modulo 4 i.e. \mathbb{Z}_4 = $\{[0]_4, [1]_4, [2]_4, [3]_4\}$ are:

A) Integers.
B) Non-negative integers.
C) Sets or subsets of integers.
D) None of the options A), B), C) is correct.

Total Results: 0

Powered by Poll Everywhere

When poll is active, respond at PollEv.com/andrejgajdos114

Match the function formula to the correct graph:

A) $y = x - 5$
B) $-3x + 4 = y$
C) $5 = y$
D) $y = -4x - 5$
E) $y = x + 6$
F) $y = x/2$

Total Results: 0

Powered by Poll Everywhere

Figure 3. Multiple-choice questions in an audience response system Poll Everywhere (<http://polleverywhere.com>)

B. SageMath Delivery Technology – Local Installations and Cloud Services

From the point of view of a researcher, teacher or student, a natural question arises about how to install, run, or effectively use SageMath. Despite the fact that Sage is a freely available software for Linux, Windows and Mac OS, during a local installation, it requires at least 2G of operating memory and 6 GB of free space on the HDD. The installation file itself is around 800 MB. We also have an experience that students have various laptops with different operating systems, and it is really not easy to help them if they run into installation or software problems. Similarly, the installation process can be time-consuming and problematic, if we want local installations in university PC rooms.

So it is beneficial to know that using cloud services, Sage can be run in multiple ways as needed, without any installation on a user computer.

SageMath Cell (<https://sagecell.sagemath.org/>)

This free cloud service [34], which is provided by SageMath, Inc., offers an easy-to-use web interface, the

so-called single-cell calculator SageMathCell (see Fig. 2, upper part). It is suitable for quick, one-time calculations (it can also be used on a smartphone). It also allows you to create separate html versions of Sage notebooks through offered Notebook player, which run locally without a Sage installation via an internet connection to the SageMathCell server.

Binder + GitHub (<https://mybinder.org/>)

It is possible to prepare your own GitHub repository with Jupyter notebooks and requirements for a SageMath environment (or other kernels for Jupyter). Then free cloud service Binder [35] builds and runs a live Sage from the given GitHub repository in a modern browser as it would be locally installed. This way works online without the need to log in and is suitable for occasional individual use at home, but also at university (e.g. during exams).

However, it is necessary to download the Jupyter notebook with our work and upload it later to Binder if we want to continue in our work. We have a GitHub repository (<https://bit.ly/UPJS-Sage>, see Fig. 4, upper part) which runs Jupyter notebooks on a Binder server with Sage, Python, Julia, Octave and JavaScript as computational kernels. CoCalc also offers an alternative, free and very similar anonymous method (<https://cocalc.com/auth/try>, see Fig. 4, middle part).

Commercial and Open CoCalc (<https://mybinder.org/>)

The SageMath, Inc. offers a virtual online workspace Cocalc, which allows to run Jupyter notebooks with many kernels for scientific calculations, research, collaboration and authoring documents. It is possible to run CoCalc anonymously or as a trial on a free CoCalc server without any payment. Or users can buy a subscription to run CoCalc service on a member CoCalc server with guaranteed hardware parameters if they want more stable or powerful calculations. Finally, there is also the possibility to install an open CoCalc service on own laptop or server (see Fig. 4, lower part).

Windows 11 subsystem for Linux and Chrome OS Flex

The underlying technology changes so rapidly that the current software development offers us other interesting and challenging possibilities for installing and using Sage locally. In October 2021, Microsoft introduced a Windows Store version of the windows subsystem of Linux [36], so it is possible to install a Linux version of Sage (9.7 in September 2022) in Windows, which is the newest of all offered versions for various OS.

In July 2022, Google announced a Linux-based OS called Chrome OS Flex [37] — the cloud-first, easy-to-manage and fast operating system — which turns even ten-year-old computers and laptops into sleek, fast Chromebooks. Since Chromebooks allow installing a Linux version of Sage, it will be pretty interesting how smoothly it will run in such OS within older devices.

Finally, it is interesting to mention that sometimes there is only a need for a quick look on Jupyter notebooks as static webpages. In such case, we can use a web notebook viewer [38] or a local notebook viewer, which is a Chrome extension [39].

Figure 4. Two free cloud-service ways to run Jupyter Notebooks with SageMath – Binder (+GitHub) and CoCalc

III. CONCLUSIONS

In conclusion, we would like to emphasize that according to our five-years experience Jupyter notebooks with Sage appear to be a powerful and universal digital scientific environment for both education and research.

In mathematical education, the combination with blended learning and STEM education permitted to teach mathematics in a more effective, engaging, motivational and practical way, especially during the COVID 19 pandemics. In our research dealing with statistics, time series and metrology [40], [41], Sage has allowed us to naturally connect many open digital tools into one coherent framework for scientific computations, simulations and numerical research. Moreover, it seems that students who learn to work with Sage in the mentioned way, gain essential research skills and can be immediately engaged in various research tasks and projects.

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