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The disruption of planetary satellites and the creation of planetary rings

J. E. Colwell

Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder, CO 80309-0392, U.S.A.

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Abstract. The Voyager spacecraft discovered that small moons orbit within all four observed ring systems coincident with the discovery of narrow and dusty rings around Jupiter, Saturn, Uranus and Neptune. These moons can provide the source for new rings if they are catastrophically disrupted by a comet or large meteoroid impact. This hypothesis for ring origins provides a natural mechanism for the ongoing creation of planetary rings. While it relieves somewhat the problem of explaining the continued existence of rings with apparently short evolutionary lifetimes, it raises the problem of explaining the continued existence of small moons, and the coexistence of moons and rings at comparable locations within the Roche zones of the giant planets. This problem has been studied in some detail recently, and the present work is a review of our current understanding of the processes in satellite disruption that pertain to the creation of planetary rings and the collisional cascade of circumplanetary bodies. Significant progress has been made. Narrow rings are produced by disruption of small moons in numerical simulations, and a self-consistent model of the collisional cascade can explain present-day moon populations. Absolute timescales and initial moon populations remain uncertain due to our poor knowledge of the impactor population and uncertainties in the strength of planetary satellites. More pressing are the qualitative issues that remain to be resolved including the nature of reaccretion of the debris and the origin of Saturn's rings.

1. Introduction

The work reviewed in this paper deals with two fundamental questions: what are the origins and ages of planetary rings, and what are the origin and ages of small ($R \lesssim 100$ km) planetary satellites. The answers to these questions are not independent of each other. The four planetary ring systems are diverse in nature but all share the property that the time for the rings' observable characteristics to change significantly is much less than the age of the solar system. This leads immediately to the conclusion that planetary rings are highly evolved structures, although it does not say anything about the age of the material that comprises the rings or what form that material has taken over the history of the solar system. Specific examples of ring evolution argue more forcefully that some of the observed rings were recently created from a single intact parent body. In the next section I will review the arguments for "young" rings and discuss how they pertain to the question of the origin of rings from satellite disruption. Models for the origin of planetary rings were reviewed by Harris (1984) who favored satellite disruption for ring origins. A recent review of planetary rings is given by Esposito (1993).

The observation of moons much smaller than craters on the larger satellites raises an independent question: what happens to these moons under 4.5 billion years of bombardment by comets and meteoroids? Either the satellites are primordial and are not disrupted by the impacting flux, or they are a source of circumplanetary debris. This debris could take the form of visible rings. There is enough compelling evidence for the disruption of moons that the question of ring origins almost becomes redundant. At each of the giant planets, craters are observed on moons up to the theoretical upper limit for crater size before destruction occurs. It would be too convenient for nature to arrange for craters almost but not quite big enough to destroy moons. The capture of Triton by Neptune led to the mutual collisional destruction of any other Neptunian satellites present at the time (Banfield and Murray, 1992). According to this model, the Neptunian satellites orbiting interior to Triton today have reaccreted from a debris disk after the capture of Triton by Neptune. The destruction of moons therefore provides a convenient source of material for planetary rings and helps alleviate some of the short evolutionary timescale problems of planetary rings. The critical remaining questions are what is the rate of the disruption of planetary satellites and how

Correspondence to: J. E. Colwell

has that rate evolved over the history of the solar system; and what is the physics that allows small moons to co-exist with planetary rings within the Roche limit?

2. Evolution of planetary rings

There is relatively abundant circumstantial evidence that if planetary rings were not formed recently, that they at least underwent some physical process recently which reset their apparent age. The distinction is subtle, but important. The hypothesis that the rings formed recently requires delivery of material into the ring zone at a rate which would lead to the continued existence of planetary rings over the lifetime of the solar system, while the second hypothesis involves recycling the same material through repeated lifecycles, for example, from moons, to rings, and back to moons again. In the case of Saturn's rings, the mass of the rings is greater than the mass of all the moons within the ring region. Thus, the new ring material hypothesis requires either that we were fortunate enough to catch the rings before they disappeared forever, or that ring material is supplied to the circum-Saturnian environment from the interplanetary medium. Dones (1991) proposed just such a model for the origin of Saturn's rings. In his model, the rings could be the remnants of a tidally disrupted comet or asteroid, some 3 times larger than Chiron.

Arguments for the rings being young generally rely on extrapolation of some observed evolutionary process backward in time to some "initial" state. The timescales for these evolutionary processes are often much less than the age of the solar system. One example of this is the computation of the time for a satellite that is shepherding a ring edge to orbitally evolve to its present position from an initial position adjacent to the ring. For Saturn's A Ring this leads to an age on the order of 100 million years (e.g. Goldreich and Tremaine, 1982; Borderies *et al.*, 1984; Esposito, 1986). Similarly, Esposito and Colwell (1989) calculated that Ophelia could have shepherded the inner ε ring edge for no more than 6×10^8 years. These calculations place meaningful limits on the length of the time that the shepherding has endured, but not directly on the age of the ring itself.

A more direct calculation of ring age is the time for a ring to double in width due to collisional dissipation of orbital energy, or viscous spreading. Poynting–Robertson light drag, exospheric drag, and plasma drag also cause a ring to spread (e.g. Goldreich and Tremaine, 1979, 1982; Dermott, 1984). For the B and C Rings of Saturn this "age" is $\sim 10^{12}$ years (Esposito, 1986), considerably longer than the age of the solar system. For the A Ring it is only $\sim 10^7$ years. Thus, collisional and gravitational spreading calculations argue for a young A Ring, but place no serious constraints on the bulk of the material in Saturn's rings (the B Ring). The narrow F Ring of Saturn and the rings of Uranus and Neptune have short spreading timescales, however. For these narrow rings in the absence of confinement their doubling time is on the order of 10^7 years (Goldreich and Tremaine, 1982; Dermott, 1984).

More recent work suggests that even the B Ring is no more than $\sim 10^8$ years old, however. The ballistic transport calculations of Durisen *et al.* (1992) show that transport of micro-meteoroid ejecta can explain features in Saturn's B and A Ring inner edges if a simple model profile is bombarded with a model flux for $\sim 10^8$ years. Those calculations are frought with poorly determined ejecta yields, ring viscosities and impactor fluxes, so that it is possible that the ballistic transport timescale is billions of years. Micrometeoroid bombardment of the C Ring has a more direct impact on the age of the ring. Mass loading of the C Ring with an assumed micrometeoroid flux like that at 1 AU (Humes, 1980) will cause it to fall into Saturn in ~ 1–2×10^8 years (Cuzzi and Durisen, 1990). Small grains (less than a micron in size) ejected by hypervelocity impacts onto ring particles have short lifetimes due to a number of processes (e.g. Burns *et al.*, 1984). In the midst of an optically thick ring of macroscopic particles, however, the dominant loss mechanism for these dust particles is swept up by the larger ring particles (e.g. Colwell and Esposito, 1990a, b). Thus, drag processes are not an efficient loss mechanism for dust particles from Saturn's B Ring and the ε Ring of Uranus, each planet's most massive ring. The dust can, however, become charged by the plasma created in the impact. These charged particles can then precipitate into the atmosphere along magnetic field lines from Saturn's C Ring, leading to complete loss of the ring in $\sim 10^{7\text{-}8}$ years (Northrop and Connerney, 1987).

Another effect of the continuing bombardment of rings is the change in composition resulting from adding dark meteoritic material to icy ring particles. Doyle *et al.* (1989) find that Saturn's rings would be darkened to their current albedo from a pure ice initial condition in only ~ 1–2×10^8 years. The spokes in Saturn's rings are definitive examples of short-lived (hours to days) phenomena. They too are created by impacts onto the rings.

The arrangement of the arcs within Neptune's Adams Ring also suggests an origin by satellite fragmentation. Material in the arcs are confined from spreading azimuthally around the ring by Galatea, a satellite orbiting less than 1000 km away from the ring's inner edge. Porco (1991) showed that the arcs occupy seven of the 86 stable corotation resonance locations within the ring. These seven sites are clustered together within 10 consecutive sites of the possible 86, meaning that the arcs are statistically unlikely (probability of 10^{-8} (Esposito and Colwell, 1992)) to have been created separately. Because the arcs are not randomly distributed around the circumference of the Adams Ring, it is unlikely that Galatea evolved to a position where it sculpted the arcs out of a pre-existing ring in their current configuration. Creation of the arcs by the disruption of a single moon seems the most likely explanation based on the arcs' observed configuration and the corotation resonance confinement model.

If Saturn's rings did not exist, there would probably be little debate over the validity of the satellite fragmentation origin of planetary rings. The origins of at least some rings are most easily explained by satellite fragmentation. The optically thick rings of Uranus and Neptune each contain no more mass than could be contained in the smallest of the observed ring moons. Most could have been created by the disruption of a moon as small as 1 km in radius. The narrow, inner rings of Uranus are also those for which evolutionary times seem inescapably short. The combination of small ring mass and the relatively dense exosphere of Uranus leads to orbital decay of all but the ε Ring in much less than 10^8 years (Esposito *et al.*, 1991).

Saturn's rings however, are too massive to be continually replenished by satellite disruption on a 10^8 year timescale. There are not enough moons, and the disruption probabilities are too low for moons massive enough to make the rings (Lissauer *et al.*, 1988). There are several possible explanations for the problem of Saturn's massive and outwardly youthful rings. They may have been created recently by a rare impact that disrupted the one moon that was large enough to produce the observed ring system. They may have been created recently by the tidal capture and disruption of a 300 km radius comet (Dones, 1991). The rings may be primordial (date from the era of planet formation), and have evolved through several semi-stable configurations. (This last possibility fails to explain the short albedo darkening timescale of Doyle *et al.* (1989).) Or the ring system may have many different ages and modes of origin. The B Ring could be the remnants of a comet capture and disruption, à la Dones, while the A Ring could be the fragments from the recent disruption of a modest 50 km radius moon. It is also important to realize that a recent, rare, catastrophic fragmentation of a large satellite is not outside the realm of possibility for the origins of Saturn's rings. Lissauer *et al.* (1988) found the probability of disruption of a moon large enough to create Saturn's rings to be roughly one per 4×10^9 years based on crater records on the larger Saturnian satellites. Perhaps one moon existed near where Saturn's ring are now located, and was disrupted only sometime in the last few hundred million years. This raises the problem of our luck in arriving on the scene after the disruption occurred, because prior to the rare event, Saturn would have had only a modest ring system like those of the other giant planets. However, we have four giant planets that could conceivably have ended up with the spectacular ring system that Saturn now displays. This makes us four times less lucky in seeing Saturn's spectacular rings, since any of the other planets could have produced the same show, but didn't. A specific example where we have arrived on the scene in just the nick of time is Mars's moon Phobos. If the solar system were only 1 or 2 per cent older, Phobos would have hit the surface of Mars, and we would have missed it. But we got lucky, and got to Mars before Phobos.

Reality is more complicated than any single hypothesis. The rings are sculpted on many different size and timescales by impacts, including the creation of new ring particles by the disruption of small moons in and near the rings. Whether this is the sole mode of ring formation is an unanswerable question.

3. Planetary satellite disruption

Planetary satellite disruption is significantly different from asteroid disruption. The most important difference stems from the location of the satellite deep within a gravitational well. Planetary satellites nearly fill their Hill spheres, and the orbital timescale is hours. The first fact is another way of saying that ring moons are in a tidal environment that inhibits reaccretion of fragments. The second fact, on the other hand, means that no matter what the fragment velocities are, the fragments will reencounter each other and collide with each other repeatedly on the short orbital timescale. The presence of ring moons close to a planet also affects the speed at which impactors typically strike the moons. Impactors gain energy as they fall into the planetary gravitational field, and the relative velocity of the impactors and satellites are higher than for two asteroids because of the orbital velocity of the satellite around the planet. Impacts can occur at speeds as high as 80 km s^{-1}, compared to the average 5–6 km s^{-1} impact speed in the asteroid belt. This means that the mass ratio of impactor to target that results in a disruption can be much smaller than in the asteroid belt. Planetary satellites also tend to be colder, less dense, and of different composition than asteroids. These differences have not yet been fully accounted for in experimental and scaling analysis of catastrophic fragmentation (cf. McKinnon *et al.* (1991)).

Two steps are necessary to create a ring from the debris of a planetary satellite. First the satellite must be fragmented by an impact. Second, the fragments must be dispersed onto individual orbits and not reaccrete into a gravitationally bound rubble pile. The first step is the more straightforward of the two. We must first ask whether or not small satellites are likely to be catastrophically fragmented by interplanetary impactors. For ring creation we are concerned with moons of radius smaller than about 100 km. As a matter of fact, any ring except those of Saturn (admittedly a big exception) could be the result of the disruption of a moon no larger than 10 km in radius. The population of these ring precursor satellites depends on the unknown fragmentation properties of the largest moon likely to have been disrupted in the lifetime of the solar system.

3.1. *Disruption probability*

We can derive a simple relationship for the disruption probability on satellite size with a prescription for the dependence of Q^* on size. The measure of an impact can be expressed by the specific energy of the impact, Q, equal to the impactor's kinetic energy divided by the mass of the target. At some critical value, $Q = Q^*$, and the impact results in the target being fragmented with the largest fragment one half as massive as the original target. The probability of disruption of a satellite of radius R per unit time is given by

$$P_d(R) \propto \sigma F(Q > Q^*), \tag{1}$$

where σ is the cross-section of the satellite to the impactor flux and $F(Q > Q^*)$ is the flux of impactors with specific impact energy greater than the threshold value needed to disrupt the satellite. Small satellites close to the primary produce negligible gravitational acceleration of interplanetary impactors so $\sigma \propto R^2$. The flux of impactors larger than radius R_i can be characterized by a power-law:

$$F(>R_i) \propto R_i^{-d}. \quad (2)$$

This parameterization of the interplanetary impactor population is supported by *in situ* measurements of the population at 1 AU and crater distributions (Dohnanyi, 1972; Grün *et al.*, 1985). Then the specific impact energy is

$$Q \propto \left(\frac{R_i}{R}\right)^3. \quad (3)$$

In general Q is a function of the impactor velocity and impactor and target material properties as well. I have assumed these other variables are constant here in order to determine the dependence of disruption probability on target size alone.

The size dependence of Q^* has two distinct regimes: the strength regime which dominates at radii less than about 1–10 km, and the gravity regime which dominates at radii larger than 10–100 km. In the strength regime, the target's resistance to fragmentation is due solely to its dynamic tensile and compressive strength, that is, the strength of the target material itself. In the gravity regime the gravitational overburden of the target dominates the target's resistance to fragmentation. The exact location of this transition is somewhat uncertain and depends in part on the size dependence of Q^* in the strength regime. Housen and Holsapple's (1990) scaling analysis suggests $Q^* \propto R^{-1/4}$ in the strength regime while a theoretical limit is simple energy scaling which gives $Q^* =$ constant in the strength regime (e.g. Davis *et al.*, 1985). Recent attempts to quantify Q^* with hydrocode calculations starting from physical models of rock fracture suggest a much steeper dependence of Q^* on R in the strength regime (Ryan, 1993), with $Q^* \propto R^{-0.6}$. For the purpose of this heuristic derivation, I will use the Housen and Holsapple (1990) scaling analysis result. More recent work shows a larger value of Q^* than that of Housen and Holsapple (1990), but with virtually the same R dependence (Housen *et al.*, 1991; Holsapple, 1993).

In the strength regime, for impacts that are barely shattering, $Q = Q^*$ and the corresponding impactor radius is

$$R_i^{*3} \propto R^3 R^{-1/4}, \quad R_i^* \propto R^{11/12}. \quad (4)$$

The flux of impactors that can shatter a satellite of radius R is then given by (2) and (4):

$$F(>R_i^*) \propto R^{-0.92d}. \quad (5)$$

In the gravity regime, Housen and Holsapple find $Q^* \propto R^{1.65}$. Combining these expressions with (1) gives

$$P_d(R) \propto R^{2-0.92d} \quad (6a)$$

for the strength regime and

$$P_d(R) \propto R^{2-1.55d} \quad (6b)$$

for the gravity regime. This means that if $d \lesssim 2.2$ that the probability of disruption actually increases with target size in the strength regime. In the gravity regime the probability of disruption increases with R if $d \lesssim 1.3$. A value of $d = 1.3$ in the size range necessary to disrupt gravity regime satellites is unlikely based on observational constraints of the cometary population and crater statistics (Colwell and Esposito, 1992). In the strength regime, however, it is not as clear. Colwell and Esposito (1992) used values of d ranging from 2.3 to 2.6 based on crater diameter scaling laws, crater size distributions, and the observed meteoroid population (Grün *et al.*, 1985). Regardless of the exact value of d, it is clear from this simple derivation that in the strength regime there is a weak dependence of P_d on satellite size. The ramification of this is that the normalization of P_d to an absolute number with real target strengths and impactor fluxes is crucial in determining the qualitative outcome of satellite bombardment. If the flux is low enough and $b \lesssim 2.2$ then no satellites will be disrupted in the age of the solar system because $P_d(R)$ does not increase without bound at small R. If the flux is high enough, however, all the strength regime moons will undergo catastrophic fragmentation. If $b \gtrsim 2.2$ then the probability of disruption increases monotonically with decreasing target size, and there is some size corresponding to the largest satellite likely to have been disrupted during the history of the solar system.

3.2. *Gravitational focusing factor*

One of the primary differences between satellite disruption and asteroid disruption is the location of the former deep in the gravity well of a planet. This complicates the question of satellite escape velocities and the role of reaccretion (see below), and it accelerates the interplanetary impactors that lead to the disruption of the moons. This acceleration leads to higher impact velocities, and it increases the flux of impactors seen by a satellite over what it would see in the absence of the planet's gravitational field. This effect is called gravitational focusing. There is thus a dependence of P_d on the orbit radius, a, of the satellite. The farther it is from the planet, the smaller is the flux and the impact velocity, and P_d is therefore smaller.

The original expression for gravitational focusing was derived for the problem of impacts onto planets (Öpik, 1951; Safronov, 1969). This expression, and other simple expressions have been misapplied to the problem of planetary satellite cratering, so it is worth examining the problem in some detail.

The standard focusing factor can be derived from conservation of energy and angular momentum in a two-body formalism. That is, the gravitational effect of the Sun is ignored. Let b be the impact parameter of an impactor relative to the center of a planet, and V_∞ its velocity prior to any gravitational acceleration by the planet. At closest approach to the planet, the velocity is V_i and the periapse distance is a_p. The specific angular momentum of the impactor is

$$\frac{L}{m} = bV_\infty = a_p V_i. \quad (7)$$

The total specific energy of the impactor is

$$\frac{E}{m} = \frac{1}{2}V_\infty^2 = \frac{1}{2}V_i^2 - \frac{GM_P}{a_p}. \tag{8}$$

Combining (7) and (8) gives

$$b^2 = a_p^2\left(1 + \left(\frac{V_{esc}}{V_\infty}\right)^2\right), \tag{9}$$

where $V_{esc} = \sqrt{2GM_P/a_p}$. This leads to the standard expression for the gravitational focusing factor, f_p. In order for the impactor to hit the planet the periapse distance must be no greater than the planet radius. Setting $a_p = R_P$ in (9) gives the maximum value of b that an impactor can have and still hit the planet. If we consider the flux of impactors arriving from a given direction, then the number of impactors that can hit the planet per unit time is $F_\infty \pi b^2$. This number is equal to $F_P \pi R_P^2$, where F_P is the flux at the planet, and therefore

$$f_p \equiv \frac{F_P}{F_\infty} = \left(\frac{b}{R_P}\right)^2 = 1 + \left(\frac{V_{esc}}{V_\infty}\right)^2. \tag{10}$$

This expression is only appropriate for impacts onto the object that is providing the gravitational acceleration. In other words, this focusing factor is equal to the ratio of the area of a disk of radius b to the area of a disk of radius R_P. The disk of radius R_P is the target cross-section. For a non-gravitating target orbiting the planet, the target cross-section is not equal to the planet cross-section. There has been some confusion in the literature about this point. For example, Shoemaker and Wolfe (1982), Lissauer *et al.* (1988) and Cuzzi and Durisen (1990) incorrectly used (10) for impacts onto satellites and rings.

To see that (10) is not appropriate for satellites and rings, let us now derive an expression for the gravitational focusing factor for impacts onto a satellite with negligible gravity orbiting a planet with mass M_P. The mapping of the impactor flux at infinity onto this massless target is different than the mapping of the impactor flux at infinity onto a planet discussed above. This leads to a different expression for the gravitational focusing factor. We assume that the satellite's size is small compared to its orbit radius, a. If we further assume that the impactor flux is monodirectional at infinity, then it is clear that only a narrow range in b will lead to impactors crossing the satellite plane (Fig. 1). If we distribute the satellite's cross-section around its orbit and let the impactor flux at infinity approach from a perpendicular to the satellite orbit plane, then there is an annulus of width $2R$ in the orbit plane that is the target for the impactor flux. This is in contrast to the situation for planetary impacts where the target is a disk of radius R_P. The target area is now $(2\pi a)(2R) \approx 2\pi a\,da$ since $R \ll a$. The number of impactors that can hit this target area is $F_\infty 2\pi b\,db$. Again, we assume the impact occurs at periapse. Then, by analogy with (10) and differentiating (9) to get db/da, the satellite gravitational focusing factor is

$$f_s \equiv \frac{b}{a}\frac{db}{da} = 1 + \frac{V_{esc}^2}{2V_\infty^2}. \tag{11}$$

That is, the focusing factor is smaller for satellites than for planets. Morfill *et al.* (1983) derived (11) for impacts onto Saturn's rings, and Colwell and Esposito (1992, 1993) used (11) for their calculations of satellite fragmentation. With the assumptions as stated the focusing factor for rings is the same as that for satellites: $f_s \equiv f_r$.

As shown in Fig. 1, the impact does not necessarily occur at periapsis. Impacts with the idealized flux geometry outlined above cannot occur at periapsis because at periapsis the impactor will not be in the ring or satellite plane if V_∞ is normal to that plane. We can remove the assumption of impact at periapsis from (11) in the following way. For the monodirectional flux normal to the satellite plane, impact occurs when the impactor velocity vector has been deflected by the planet's gravitational field through an angle χ' (Fig. 1). This angle is (Colwell, 1993):

$$\chi' = \chi \pm \cos^{-1}\left[\frac{V_\infty^2 b^2/R - GM_P}{\sqrt{(GM_P)^2 + b^2 V_\infty^4}}\right], \tag{12}$$

where 2χ is the full deflection angle in the absence of any impacts (e.g. Fowles, 1977):

$$\chi = \sin^{-1}([1 + (bV_\infty^2/GM)^2]^{-1/2}). \tag{13}$$

The relationship between impact parameter and satellite orbit radius is given by conservation of angular momentum (like in the heuristic derivation):

$$b' = \frac{V_i}{V_\infty} R\cos\chi', \tag{14}$$

where V_i is given by equation (8), and the b' is to distinguish this impact parameter from that defined in equation (9). Because χ' depends on both b' and R, equation (14) must be solved iteratively. Solving (14) gives $b'(R)$ which can be differentiated and a new focusing factor computed from $(b'/R)(db'/dR)$. Including χ' in the calculation reduces the focusing factor by less than 20 per cent for typical values of V_∞. The change is larger for smaller values of V_∞ and smaller values of R.

Removing the other assumptions in the derivation of the focusing factors (two-body approximation, monodirectional flux) requires orbit integration of realistic impactor populations. Greenzweig and Lissauer (1992) have done just that for the case of impacts onto a protoplanet. They found a factor of three enhancement in the accretion rate for the particular cases they integrated. This large a change in the planetary case suggests that a detailed solution for the focusing factor for impacts onto satellites may produce a significantly different result than that given in (11).

3.3. *Rates of satellite fragmentation*

Shoemaker and Wolfe (1982) made the first quantitative estimates of cratering rates in the outer solar system in the Voyager era. They used the observed population of short period comets and corrected for observational selection effects and extinct comets by deriving "synthetic" comet orbits. The basic idea behind their approach is to compute the impact probability for a given orbit and then compute the impact rate from a modeled population of

Table 1. Small outer planet satellites

Satellite	Orbit (10^4 km)[a]	Orbit (Roche Radii[c])	Rings[d]	Radius (km)[b]	$N(>R)$[e]
Metis	12.80	0.76	Main Ring	20	6.7
Adrastea	12.90	0.76	Main Ring	10	9.8
Amalthea	18.13	1.06	No	100	1.7
Thebe	22.19	1.30	No	50	1.9
Pan	13.36	0.96	Encke Gap	13	5.5
Atlas	13.77	0.99	A Ring	19	3.4
Prometheus	13.94	1.00	F Ring	50	1.1
Pandora	14.17	1.02	F Ring	43	1.3
Janus	15.14	1.09	No	95	0.5
Epimetheus	15.15	1.09	No	58	0.8
Cordelia (1986U7)	4.98	0.78	ε Ring	13	24
Ophelia (1986U8)	5.38	0.85	ε Ring	16	15
Bianca (1986U9)	5.92	0.93	No	22	8.3
Cressida (1986U3)	6.18	0.97	No	33	4.2
Desdemona (1986U6)	6.27	0.99	No	29	5.0
Juliet (1986U2)	6.44	1.01	No	42	2.8
Portia (1986U1)	6.61	1.04	No	55	1.8
Rosalind (1986U4)	6.99	1.10	No	29	4.3
Belinda (1986U5)	7.53	1.18	No	34	3.0
Puck (1985U1)	8.60	1.35	No	77	0.7
Naiad (1989N6)	4.82	0.79	No	29	17
Thalassa (1989N5)	5.01	0.82	No	40	8.9
Despina (1989N3)	5.25	0.86	Leverrier Ring?	74	2.5
Galatea (1989N4)	6.19	1.01	Adams Ring	79	2.4
Larissa (1989N2)	7.35	1.20	No	96	1.4

[a] Orbital radii for the Uranian satellites are from Owen and Synnott (1987), and for Neptune from Owen *et al.* (1991). [b] Uranian satellite radii from Thomas *et al.* (1989), and for Neptune from Thomas and Veverka (1991). Jovian and Saturnian satellite radii (approximate values for triaxial ellipsoids) and orbit radii from Burns (1986), except for Pan from Showalter (1991). [c] Roche radii were calculated using a mean radius for each planet and a satellite density equal to the density of the planet. [d] All of the listed moons may have tenuous dust rings derived from micrometeoroid erosion of the moons. The listed satellite–ring associations are those which indicate a common origin and linked evolution of the ring and the moon. [e] $N(>R)$ is the number of craters expected in the last 3.5×10^9 years with diameters larger than the radius of the satellite. All values are for planet-family comets only. Asteroids and long period comets roughly double $N(>R)$ for Jovian satellites, and increase $N(>R)$ by a factor of 1.3–1.5 for Saturnian satellites (Shoemaker and Shoemaker, 1990). Planet-family comets are the dominant source of impacts for Uranus and Neptune. Jupiter values assumed 1400 Jupiter-family comets larger than 2.5 km in diameter, and a cumulative crater size frequency distribution of power-law index 2.2 (Shoemaker and Wolfe, 1982). For Saturn, 10,000 comets larger than 2.5 km in diameter (Smith *et al.*, 1982) and a crater distribution slope of 2.7 were used (Lissauer *et al.*, 1988). Uranus and Neptune satellite values are from Colwell and Esposito (1992).

impactors and distribution of those impactors' orbits in orbital element phase space. This approach was used by the Voyager imaging team to compute satellite disruption rates for the moons of Saturn (Smith *et al.*, 1982), Uranus (Smith *et al.*, 1986) and Neptune (Smith *et al.*, 1989). Lissauer *et al.* (1988) re-evaluated the Saturn impact rates. They found disruption probabilities 10–20 times lower than those of Smith *et al.* (1982). Both papers used the f_p focusing factor (equation (10)) rather than f_s (equation (11)). Colwell and Esposito (1992) used f_s from equation (11) and re-evaluated the disruption rates for Uranus and Neptune. They found disruption frequencies within a factor of two of the Shoemaker results in the Voyager imaging papers.

For Jupiter, Shoemaker and Wolfe (1982) had an observed population of "Jupiter-family" comets: more than 100 short period comets. At Saturn, crater counts were used to constrain the satellite disruption rate (Smith *et al.*, 1982; Lissauer *et al.*, 1988). Impactor populations are less certain at the orbits of Uranus and Neptune. Crater statistics at Neptune in particular are almost non-existent (Smith *et al.*, 1989). At the orbits of Uranus and Neptune cometary activity is low and the greater distance from the Sun and the Earth makes observational detection of comets difficult. However, the existence of Chiron and a handful of putative Kuiper Belt objects (e.g. Luu, 1993) help place limits on the populations of the largest planet-family comets in the outer solar system. Theoretical models of the origin of the short-period comets from the Kuiper belt provide the best quantitative estimates of the population of planet-family comets beyond the orbit of Jupiter. Colwell and Esposito (1992) derived a flux for the Uranus and Neptune regions by using the Shoemaker and Wolfe (1982) approach and linking it to micrometeoroid fluxes measured at 1 AU and out to 19 AU by Pioneer 10 (Humes, 1980).

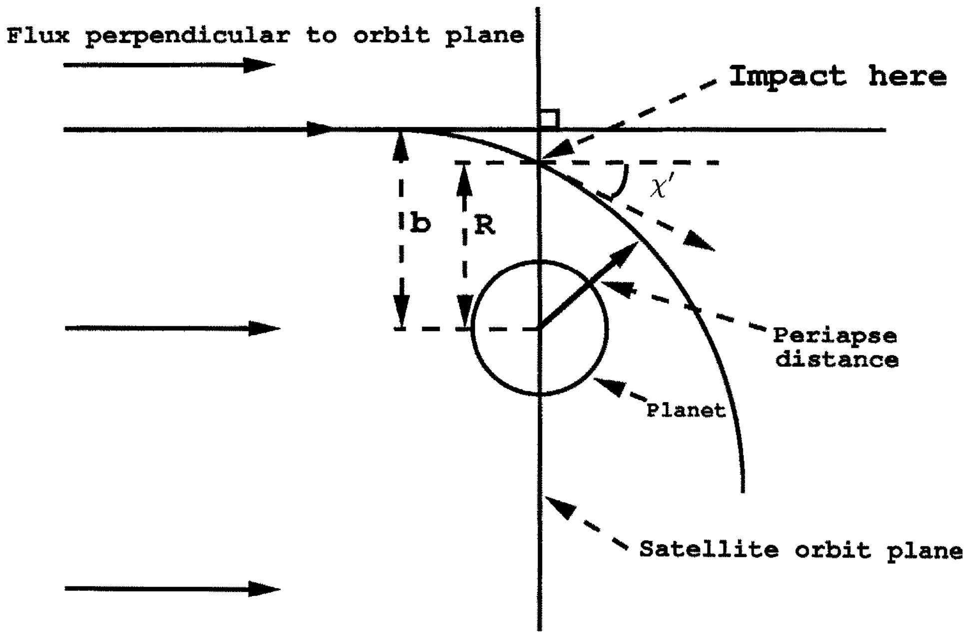

Fig. 1. Idealized geometry for an interplanetary impactor hitting a planetary satellite or ring particle. In the heuristic derivation of the gravitational focusing factor, the impact occurs at periapse, which cannot be true for this idealized geometry at all points in a satellite's orbit. Instead, the impact occurs after the impactor's velocity vector has been deflected toward the planet through an angle χ'

While the planet-family comets can provide an estimate of the population at one size, they say nothing about the size distribution. Shoemaker and Wolfe (1982) and Colwell and Esposito (1992) estimated size distributions from the distribution of crater diameters on the larger satellites of the planet in question. Using this technique Colwell and Esposito (1992) estimated a cometary flux at Uranus and Neptune that matched the magnitude and slope of the micrometeoroid flux (Fig. 2). This approach fails if the crater diameter distribution is affected by circumplanetary debris. This model impactor flux satisfies

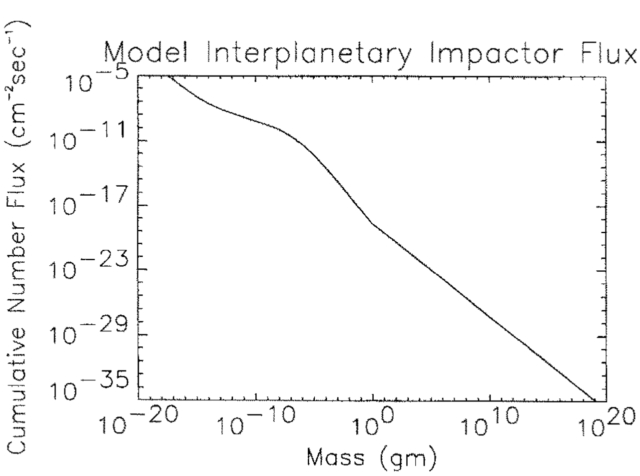

Fig. 2. The model cumulative number flux of interplanetary impactors used by Colwell and Esposito (1992, 1993) and used in this paper for modeling the collisional cascade of planetary satellites. The distribution up to 1 g is that of Grün *et al.* (1985) from measurements at 1 AU. Above 1 g, a simple power-law extrapolation based on observed comet distributions and crater distributions was adopted

the minimal requirement of being consistent with observations and our current understanding of the population and size distribution of outer-solar-system, planet-family comets. It is, however, only a best guess estimate, and could conceivably be off by orders of magnitude at any given impactor size.

Using this model flux and the scaling law value of Q^* from Housen and Holsapple (1990), Colwell and Esposito (1992) derived fragmentation rates for the moons of Uranus and Neptune. They found that the ten innermost moons of Uranus and five innermost moons of Neptune are likely to have been catastrophically fragmented at least once during the last 3.5×10^9 years. The fragmentation probability as a function of satellite size is shown in Fig. 3 for moons of Uranus with their model. In order for the smallest Uranian satellite to be unfractured and primordial, the flux estimate would have to be too high by a factor of 20. It is unlikely that the value of Q^* at the size of Cordelia ($R = 13$ km) from Housen and Holsapple (1990) is too low by more than a factor of two or three, because energy scaling provides a theoretical upper limit to $Q^*(R)$. In fact, hydrocode calculations suggest that it may be much lower, perhaps by as a much as a factor 5 (Ryan, 1993).

The conclusion, therefore, is that either the moons are collisional fragments from some initial population of larger moons, or they are multiply fractured rubble piles, somehow held together despite their presence in their planet's Roche zone.

3.4. *Collisional cascade of planetary satellites*

The definition of Q^* is the specific impact energy to fragment an object so that the largest fragment has one half

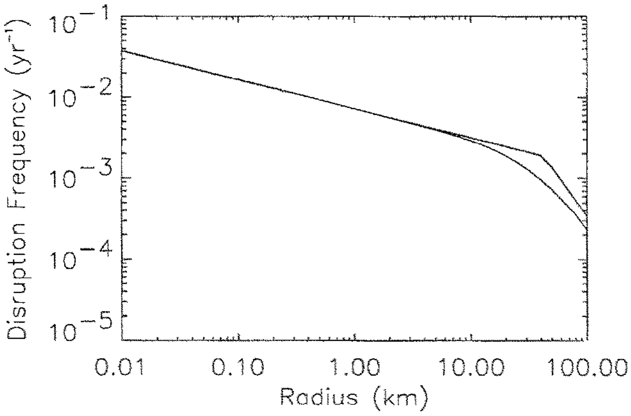

Fig. 3. The fragmentation frequency is plotted as a function of satellite size for moons of Uranus at the orbit of Cordelia, the innermost satellite of Uranus, and the heuristic expression for the fragmentation frequency is also shown with the value $b = 2.565$ (Colwell and Esposito, 1992, 1993)

the mass of the original target. The consequences of using Q^* without any additional criteria for fragment dispersal are that the satellite systems of Uranus and Neptune will collisionally evaporate on timescales of 10^{6-7} years (Colwell and Esposito, 1992). Colwell and Esposito used a Markov Chain formalism to model the evolution of the fragment size distribution of moons of Uranus and Neptune, using the model impactor flux of Fig. 2. Figure 4 shows the probability of each of the 10 small Uranian satellites remaining unfragmented as a function of time if a bombarding flux was turned on the present day satellite system of Uranus. The five large classical satellites of Uranus survive indefinitely and are not plotted, while the smaller moons are all likely to have been collisionally fragmented in times much less than the age of the solar system.

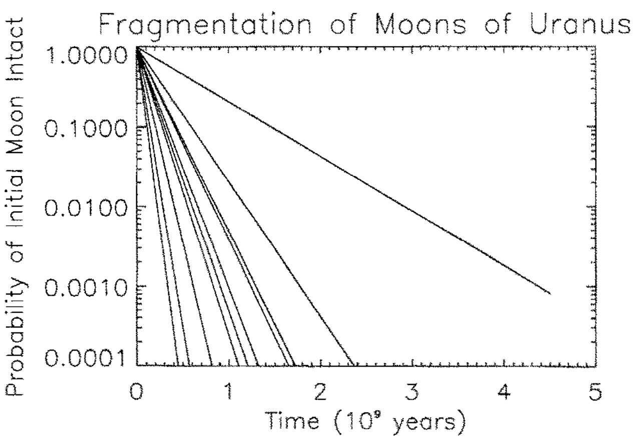

Fig. 4. The fragmentation probabilities as a function of time for the ten small Uranian satellites are plotted. Puck has the greatest probability of surviving intact, but given the model flux and fragmentation criteria of Colwell and Esposito (1992) it is likely to be fragmented in only 3×10^8 years. In that time, Cordelia (bottom curve) has a greater than 99 per cent probability of being fragmented. These probabilities are for fragmentation, not fragmentation and dispersal of the fragments. For comparison, Miranda, the next largest satellite of Uranus, has a fragmentation probability of about 10 per cent over the age of the solar system with the current flux model (Colwell and Esposito, 1992)

In the absence of any reaccretion the fragmentation of a satellite creates numerous smaller fragments, each of which can be further fragmented by subsequent impacts. This leads to a collisional cascade of the ring moons from large satellites to smaller ones. Figure 5 shows an example of this collisional cascade for Naiad, the smallest observed moon of Neptune. The distributions shown in Fig. 5 assume that all fragments created in an impact escape the parent object. However, even though the ring moons are located within their planets' Roche limits, some reaccretion can occur. Furthermore, the fragmentation criterion embodied in Q^* is for simple fragmentation with no movement of the fragments. Colwell and Esposito (1993) modeled the velocity distribution of the fragments, and computed the collisional cascade with the requirement that fragments must have escape velocity from the parent object in order to be lost from the parent. An example of the collisional cascade for Naiad with this condition is shown in Fig. 6.

This treatment of reaccretion of the fragments is incomplete, however, because escape velocity from the parent object is the appropriate criterion only if the objects are not embedded in another gravitational field. Since the moon and its fragments are gravitationally bound to the planet, they re-encounter each other on the short orbital timescale (~ 10 h). This short timescale leads to rapid reaccretion of all fragments in the absence of tidal forces (Canup and Esposito, 1992). Thus we are faced with the problem of explaining the continued existence of rings and moons in the same tidal environment at each of the giant planets. One possibility is size-dependent accretion. In this hypothesis, the size distribution of the fragments from a catastrophic disruption of a satellite determines to what extent the fragments reaccrete. The physical basis for this idea is that within the Roche zone, moon fragments are physically almost as large as their Hill spheres.

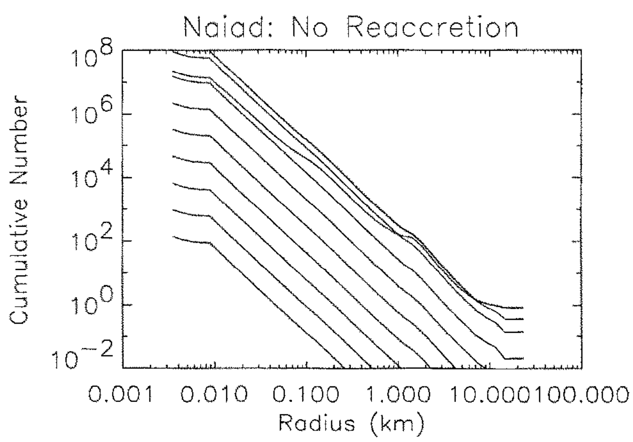

Fig. 5. An accretionless collisional cascade for Neptune's innermost satellite, Naiad, shows the number of objects larger than a given size for different times. The distributions are plotted at the following times (in descending order from the top of the plot): 10^7, 5×10^7, 10^8, 2×10^8, 3×10^8, 4×10^8, 5×10^8, 6×10^8, 7×10^8 and 8×10^8 years. After 2×10^8 years there is about a 1 per cent chance that Naiad would be intact, and the expectation value for objects larger than 1 km in radius is about 20

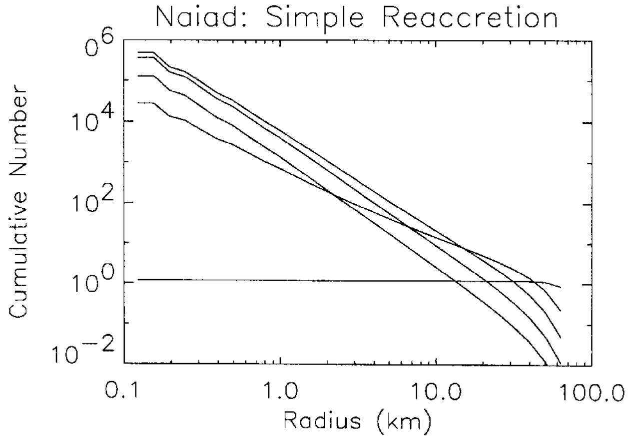

Fig. 6. The collisional cascade for a moon at the orbit of Naiad is shown where only fragments that exceed the escape velocity of the moon are considered lost from the moon. The distribution is plotted at times of 10^8, 10^9, 2×10^9, 3×10^9 and 4×10^9 years. The slope of the distribution gets progressively steeper with time, and the number (expectation value) of objects at the size of the original object (63 km radius) decreases with time. After 4×10^9 years (bottom curve) the expectation value for the number of objects larger than 1 km in radius is about 1000

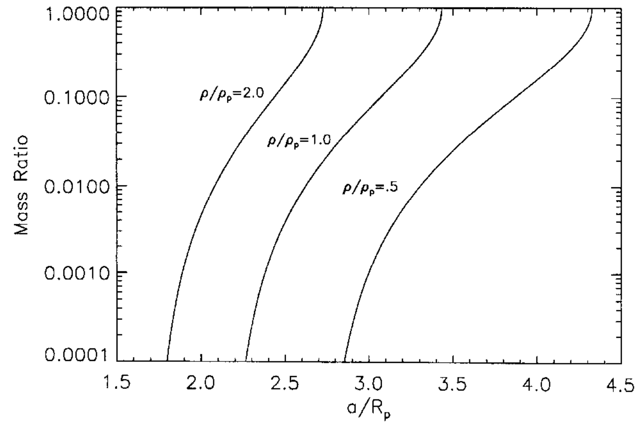

Fig. 7. Critical curves for accretion in the Roche zone from the model of Canup and Esposito (1993). Each curve is for a specific density of the accreting objects relative to the density of the planet. To the left of a curve accretion does not occur because the accreting objects overflow each others' mutual Hill sphere. To the right of a curve accretion can occur if the rebound velocities are low enough to prevent escape. At 2.5 planetary radii, for example, and with a density equal to the density of the planet, accretion can only occur for objects that have a mass ratio of 0.003 or smaller

Thus, a single large fragment, such as might occur from a marginally catastrophic disruption, can reaccrete the remainder of the fragments if they are small enough to fit inside the largest fragment's Hill sphere. A super-catastrophic disruption, on the other hand, would create fragments of similar size that could not fit within each other's mutual Hill spheres. The result of such an impact would be a planetary ring or moonlet belt. The result of a marginal disruption would be a (mostly) reaccreted rubble pile.

Canup and Esposito (1993, 1995, personal communication) have developed a numerical approach to reaccretion within the Roche zone that differs from other planetary accretion codes. They have shown that tidal forces can lead to particle size dependent accretion with the Roche zone. Figure 7 shows their result for what particles can accrete with each other as a function of particle density and orbit radius. Their model is a zero-dimensional Markov chain with a particle-in-a-box collision frequency. At some point in the velocity evolution of the swarm this collision frequency breaks down, but the preliminary results suggest a way that rings and moons can coexist at the same orbit region. Further work incorporating an explicit reaccretion model such as that of Canup and Esposito with a collisional cascade model should produce a fully self-consistent model of the collisional evolution of planetary satellites and the origin of planetary rings.

3.5. *Initial ring profiles*

Catastrophic disruption of satellites occurs when a relatively small impactor travelling at speeds in excess of 10 km s^{-1} strikes a satellite. The amount of energy delivered to the satellite for fragmentation is $\sim 10^{6-7}$ erg g^{-1} for small ring moons. If the satellite were comminuted into pieces 1 g in mass, and all the kinetic energy of the impact were transmitted to the kinetic energy of the fragments, the fragment speeds would be $\sim$ 14–45 m s^{-1}. Experiments where the fragment velocity distribution is measured show that larger fragments have lower velocities (Nakamura and Fujiwara, 1991; Nakamura *et al.*, 1992). Since many of the fragments will be considerably larger than 1 g, and since only about 1/10 of the impactor kinetic energy is partitioned into the fragments' kinetic energy, the true fragment speeds are considerably lower than 10 m s^{-1} for the bulk of the fragments.

Orbital velocities of ring moons are typically on the order of 10 km s^{-1}. The small fragment speeds thus imply an orbital eccentricity of only $\sim 10^{-3}$. For ring moons of Uranus at Neptune at orbits of 40,000–60,000 km satellite disruption therefore naturally leads to a ring with a characteristic width of about 50 km. Colwell and Esposito (1993) verified this heuristic result with detailed calculations of the fragment mass and velocity distribution. An example of a debris belt radial profile after the fragments have spread into a complete ring around the planet is shown in Fig. 8. The ring will evolve due to interparticle collisions and gravitational perturbations from any nearby moons, but the initial condition from the satellite disruption is consistent with the development of a narrow ring. Specifically, it is hard to imagine the initial distribution contracting in width so that a minimum requirement for the hypothesis that narrow rings are created from disrupted satellites is a narrow initial distribution. Figure 8 shows that this requirement is satisfied to first order by current models of fragmentation.

4. Summary and discussion

Our present understanding of the disruption of planetary satellites and the origins of planetary rings is incomplete,

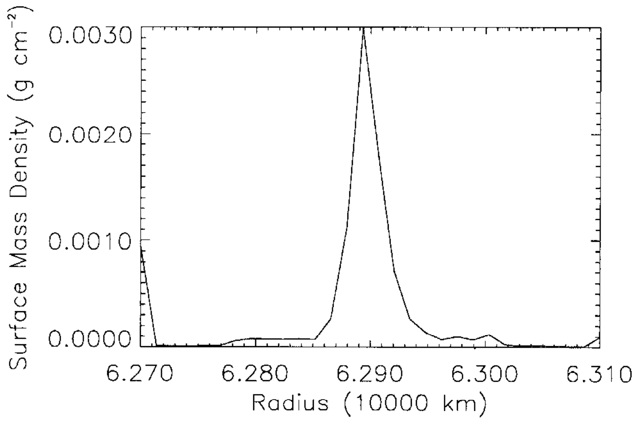

Fig. 8. An azimuthally averaged radial profile of the surface mass density of the debris from the disruption of 1 km moon is plotted after the fragment orbits have sheared enough to form a uniform ring around the planet. The fragment mass and velocity distributions were calculated as in Colwell and Esposito (1993)

but rapidly evolving. It is almost certain that the small moons ($R \lesssim 100$ km) have at least been shattered one or more times over the age of the solar system. This conclusion is uncertain only to the extent that the flux of cometary impactors in the outer solar system is not well determined. The smallest moons ($R \lesssim 50$ km) have probably been multiply shattered and dispersed, and are either reaccreted rubble piles or fragments of larger precursor moons. Small fragments ($R \lesssim 10$ km) from the disruption of larger satellites have short lifetimes against disruption, and create narrow planetary rings when disrupted. The subsequent evolution of these rings depends on the details of the ring environment. Nearby moons may shepherd the new ring edges, or the ring may rapidly spread due to collisions and exospheric drag to the point where its optical depth is below detection thresholds. Such rings may be what are called moonlet belts: optically thin rings of meter-sized and larger particles that are the source of the dust rings at each of the ringed planets.

Using the escape velocity criterion for dispersal of fragments, Colwell and Esposito (1993) estimated the total number of ring moons larger than 1 km in radius at Uranus and Neptune at the present time by matching the present observed population to a model collisional cascade after 4.5×10^9 years of bombardment. Their results were that the population of 1 km ring precursor satellites is $\gtrsim 100$ at each planet. Their Markov chain calculation produces only average distributions over an infinite ensemble of identical systems, not an individual history. Furthermore, the escape velocity is an oversimplification of the true accretion processes acting in the Roche zone. Nevertheless, the existence of the sharp-edged and narrow rings of Uranus implies an unseen population of moons. With the exception of the ε Ring, the individual ring masses of the Uranian rings correspond to moons in the 1 km size range. The dust rings of Uranus and Neptune require a population of unseen moonlets as a source of the short-lived dust particles (Colwell and Esposito, 1990a, b). The structure and evolution of rings are intimately linked with the small ring moons and their collisional evolution. Resonance interactions between the rings and moons further affect their evolution.

We are now at the point where the basic idea of creating rings from moons has been shown to be feasible, and even likely. The next step is to understand how the systems evolve qualitatively and quantitatively over long time scales, and link the observed systems to the initial conditions at each planet. This last step may prove impossible if the systems have lost their memory of their initial conditions. Detailed simulations of individual systems will resolve this question.

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References

Banfield, D. and Murray, N., A dynamical history of the inner Neptunian satellites. *Icarus* **99,** 390–401, 1992.

Borderies, N., Goldreich, P. and Tremaine, S., Unsolved problems in planetary ring dynamics, in *Planetary Rings* (edited by R. Greenberg and A. Brahic), pp. 713–734. University of Arizona Press, Tucson, 1984.

Burns, J. A., Some background about satellites, in *Planetary Rings* (edited by J. A. Burns and M. S. Matthews), pp. 1–38. University of Arizona Press, Tucson, 1986.

Burns, J. A., Showalter, M. R. and Morfill, G. E., The ethereal rings of Jupiter and Saturn, in *Planetary Rings* (edited by R. Greenberg and A. Brahic), pp. 200–272. University of Arizona Press, Tucson, 1984.

Canup, R. M. and Esposito, L. W., The re-accretion of disrupted planetary satellites. *Bull. Amer. Astron. Soc.* **24,** 937, 1992.

Canup, R. M. and Esposito, L. W., Accretion in the Roche zone. *Bull. Amer. Astron. Soc.* **25,** 1102, 1993.

Canup, R. M. and Esposito, L. W., Accretion in the Roche zone, 1, co-existence of rings and ringmoons. *Icarus*, in press, 1995.

Colwell, J. E., A general formulation for the distribution of impacts and ejecta from small planetary satellites. *Icarus* **106,** 536–548, 1993.

Colwell, J. E. and Esposito, L. W., A numerical model of the Uranian dust rings. *Icarus* **86,** 530–560, 1990a.

Colwell, J. E. and Esposito, L. W., A model of dust production in the Neptune ring system. *Geophys. Res. Lett.* **17,** 1741–1744, 1990b.

Colwell, J. E. and Esposito, L. W., Origins of the rings of Uranus and Neptune 1. Statistics of satellite disruptions. *J. geophys. Res.* **97,** 10,227–10,241, 1992.

Colwell, J. E. and Esposito, L. W., Origins of the rings of Uranus and Neptune 2. Initial conditions and ring moon populations. *J. geophys. Res.* **98,** 7387–7401, 1993.

Cuzzi, J. N. and Durisen, R. H., Bombardment of planetary rings by meteoroids: general formulation and effects of Oort cloud projectiles. *Icarus* **84,** 467–501, 1990.

Davis, D. R., Chapman, C. R., Weidenschilling, S. J. and Greenberg, R., Collisional history of asteroids: evidence from Vesta and the Hirayama families. *Icarus* **62,** 30–53, 1985.

Dermott, S. F., Dynamics of narrow rings, in *Planetary Rings* (edited by R. Greenberg and A. Brahic), pp. 589–637. University of Arizona Press, Tucson, 1984.

Dohnanyi, J. S., Interplanetary objects in review: statistics of their masses and dynamics. *Icarus* **17,** 1–48, 1972.

Dones, L., A recent cometary origin for Saturn's rings? *Icarus* **92**, 194–203, 1991.

Doyle, L. R., Dones, L. and Cuzzi, J. N., Radiative transfer modeling of Saturn's outer B Ring. *Icarus* **80**, 104–135, 1989.

Durisen, R. H., Bode, P. W., Cuzzi, J. N., Cederbloom, S. E. and Murphy, B. W., Ballistic transport in planetary ring systems due to particle erosion mechanisms 2. Theoretical models for Saturn's A- and B-Ring inner edges. *Icarus* **100**, 364–393, 1992.

Esposito, L. W., Structure and evolution of Saturn's rings. *Icarus* **67**, 345–357, 1986.

Esposito, L. W., Understanding planetary rings. *Ann. Rev. Earth Planet. Sci.* **21**, 487–523, 1993.

Esposito, L. W. and Colwell, J. E., Creation of the Uranus rings and dust bands. *Nature* **339**, 605–607, 1989.

Esposito, L. W. and Colwell, J. E., Neptune's rings and satellite system: collisional origin and evolution. *Eos* **73**, 177, 1992.

Esposito, L. W., Brahic, A., Burns, J. A. and Marouf, E. A., Particle properties and processes in Uranus' rings, in *Uranus* (edited by J. T. Bergstralh, E. D. Miner and M. S. Matthews), pp. 410–465. University of Arizona Press, Tucson, 1991.

Fowles, G. R., *Analytical Mechanics*, third edition, pp. 154–158. Holt, Reinhart and Winston, New York, 1977.

Goldreich, P. and Tremaine, S., Towards a theory for the Uranian rings. *Nature* **277**, 97–99, 1979.

Goldreich, P. and Tremaine, S., The dynamics of planetary rings. *Ann. Rev. Astron. Astrophys.* **20**, 249–283, 1982.

Greenzweig, Y. and Lissauer, J. J., Accretion rates of protoplanets 2. Gaussian distributions of planetesimal velocities. *Icarus* **100**, 440–463, 1992.

Grün, E., Zook, H. A., Fechtig, H. and Giese, R. H., Collisional balance of the meteoritic complex. *Icarus* **62**, 244–272, 1985.

Harris, A. W., The origin and evolution of planetary rings, in *Planetary Rings* (edited by R. Greenberg and A. Brahic), pp. 641–659. University of Arizona Press, Tucson, 1984.

Holsapple, K. A., The scaling of impact processes in planetary sciences. *Ann. Rev. Earth Planet. Sci.* **21**, 333–373, 1993.

Housen, K. R. and Holsapple, K. A., On the fragmentation of asteroids and planetary satellites. *Icarus* **84**, 226–253, 1990.

Housen, K. R., Schmidt, R. M. and Holsapple, K. A., Laboratory simulations of large scale fragmentation events. *Icarus* **94**, 180–190, 1991.

Humes, D. H., Results of Pioneer 10 and 11 meteoroid experiments: interplanetary and near Saturn. *J. geophys. Res.* **85**, 5841–5852, 1980.

Lissauer, J. J., Squyres, S. W. and Hartmann, W. K., Bombardment history of the Saturn system. *J. geophys. Res.* **93**, 13,776–13,804, 1988.

Luu, J. X., The Kuiper belt comets. *Abstracts for IAU Symposium* 160: *Asteroids, Comets, and Meteors*, 196, 1993.

McKinnon, W. B., Chapman, C. R. and Housen, K. R., Cratering of the Uranian satellites, in *Uranus* (edited by J. T. Bergstralh, E. D. Miner and M. S. Matthews), pp. 629–692. University of Arizona Press, Tucson, 1991.

Morfill, G. E., Fechtig, H., Grün, E. and Goertz, C. K., Some consequences of meteoroid impacts on Saturn's rings. *Icarus* **55**, 439–447, 1983.

Nakamura, A. and Fujiwara, A., Velocity distribution of fragments formed in a simulated collisional disruption. *Icarus* **92**, 132–146, 1991.

Nakamura, A., Suguiyama, K. and Fujiwara, A., Velocity and spin of fragments from impact disruptions. *Icarus* **100**, 127–135, 1992.

Northrop, T. G. and Connerney, J. E. P., A micrometeorite erosion model and the age of Saturn's rings. *Icarus* **70**, 124–137, 1987.

Öpik, E. J., Collision probabilities with the planets and distribution of interplanetary matter. *Proc. R. Ir. Acad. Sect. A* **54A**, 165–199, 1951.

Owen, W. M., Jr and Synnott, S. P., Orbits of the ten small satellites of Uranus. *Astron. J.* **93**, 1268–1271, 1987.

Owen, W. M., Jr, Vaughan, R. M. and Synnott, S. P., Orbits of the six new satellites of Neptune. *Astron. J.* **101**, 1511–1515, 1991.

Porco, C. C., An explanation for Neptune's ring arcs. *Science* **253**, 995–1001, 1991.

Ryan, E., Asteroid collisions: target size effects and resultant velocity distributions. *Lunar Planet. Sci. Conf.* XXIV, 1227–1228, 1993.

Safronov, V. S., *Evolution of the Protoplanetary Cloud and Formation of the Earth and Planets.* Nauka Press, Moscow, 1969; also NASA TTF-677, 1972.

Shoemaker, E. M. and Shoemaker, C. S., The collision of solid bodies, in *The New Solar System* (edited by J. K. Beatty and A. Chaikin), pp. 259–274. Cambridge University Press, Cambridge, 1990.

Shoemaker, E. M. and Wolfe, R. F., Cratering time scales for the Galilean satellites, in *Satellites of Jupiter* (edited by D. Morrison), pp. 277–339. University of Arizona Press, Tucson, 1982.

Showalter, M. R., Visual detection of 1981S13, Saturn's eighteenth satellite, and its role in the Encke gap. *Nature* **351**, 709–713, 1991.

Smith, B. A., *et al.*, A new look at the Saturn system: the Voyager 2 images. *Science* **215**, 504–537, 1982.

Smith, B. A., *et al.*, Voyager 2 in the Uranian system: imaging science results. *Science* **233**, 43–64, 1986.

Smith, B. A., *et al.*, Voyager 2 at Neptune: imaging science results. *Science* **246**, 1422–1449, 1989.

Thomas, P. and Veverka, J., Neptune's small inner satellites. *J. geophys. Res.* **96**, 19,261–19,268, 1991.

Thomas, P., Weitz, C. and Veverka, J., Small satellites of Uranus: disk-integrated photometry and estimated radii. *Icarus* **81**, 92–101, 1989.