

Stat 104: Quantitative Methods for Economics

Homework 3: Due Friday, September 26

Homework policy: Homework must be submitted into the homework submission cabinet next to Science Center Room 300 by 4:00 pm on the due date. Late homework will not be accepted. You are encouraged to discuss homework problems with other students (and with the instructor and TFs, of course), but you must write your final answer in your own words. Solutions prepared “in committee” or by copying someone else’s paper are not acceptable.

- 1) A department store manager has monitored the numbers of complaints received per week about poor service. The probabilities for numbers of complaints in a week, established by this review, are shown in the table. Let A be the event "There will be at least one complaint in a week," and B the event "There will be less than 10 complaints in a week."

NUMBER OF COMPLAINTS	0	1-3	4-6	7-9	10-12	More than 12
PROBABILITY	.14	.39	.23	.15	.06	.03

- Find the probability of A .
- Find the probability of B .
- Find the probability of the complement of A .
- Find the probability of A or B .
- Find the probability of A and B .
- Are A and B mutually exclusive?
- Are A and B collectively exhaustive?

- 2) When 2 dice are rolled, find the probability of the following situations (the sample space for this is shown below and might help you answer the question).

		FIRST DIE					
		1	2	3	4	5	6
SECOND DIE	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- a) A sum of 5 or 6
 - b) A sum greater than 9
 - c) A sum less than 4 or greater than 9
 - d) A sum that is divisible by 4
 - e) A sum of 14
 - f) A sum less than 13
- 3) A silver dollar is flipped twice. Calculate the probability of each of the following occurring:
- a) a head on the first flip
 - b) a tail on the second flip given that the first toss was a head
 - c) two tails
 - d) a tail on the first and a head on the second
 - e) a tail on the first and a head on the second or a head on the first and a tail on the second
 - f) at least one head on the two flips

- 4) A local postal carrier distributes first class letters, advertisements, and magazines. For a certain day, she distributed the following numbers of each type of item.

Delivered to	First-class letters	Ads	Magazines
Home	325	406	203
Business	732	1021	97

If an item of mail is selected at random, find these probabilities.

- The item went to a home.
 - The item was an ad, or it went to a business.
 - The item was a first-class letter, or it went to a home.
- 5) Two T-shirt printing factories produce T-shirts for a local sports team. Factory A produces 60% of the shirts and factory B produces 40%. Five percent of the shirts from factory A are defective, and 6% of the shirts from factory B are defective. Choose 1 shirt at random. Given that the shirt is defective, what is the probability that it came from factory A?
- 6) A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Find these probabilities.

- The respondent answered yes, given that the respondent was a female.
 - The respondent was a male, given that the respondent answered no.
- 7) Read the pdf document on the website entitled Birthday Problems. Then answer the following question (question 3 on page 199 of the document):

A small class contains 6 students. What is the chance that at least two have the same *birthmonth*?

- 8) Records of a midwestern university show that in one semester, 42% of students received an "A" in math, 34% received an "A" in chemistry, and 15% received an "A" in both.
- If a student received an "A" in chemistry, what is the probability that he or she received an "A" in math as well?
 - What is the probability that a randomly selected student received an "A" in either math or chemistry (or both)?
 - Are the events "a student received an 'A' in math" and "a student received an 'A' in chemistry" independent events? Explain.
- 9) At a large factory, the employees were surveyed and classified according to their level of education and whether they attend a sports event at least once a month. The data are shown in the following table.

Sports event	Educational level		
	High school graduate	Two-year college degree	Four-year college degree
Attend	16	20	24
Do not attend	12	19	25

If an employee is selected at random, find the probability that

- The employee attends sports events regularly, given that he or she graduated from college (2- or 4-year degree)
 - Given that the employee is a high school graduate, he or she does not attend sports events regularly
 - Are "sports event" and "educational level" independent or dependent events?
- 10) A traveling salesman routinely takes a morning flight from New York to Washington, DC. The probability he flies on Continental is 0.35, on Delta it is 0.40, and on United 0.25. The salesman prefers an aisle seat, but cannot always get one. Ten percent of the time he is able to sit on the aisle while flying Continental, 12% of the time with Delta, and 25% of the time with United. Suppose the salesman travels from New York to Washington, DC.
- Find the probability the salesman flies on United and gets an aisle seat.
 - Find the probability the salesman gets an aisle seat.
 - Suppose the salesman got an aisle seat, what is the probability he is flying on United?

- 11) The Campus Bookstore at the University of Northern South Dakota must decide how many of Professor Mankiw's economics textbooks to order for the next semester's class. The bookstore believes that seven, eight, nine or ten sections of the course will be offered next semester; each section contains 40 students. The publisher is offering bookstores a discount if they place their orders early. If the bookstore orders too few texts and runs out, the publisher will air express additional books at the bookstore's expense. If it orders too many texts, the store can return unsold texts to the publisher for a partial credit. The bookstore is considering ordering either, 280, 320, 360 or 400 texts in order to get the discount. Taking into account the discounts, air express expenses, and credits for returned texts, the bookstore manager estimates the following resulting profits.

Decision Alternatives	# of Textbooks to Order	The Number of Introductory Economics Classes Offered				States Of Nature
		7	8	9	10	
	280	2800	2720	2640	2480	
	320	2600	3200	3040	2880	
	360	2400	3000	3600	3440	
	400	2200	2800	3400	4000	

- a) What is the maximax decision ?
b) What is the minimax decision?
- 12) Consider the data given in problem 11 for the Campus Bookstore. Based on conversations with the chair of the economics department, suppose the bookstore manager believes that the following probabilities hold:

- $P(7 \text{ classes offered}) = .10$
- $P(8 \text{ classes offered}) = .30$
- $P(9 \text{ classes offered}) = .40$
- $P(10 \text{ classes offered}) = .20$

Using the expected value criterion, determine how many economics books the bookstore manager should purchase in order to maximize the store's expected profit.

- 13) The manager of Richard's Café, Tony, is to decide how many pounds of garden salad should be ordered each day. According to the past records, daily demand of garden salad is between 10 and 14 pounds, with probabilities 0.1 for 10 lb, 0.2 for 11 lb, 0.4 for 12 lb, 0.1 for 13 lb, and 0.2 for 14 lb. A pound of garden salad costs \$3, and is sold at \$7. The leftover salad at the end of a day will be discarded. Stock-out, on the other hand, damages the café's reputation and weakens its competitiveness. Tony estimates that the penalty of stock-out is \$2 for each pound that is short. Fill in the following table and determine the amount Tony should order to maximize his expected net profit.

[I've given you two values so you can check your work. If Tony orders 10 pounds and the demand is 10 pounds his net profit is \$40. If Tony orders 14 pounds and demand is 13 pounds, his net profit is \$49.]

		State of Nature: Daily demand (pounds)					EMV
		<i>0.1</i>	<i>0.2</i>	<i>0.4</i>	<i>0.1</i>	<i>0.2</i>	
		10	11	12	13	14	
Decision Alternatives	10	40					
	11						
	12						
	13						
	14				49		

14) Ronald Lau, chief engineer at South Dakota Electronics, has to decide whether to build a new state-of-the-art processing facility. If the new facility works, the company could realize a profit of \$200,000. If it fails, South Dakota Electronics could lose \$180,000. At this time, Lau estimates a 60% chance that the new process will fail.

The other option is to build a pilot plant and then decide whether to build a complete facility. The pilot plant would cost \$10,000 to build. Lau estimates a 50-50 chance that the pilot plant will work. If the pilot plant works, there is a 90% probability that the complete plant, if it is built, will also work. If the pilot plant does not work, there is only a 20% chance that the complete project (if it is constructed) will work. Lau faces a dilemma. Should he build the plant? Should he build the pilot project and then make a decision? Help Lau by analyzing this problem and finding the optimal strategy.

[I have outlined the start of the decision tree below.]

