

Boolean Networks in Life Sciences

Exercise Sheet 7: Model Checking

Friday 19th December, 2025

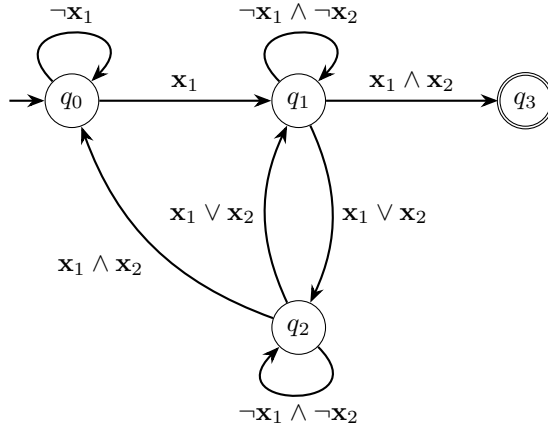
Exercise 1 The following LTL formulae define safety properties L_{safe}^1 and L_{safe}^2 , construct NFAs accepting the languages $\text{MinBad}(L_{safe}^1)$ and $\text{MinBad}(L_{safe}^2)$.

1. $\mathbf{x}_1 \Rightarrow \mathbf{X}(\neg \mathbf{x}_1 \mathbf{U} \mathbf{x}_2)$;
2. $\mathbf{G}(\varphi) \vee (\varphi \mathbf{U} \mathbf{x}_3)$ where
 $\varphi = (\mathbf{x}_1 \wedge \neg \mathbf{x}_2 \wedge \mathbf{X}(\neg \mathbf{x}_1 \wedge \mathbf{x}_2)) \vee (\neg \mathbf{x}_1 \wedge \mathbf{x}_2 \wedge \mathbf{X}(\mathbf{x}_1 \wedge \neg \mathbf{x}_2))$;

Exercise 2 Consider the transition system of the following Boolean network of dimension 2 given by fully asynchronous semantics, extended to a Kripke structure \mathcal{T} with $I = \{00\}$ and atomic propositions consisting of propositional formulae on the variables \mathbf{x}_1 and \mathbf{x}_2 with the obvious interpretation.

$$f_1(\mathbf{x}) = \mathbf{x}_2; f_2(\mathbf{x}) = \neg \mathbf{x}_1 \vee \neg \mathbf{x}_2$$

Consider further the following NFA \mathcal{A} and construct the Kripke structure $\mathcal{T} \otimes \mathcal{A}$.



Exercise 3 The following LTL formula defines an ω -regular property, construct a Büchi automaton accepting the property.

$$(\neg \mathbf{x}_1 \mathbf{U} (\mathbf{G}(\mathbf{x}_2))) \vee ((\mathbf{x}_2 \vee (\mathbf{x}_1 \wedge \mathbf{X}(\neg \mathbf{x}_1))) \mathbf{U} (\mathbf{G}(\mathbf{x}_2)))$$

Exercise 4 Consider the following Büchi automata and characterise the properties they accept, try to find LTL formulae that define these properties.

