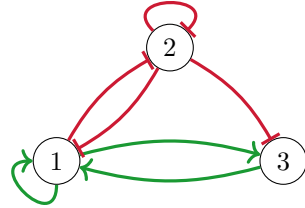


Boolean Networks in Life Sciences

Exercise Sheet 6: Model Verification

Friday 12th December, 2025

Exercise 1 Consider the following Boolean network of dimension 3.



$$f_1(\mathbf{x}) = \neg \mathbf{x}_2 \vee (\mathbf{x}_1 \wedge \mathbf{x}_3)$$

$$f_2(\mathbf{x}) = \neg \mathbf{x}_1 \vee \neg \mathbf{x}_2$$

$$f_3(\mathbf{x}) = \mathbf{x}_1 \vee \neg \mathbf{x}_2$$

Characterise the set $\mathcal{S}(000)$ using the maximal traces.

Exercise 2 Sketch the execution tree rooted in 000 for the Boolean network from Exercise 1.

Exercise 3 Determine the validity of the following LTL formulae in all configurations of the Boolean network from Exercise 1:

1. $\mathbf{X}(\mathbf{x}_3)$;
2. $\mathbf{XXX}(\mathbf{x}_3)$;
3. $\mathbf{G}(\mathbf{F}(\mathbf{x}_2))$;
4. $\mathbf{G}(\mathbf{x}_1 \mathbf{U} \mathbf{x}_2)$;
5. $\mathbf{F}(\mathbf{x}_1 \mathbf{U} \mathbf{x}_2)$;

Exercise 4 Determine the validity of the following CTL formulae in all configurations of the Boolean network from Exercise 1:

1. $\exists \mathbf{X}(\mathbf{x}_3)$;
2. $\forall \mathbf{X} \exists \mathbf{X} \forall \mathbf{X}(\mathbf{x}_3)$;

3. $\exists \mathbf{x}_1 \mathbf{U} \mathbf{x}_3;$
4. $\forall \mathbf{x}_1 \mathbf{U} \mathbf{x}_3;$
5. $\forall \mathbf{x}_1 \mathbf{U} (\exists \mathbf{x}_3 \mathbf{U} \mathbf{x}_2);$

Exercise 5 Assume a Boolean network of dimension at least 3. Write LTL formulae that best capture the following properties:

1. Eventually, either 1 activates or 2 stays active forever;
2. It remains possible to activate 3 at least until simultaneous activation of both 1 and 2;
3. 1 cannot activate until any activation of 2 causes immediate deactivation of 3;

Exercise 6 Assume a Boolean network of dimension at least 2. Write CTL formulae that best capture the following properties:

1. There exists a successor from which 1 is always active until it becomes possible to immediately activate 2;
2. It is always possible to eventually ensure that 1 will always be active infinitely often;
3. Exactly one of 1 and 2 is active across all successors, and there is always a possibility for the active one to stay active forever;