Boolean Networks in Life Sciences
Exercise Sheet 7: Model Checking

Friday 9th January, 2026

Exercise 1 *The following LTL formulae define safety properties L^1_{safe} and L^2_{safe}, construct NFAs accepting the languages $\mathtt{MinBad}(L^1_{safe})$ and $\mathtt{MinBad}(L^2_{safe})$.*

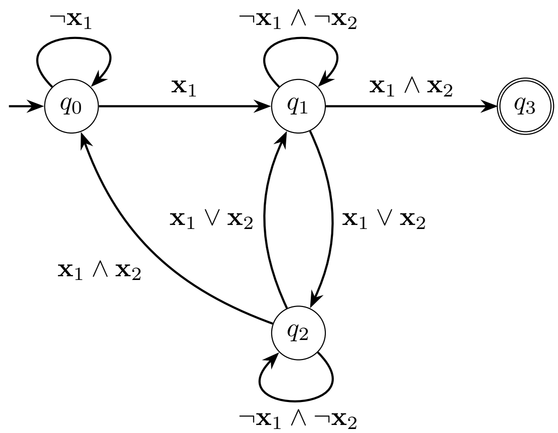
1. $\mathbf{x}_1 \Rightarrow \mathbf{X}(\neg\mathbf{x}_1 \,\mathbf{U}\, \mathbf{x}_2)$;

2. $\mathbf{G}(\varphi) \vee (\varphi \,\mathbf{U}\, \mathbf{x}_3)$ *where*
   $\varphi = (\mathbf{x}_1 \wedge \neg\mathbf{x}_2 \wedge \mathbf{X}(\neg\mathbf{x}_1 \wedge \mathbf{x}_2)) \vee (\neg\mathbf{x}_1 \wedge \mathbf{x}_2 \wedge \mathbf{X}(\mathbf{x}_1 \wedge \neg\mathbf{x}_2))$;

Exercise 2 *Consider the transition system of the following Boolean network of dimension 2 given by fully asynchronous semantics, extended to a Kripke structure $\mathcal{T}$ with $I = \{00\}$ and atomic propositions consisting of propositional formulae on the variables $\mathbf{x}_1$ and $\mathbf{x}_2$ with the obvious interpretation.*

$$f_1(\mathbf{x}) = \mathbf{x}_2; f_2(\mathbf{x}) = \neg\mathbf{x}_1 \vee \neg\mathbf{x}_2$$

Consider further the following NFA $\mathcal{A}$ and construct the Kripke structure $\mathcal{T} \otimes \mathcal{A}$.

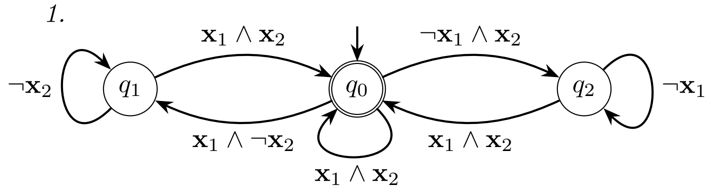


Exercise 3 *The following LTL formula defines an ω-regular property, construct a Büchi automaton accepting the property.*

$$(\neg\mathbf{x}_1 \,\mathbf{U}\, (\mathbf{G}(\mathbf{x}_2))) \vee ((\mathbf{x}_2 \vee (\mathbf{x}_1 \wedge \mathbf{X}(\neg\mathbf{x}_1))) \,\mathbf{U}\, (\mathbf{G}(\mathbf{x}_2)))$$

Exercise 4 *Consider the following Büchi automata and characterise the properties they accept, try to find LTL formulae that define these properties.*

1.



2.