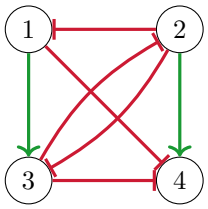
Boolean Networks in Life Sciences
Exercise Sheet 5: Most Permissive Semantics

Friday 28th November, 2025

Exercise 1 *Consider the following Boolean network of dimension* 4.

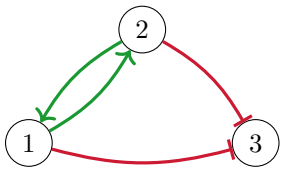


$$f_1(\mathbf{x}) = \neg\mathbf{x}_2$$
$$f_2(\mathbf{x}) = \neg\mathbf{x}_3$$
$$f_3(\mathbf{x}) = \mathbf{x}_1 \wedge \neg\mathbf{x}_2$$
$$f_4(\mathbf{x}) = \neg\mathbf{x}_1 \wedge \mathbf{x}_2 \wedge \neg\mathbf{x}_3$$

Find all configurations reachable from 0000 *under the generalised asynchronous semantics, and show that the configuration* 1111 *is reachable from* 0000 *with most permissive semantics by constructing a trace in* $\xrightarrow{mp}$.

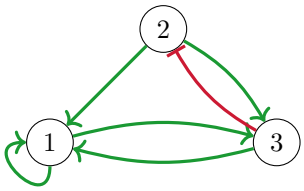
Exercise 2 *Let* $\mathcal{R}_s(\mathbf{x}) = \left\{\mathbf{y} \in \mathbb{B}^n \mid \mathbf{x} \xrightarrow{s} \mathbf{y}\right\}$ *be the set of all configurations reachable from a configuration* $\mathbf{x} \in \mathbb{B}^n$ *under the semantics* $s \in \{sync, async, gen, mp\}$.

Consider the following Boolean network of dimension 3, *and compare the set* $\mathcal{R}_s(\mathbf{x})$ *for each configuration* $\mathbf{x} \in \mathbb{B}^3$ *and all the different semantics.*



$$f_1(\mathbf{x}) = \mathbf{x}_2$$
$$f_2(\mathbf{x}) = \mathbf{x}_1$$
$$f_3(\mathbf{x}) = \neg\mathbf{x}_1 \wedge \neg\mathbf{x}_2$$

Exercise 3 *Consider the following Boolean network of dimension* 3.



$$f_1(\mathbf{x}) = \mathbf{x}_1 \wedge (\mathbf{x}_2 \vee \mathbf{x}_3)$$
$$f_2(\mathbf{x}) = \neg\mathbf{x}_3$$
$$f_3(\mathbf{x}) = \mathbf{x}_1 \vee \mathbf{x}_2$$

Find all the trap spaces of f and identify the minimal ones, which correspond to the attractors of the most permissive semantics.