

# Boolean Networks in Life Sciences

## Exercise Sheet 3: Transition Systems

Friday 14<sup>th</sup> November, 2025

**Exercise 1** Construct the transition system of the following Boolean networks, for all of synchronous, fully asynchronous and generalised asynchronous semantics.

1.  $f_1(\mathbf{x}) = f_2(\mathbf{x}) = \mathbf{x}_1 \Leftrightarrow \mathbf{x}_2;$
2.  $f_1(\mathbf{x}) = \neg \mathbf{x}_3, f_2(\mathbf{x}) = \neg \mathbf{x}_1, f_3(\mathbf{x}) = \neg \mathbf{x}_2;$
3.  $f_1(\mathbf{x}) = (\mathbf{x}_1 \Leftrightarrow \mathbf{x}_2) \vee (\mathbf{x}_2 \oplus \mathbf{x}_3), f_2(\mathbf{x}) = (\neg \mathbf{x}_1 \wedge \mathbf{x}_2) \vee (\mathbf{x}_1 \oplus \mathbf{x}_3), f_3(\mathbf{x}) = \neg \mathbf{x}_2 \vee (\mathbf{x}_1 \Leftrightarrow \mathbf{x}_3);$

**Exercise 2** Let  $\mathcal{R}_s(\mathbf{x}) = \left\{ \mathbf{y} \in \mathbb{B}^n \mid \mathbf{x} \xrightarrow{s}^* \mathbf{y} \right\}$  be the set of all configurations reachable from a configuration  $\mathbf{x} \in \mathbb{B}^n$  under the semantics  $s \in \{\text{sync, async, gen}\}$ .

For all Boolean networks from Exercise 1, and for each configuration  $\mathbf{x} \in \mathbb{B}^2$ , resp.  $\mathbf{x} \in \mathbb{B}^3$ , compare the sets  $\mathcal{R}_s(\mathbf{x})$  across all of synchronous, fully asynchronous and generalised asynchronous semantics.

**Exercise 3** Find all attractors of the Boolean networks from Exercise 1 for all of synchronous, fully asynchronous and generalised asynchronous semantics. Compare the discovered attractors against minimal trap spaces.

**Exercise 4** Characterise the weak and strong basins of attraction of all attractors discovered in Exercise 3.