Mergesort

Merge Sort

- Merge sort is a comparison sorting technique.
- This technique follows the divide-and-conquer approach.
- It maintains the following 3 steps:

  1. Divide: Divide N-element sequence to be sorted into two subsequences of about N/2 elements each and sort the two subsequences recursively .

  2. Conquer: Merge the two sorted subsequences to produce the sorted result.

- Merge sort uses the “merge” step to combine two sorted sublists to create one single sorted list.
- Suppose A is a sorted list with R elements and B is another sorted list with S elements. After merging there is only a single sorted list C with N=R+S elements.

Mergesort Algorithm

Mergesort(List, N)

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(1) Msort (List, TempList, 0, N-1)
(2) End
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Msort(List, TempList, Left, Right)

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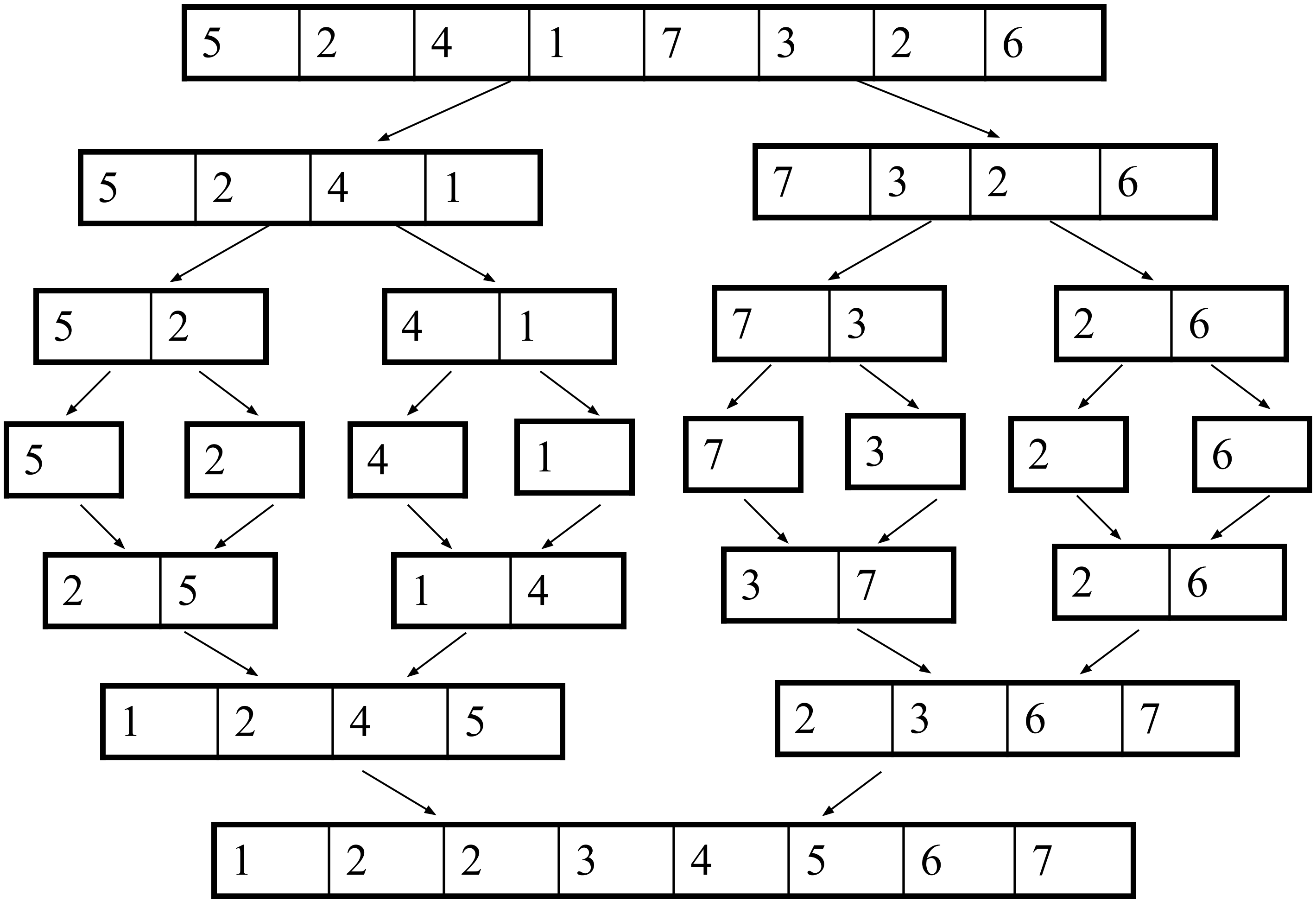
(1) If Left < Right do steps 2 to 5
(2)     Set Center = (Left+Right)/2
(3)     Msort (List, Temp List,Left,Center)
(4)     Msort (List,TempList,Center+1,Right)
(5)     Merge(List,TempList,Left,Center+1,Right)
(6)     End
```

Merge(List,TempList,Lpos,Rpos,RightEnd)

```
(1) Set LeftEnd = Rpos-1 and TmpPos = Lpos
(2) NumElement = RightEnd – Lpos + 1
(3) While Lpos<=LeftEnd && Rpos<=RightEnd
(4)      If List [Lpos] <= List [Rpos] then
(5)          TmpList[TmpPos++]=List [Lpos++]
(6)      Else
(7)          TmpList[TmpPos++]=List [Rpos++]
(8)  While Lpos <= LeftEnd
(7)      TmpList[TmpPos++] = List [Lpos++]
(10) While Rpos <= RightEnd
(11)         TmpList[TmpPos++] = List [Rpos++]
(12)     For I = 0 to NumElement do step 13
(13)         List [RightEnd--]=TmpList [RightEnd--]
(14)     End
```

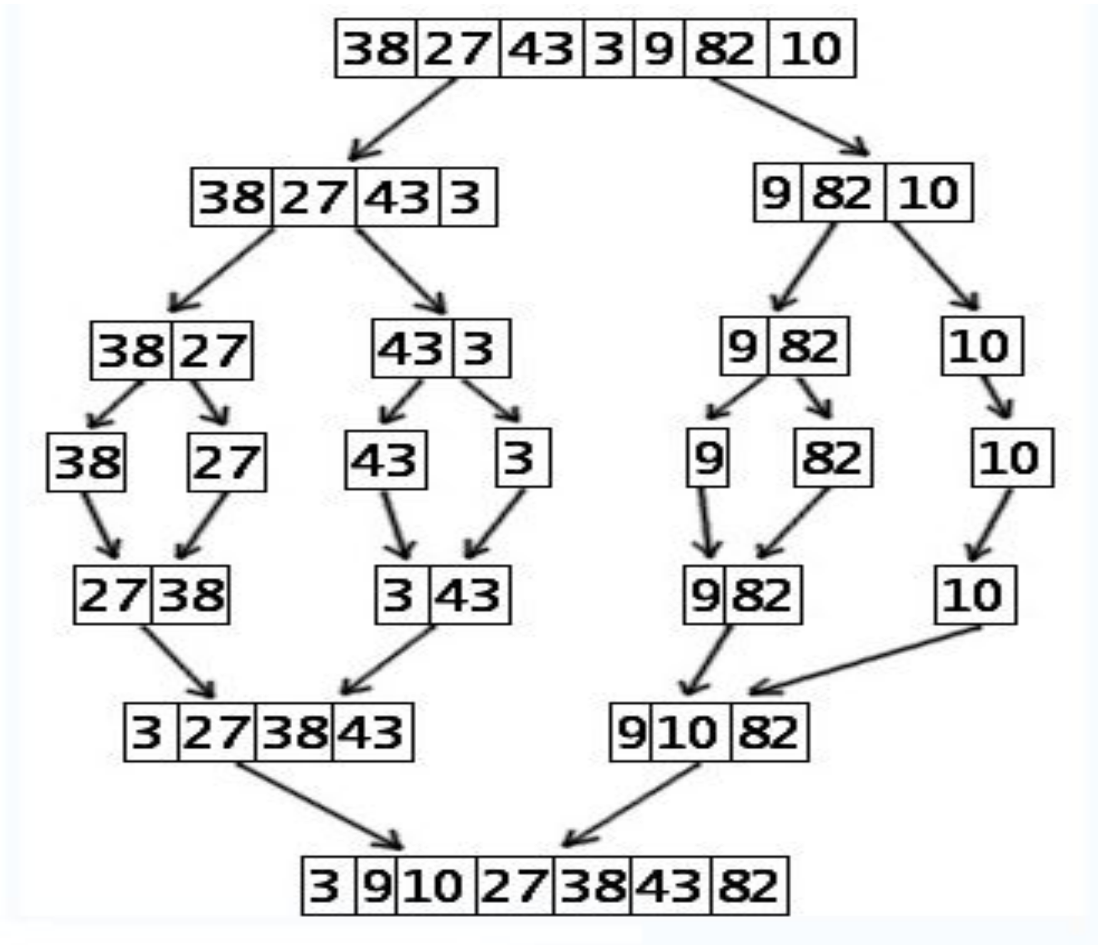
Example:

Suppose Array A = 5, 2, 4, 7, 1, 3, 2, 6. Sort the array using Mergesort.



Example:

Suppose Array A = 38, 27, 43, 3, 9, 82 and 10. Sort the array using Mergesort.



Complexity of Merge-Sort

Let T(N) be the number of comparisons needed to sort N elements using merge sort. This algorithm requires at most logN passes. Moreover, each pass merges a total of N elements and each pass will require at most N comparisons. So, for all cases, T(N) = 0(NlogN).

Recurrence Relation for Mergesort

$T(1) = 1$ For $N = 1$

$T(N) = 2T(N/2) + N$ Otherwise

$T(N) = 2T(N/2) + N$ is equivalent to $O(N\log_2 N)$

Solution:

$$T(N) = 2T(N/2) + N$$

$$T(N/2) = 2T(N/4) + N/2$$

$$T(N/4) = 2T(N/8) + N/4$$

……………………..

$$T(2) = 2T(1) + 2$$

$$T(N) = 2^K T(N/2^K) + K.N$$

By using $2^K = N$, it is obtained as

$$T(N) = NT(1) + NLog_2 N = N + NLog_2 N = O(NLog_2 N)$$

So, $T(N) = 2T(N/2) + N$ is equivalent to $O(N\log_2 N)$.

END